Advanced Quantitative Research Methodology, Lecture Notes: Single Equation Models[1]

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Binary Variable Regression Models

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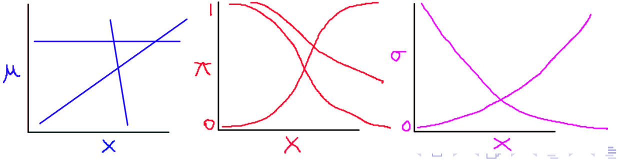
(The graph in the middle:)



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What do we do with this?

Interpreting Functional Forms

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   (d) Average effects: compute effects for every observation and average

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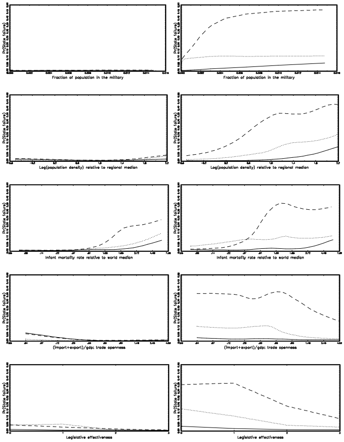

Example Marginal Effect Plot of a neural network model (about which more later). Full democracies (dotted), partial democracies (dashed), and autocracies (solid). From Gary King and Langche Zeng. "Improving Forecasts of State Failure," *World Politics*, Vol. 53, No. 4 (July, 2001): 623-58.

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For any quantity but a probability, always also include a measure of uncertainty (standard error, confidence interval, etc.)

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Variable	From		To	First Difference
Sex	Male	$\rightarrow$	Female	.05
Age	65	$\rightarrow$	75	$-.10$
Home	NYC	$\rightarrow$	Madison, WI	.26
Income	\$35,000	$\rightarrow$	\$75,000	.14

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   (a) Max value of logit derivative: $\hat{\beta} \times 0.5(1 - 0.5) = \hat{\beta}/4$

   (b) Max value for probit $[\pi_i = \Phi(X_i\beta)]$ derivative: $\hat{\beta} \times 0.4$

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Since Y^* is unobserved anyway, define the threshold as $\tau = 0$. (Plus the same independence assumption, which from now on is assumed implicit.)

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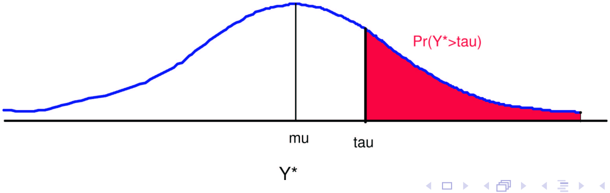

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5. $\implies$ interpret β as regression coefficients of Y^* on X: $\hat{\beta}_1$ is what happens to Y^* on average (or μ_i) when X_1 goes up by one unit, holding constant the other explanatory variables (and conditional on the model). In probit, one unit of Y^* is one standard deviation.

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- Which would enable you to choose logit over probit?

How Not to Present Statistical Results

TABLE 1
Predicting Which Ethnic Group Conquered Most of Bosnia

Attention to Bosnia crisis	.609**
Age	.007**
Education	.289**
Family income	.151**
Race (non-White/White)	.695**
Gender (female/male)	.789**
Region (South/non-South)	.076
Network coverage	.000
Education × Time	–.003*
Time in months	.078**
Constant	–9.257**
Number	7,021
–2 log-likelihood	7,215.231
Goodness of fit	6,789.45
Cox & Snell R^2	.212
Nagelkerke R^2	.295
Overall correct classification (%)	73.96

SOURCE: *Times Mirror* polls from September 1992, January 1993, September 1993, January 1994, and June 1995.
NOTE: Unstandardized coefficients for logistic regression. Dependent variable is knowledge of which group conquered most of Bosnia.
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1. This one is typical of current practice, not that unusual.

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How Not to Present Statistical Results

TABLE 1

Predicting Which Ethnic Group Conquered Most of Bosnia

Attention to Bosnia crisis	.609**
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Education	.289**
Family income	.151**
Race (non-White/White)	.695**
Gender (female/male)	.789**
Region (South/non-South)	.076
Network coverage	.000
Education × Time	–.003*
Time in months	.078**
Constant	–9.257**
Number	7,021
–2 log-likelihood	7,215.231
Goodness of fit	6,789.45
Cox & Snell R^2	.212
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Overall correct classification (%)	73.96

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4. Reading assignment: (at my web page) the handout on papers, and King, Tomz, Wittenberg, "Making the Most of Statistical Analyses: Improving Interpretation and Presentation" American Journal of Political Science, Vol. 44, No. 2 (March, 2000): 341-355.

