

Advanced Quantitative Research Methodology, Lecture

Notes: **Single Equation Models**¹

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Binary Variable Regression Models

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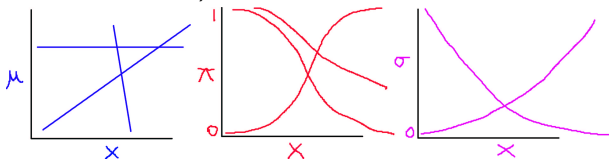
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(The graph in the middle:)



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What do we do with this?

Interpreting Functional Forms

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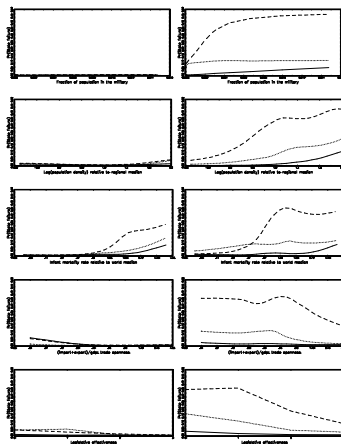
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- (d) Average effects: compute effects for every observation and average

Interpreting Functional Forms



Example Marginal Effect Plot of a neural network model (about which more later). Full democracies (dotted), partial democracies (dashed), and autocracies (solid). From Gary King and Langche Zeng. "Improving Forecasts of State Failure," *World Politics*, Vol. 53, No. 4 (July, 2001): 623-58.

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Sex	Age	Home	Income	Pr(vote)
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For any quantity but a probability, always also include a measure of uncertainty (standard error, confidence interval, etc.)

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Variable	From		To	First Difference
Sex	Male	→	Female	.05
Age	65	→	75	-.10
Home	NYC	→	Madison, WI	.26
Income	\$35,000	→	\$75,000	.14

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(b) Max value for probit $[\pi_i = \Phi(X_i\beta)]$ derivative: $\hat{\beta} \times 0.4$

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Since Y^* is unobserved anyway, define the threshold as $\tau = 0$. (Plus the same independence assumption, which from now on is assumed implicit.)

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then we get a logit model.

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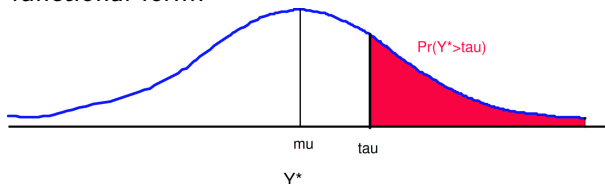
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5. $\implies$ interpret β as regression coefficients of Y^* on X : $\hat{\beta}_1$ is what happens to Y^* on average (or μ_i) when X_1 goes up by one unit, holding constant the other explanatory variables (and conditional on the model). In probit, one unit of Y^* is one standard deviation.

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- Which would enable you to choose logit over probit?

How Not to Present Statistical Results

TABLE 1
Predicting Which Ethnic Group Conquered Most of Bosnia

Attention to Bosnia crisis	.609**
Age	.007**
Education	.289**
Family income	.151**
Race (non-White/White)	.695**
Gender (female/male)	.789**
Region (South/non-South)	.076
Network coverage	.000
Education $\times$ Time	-.003*
Time in months	.078**
Constant	-9.257**
Number	7,021
-2 log-likelihood	7,215.231
Goodness of fit	6,789.45
Cox & Snell R^2	.212
Nagelkerke R^2	.295
Overall correct classification (%)	73.96

SOURCE: *Times Mirror* polls from September 1992, January 1993, September 1993, January 1994, and June 1995.

NOTE: Unstandardized coefficients for logistic regression. Dependent variable is knowledge of which group conquered most of Bosnia.

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Education $\times$ Time	-.003*
Time in months	.078**
Constant	-9.257**
Number	7,021
-2 log-likelihood	7,215.231
Goodness of fit	6,789.45
Cox & Snell R^2	.212
Nagelkerke R^2	.295
Overall correct classification (%)	73.96

SOURCE: *Times Mirror* polls from September 1992, January 1993, September 1993, January 1994, and June 1995.
NOTE: Unstandardized coefficients for logistic regression. Dependent variable is knowledge of which group conquered most of Bosnia.
* $p \leq .05$, two-tailed. ** $p \leq .01$, two-tailed.

1. This one is typical of current practice, not that unusual.
2. What do these numbers mean?
3. Why so much whitespace? Can you connect cols A and B?

How Not to Present Statistical Results

TABLE 1
Predicting Which Ethnic Group Conquered Most of Bosnia

Attention to Bosnia crisis	.609**
Age	.007**
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Family income	.151**
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6. Can you compute a quantity of interest from these numbers?

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4. Reading assignment: (at my web page) the handout on papers, and King, Tomz, Wittenberg, "Making the Most of Statistical Analyses: Improving Interpretation and Presentation" American Journal of Political Science, Vol. 44, No. 2 (March, 2000): 341-355.

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Denote the draw $\tilde{\gamma} = \text{vec}(\tilde{\beta}, \tilde{\alpha})$, which has k elements.

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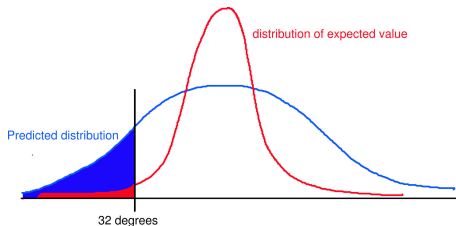
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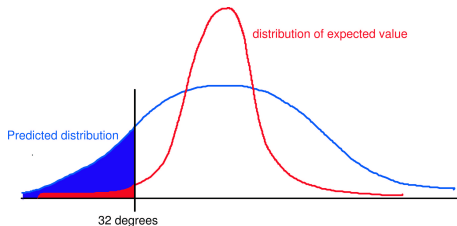
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Repeat algorithm say $M = 1000$ times, to produce 1000 predicted values. Use these to compute a histogram for the full posterior, the average, variance, percentile values, or others.

The Distribution of Expected v. Predicted Values

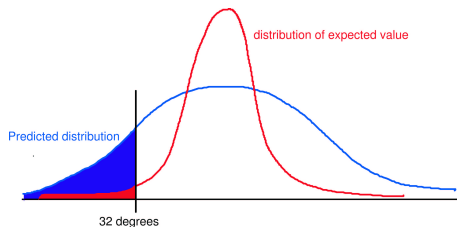


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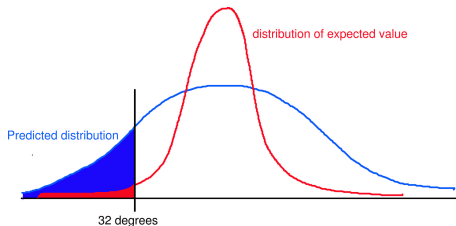
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