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1. *Estimation uncertainty*: Lack of knowledge of β and α. Vanishes as n gets larger.
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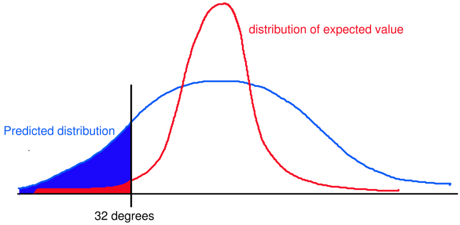
Repeat algorithm say $M = 1000$ times, to produce 1000 predicted values. Use these to compute a histogram for the full posterior, the average, variance, percentile values, or others.

How the Distribution of Expected and Predicted Values Differ



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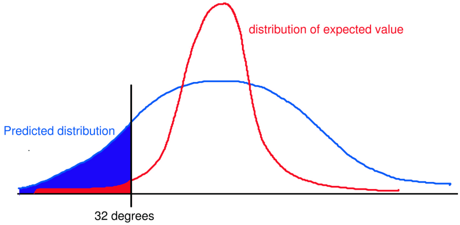

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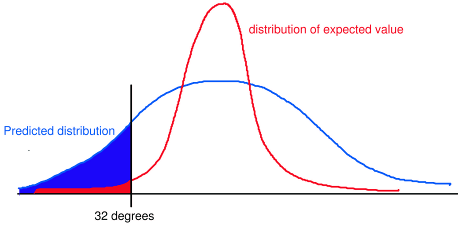

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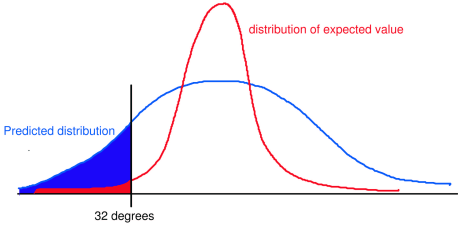

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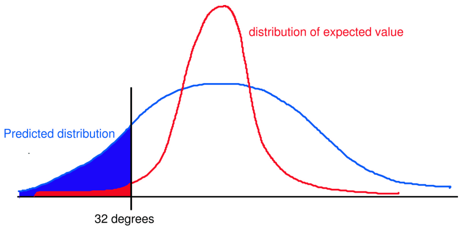

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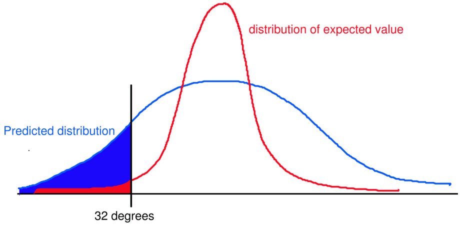
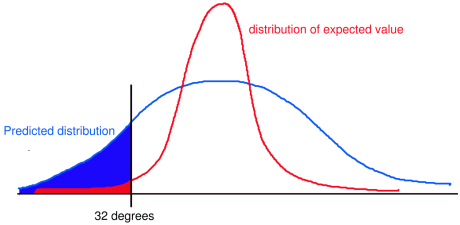

distribution of expected value
Predicted distribution
32 degrees



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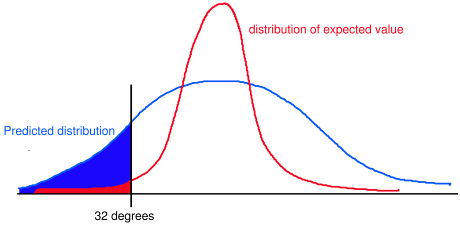

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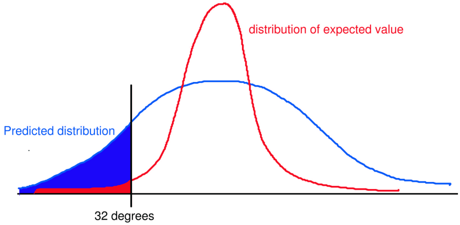

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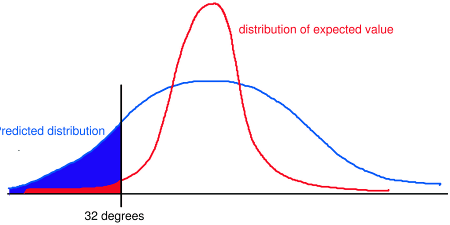

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5. Average over the fundamental uncertainty by calculating the mean of the m simulations to yield one simulated expected value $\tilde{E}(Y_c) = \sum_{k=1}^{m} \tilde{Y}_c^{(k)}/m$.

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5. When $E(Y_c) = \theta_c$, we can skip the last two steps. E.g., in the logit model, once we simulate π_i, we don't need to draw Y and then average to get back to π_i. (If you're unsure, do it anyway!)

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4. (To save computation time, and improve approximation, use the same simulated β in each.)

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   In all 3 cases, η is unbounded: estimate it, simulate from it, and reparameterize back to the scale you care about.

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5. Clarify does this all automatically in Stata. Zelig does the same and more in R.

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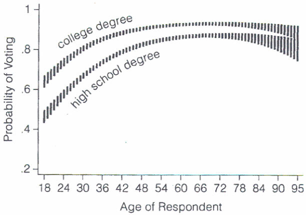
3. Use $M = 1000$ and compute 99% CI:

Figure 1 Probability of Voting by Age



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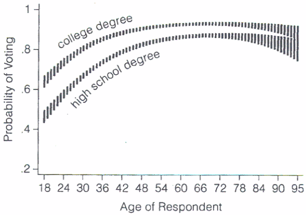

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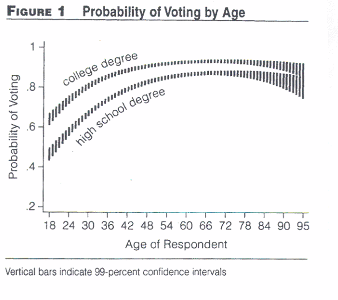

To create this graph, simulate:

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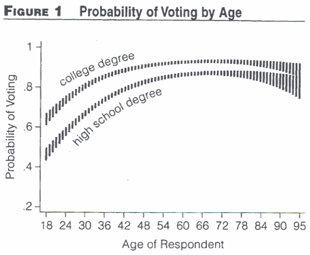

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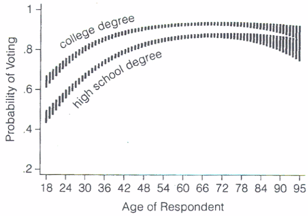

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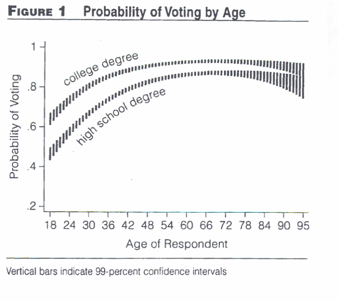

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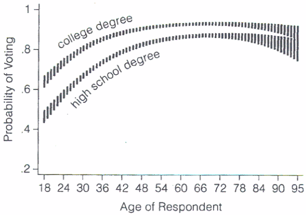

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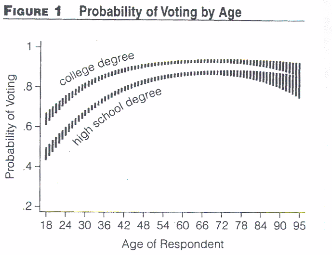

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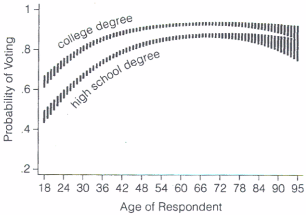

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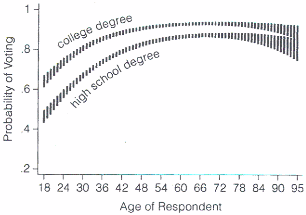

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8. Repeat for other ages and for college degree.

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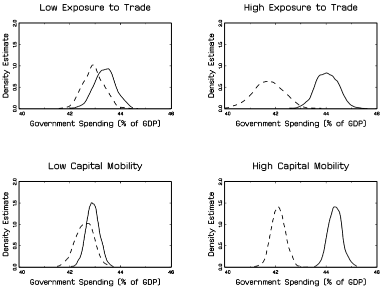

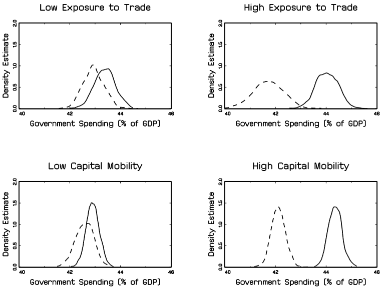

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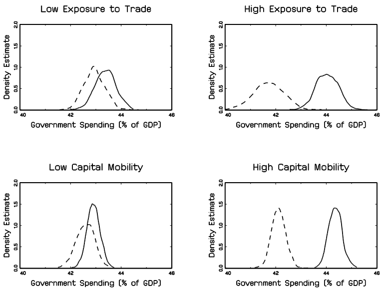

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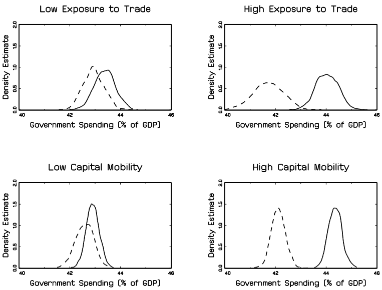

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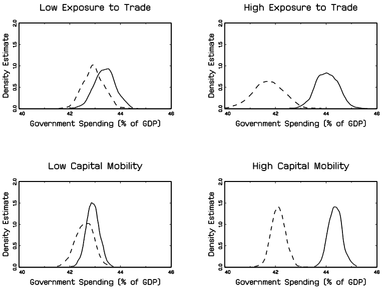

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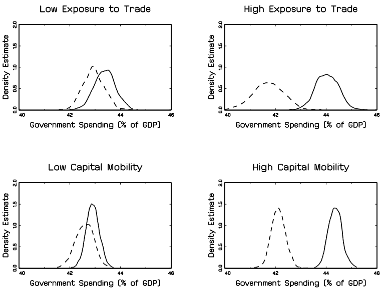

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- How could we summarize this information with less real estate?

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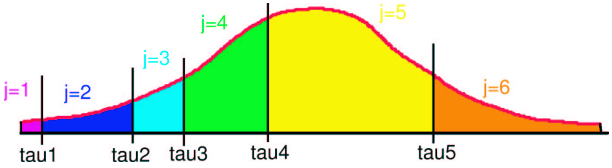

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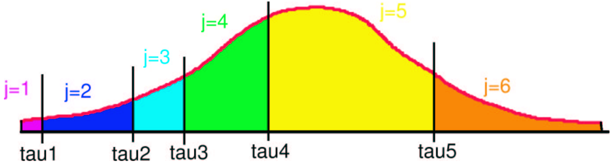

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Bracketed portion has only one factor for each i.

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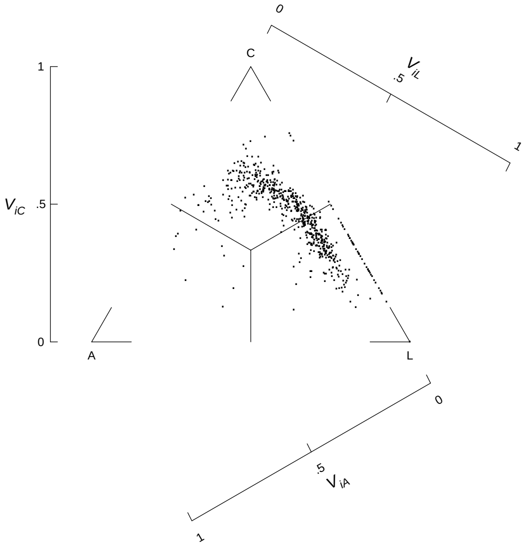
5. Can use ternary diagrams if $J = 3$

Representing 3 variables, with $Y_j \in [0,1]$ and $\sum_{j=1}^{3} Y_j = 1$



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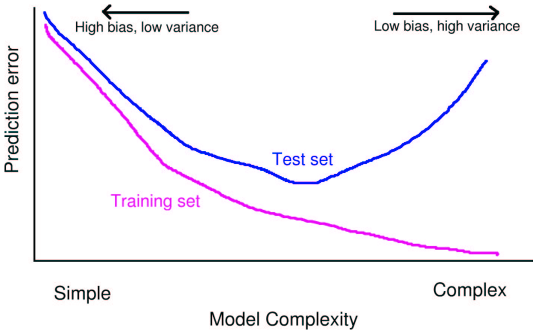
   (h) If the world changes, an otherwise good model will fail. But it's still the right test.



(See Trevor Hastie et al. 2001. *The Elements of Statistical Learning*, Springer, Chapter 7: Fig 7.1.)

(i) Binary variable predictions require a normative decision.
- Let C be number of times more costly misclassifying a 1 is than a 0.
- C must be chosen independently of the data.
- C could come from your philosophical justification, survey of policy makers, a review of the literature, etc.
- People often choose $C = 1$, but without justification.
- Decision theory: choose $Y = 1$ when $\hat{\pi} > 1/(1 + C)$ and 0 otherwise.
  - If $C = 1$, predict $y = 1$ when $\hat{\pi} > 0.5$
- Only with C chosen can we compute (a) % of 1s correctly predicted and (b) % of 0s correctly predicted, and (c) patterns in errors in different subsets of the data or forecasts.

(j) If you can't justify a choice for C, use ROC (receiver-operator characteristic) curves
- Compute %1s and %0s correctly predicted for every possible value of C
- Plot %1s by %0s
- Overlay curves for several models on the same graph.
- If one curve is above another the whole way, then that model *dominates* the other. It's better no matter your normative decision (about C)
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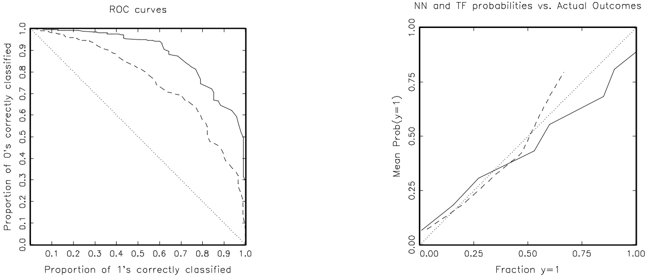
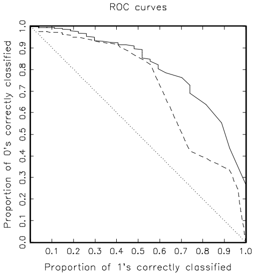
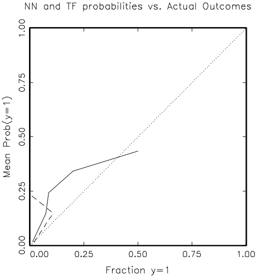


In sample ROC, on left (from Gary King and Langche Zeng. "Improving Forecasts of State Failure," World Politics, Vol. 53, No. 4 (July, 2001): 623-58)





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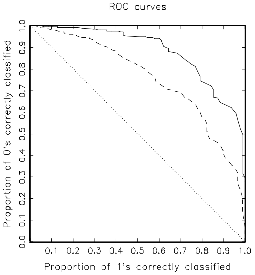

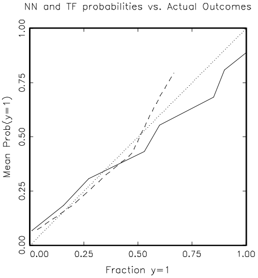

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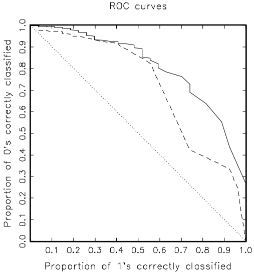

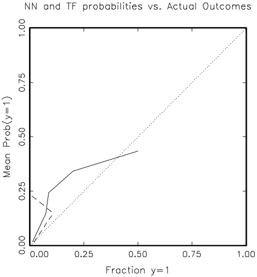

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Grouped Uncorrelated Binary Variables

Notes:

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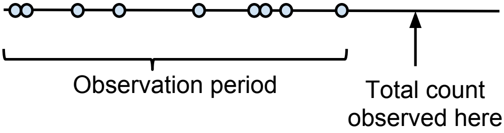

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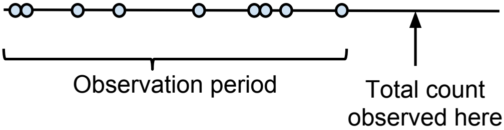

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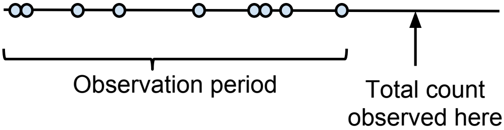

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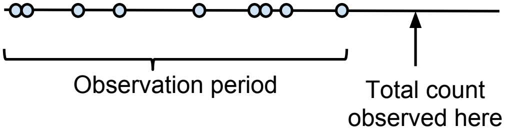

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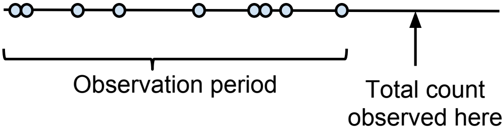

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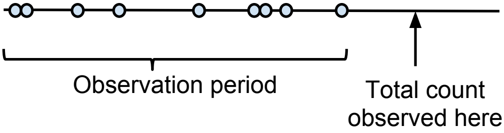

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   so we could use $\bar{y}\beta$ for an approximate linearized effect.

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   (d) Draw Y_c from Poisson$(y|\tilde{\lambda})$

3. Under the Poisson: $V(Y_i|X_i) = E(Y_i|X_i)$, which is heteroskedastic and fixed.
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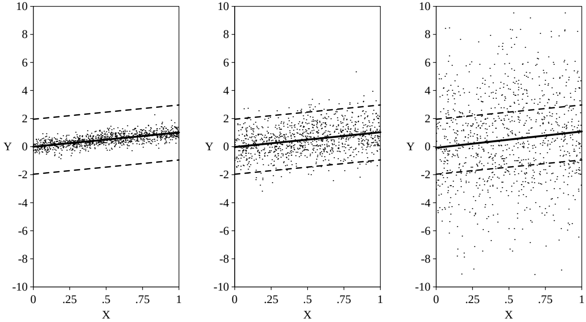

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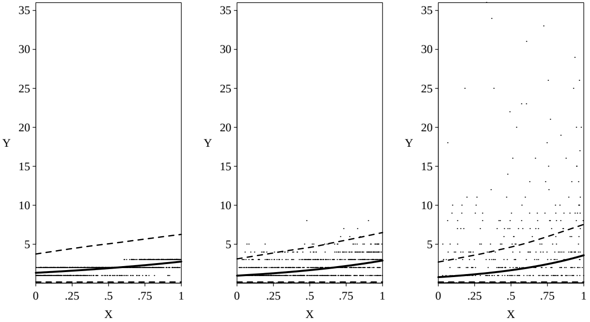

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Negative Binomial Event Count Model

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3. Test of Poisson vs NegBin: look at σ^2 (likelihood ratio doesn't work since Poisson doesn't exactly nest within Negbin)

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3. $\sigma^2 > 1$, and so we estimate γ, where $\sigma^2 = e^\gamma + 1$.

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$$x^{(m,\delta)} = \begin{cases} \prod_{i=0}^{m-1}(x+\delta i) = x(x+\delta)(x+2\delta)\cdots[x+\delta(m-1)] & m \geq 1 \\ 1 & m = 0 \end{cases}$$

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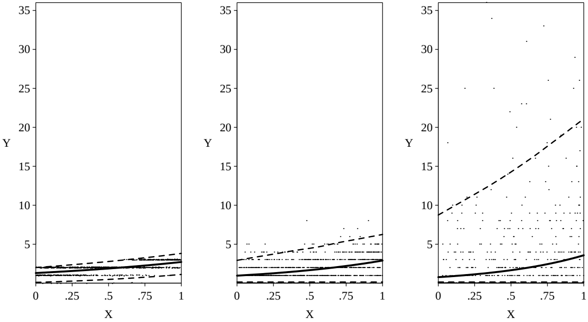
- Can simulate in three parts, indexed by σ^2

What happens with the extra parameter? GEC



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King, Gary and Curtis S. Signorino. "The Generalization in the Generalized Event Count Model" in *Political Analysis*, 6 (1996): 225-252.

King, Gary. "Variance Specification in Event Count Models: From Restrictive Assumptions to a Generalized Estimator," *American Journal of Political Science*, 33, 3 (August, 1989): 762-784.

Duration Models and Censoring: The Exponential Model

King, Gary; James Alt; Nancy Burns; and Michael Laver. "A Unified Model of Cabinet Dissolution in Parliamentary Democracies," American Journal of Political Science, Vol. 34, No. 3 (August, 1990): Pp. 846-871; Errata Vol. 34, No. 4 (November, 1990): P. 1168. (replication dataset: ICPSR s1115).

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5. We will model these observations explicitly as censored.

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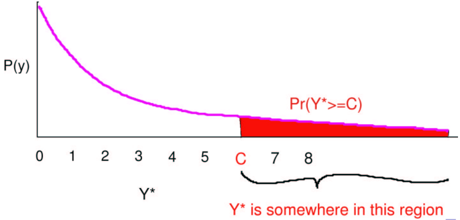

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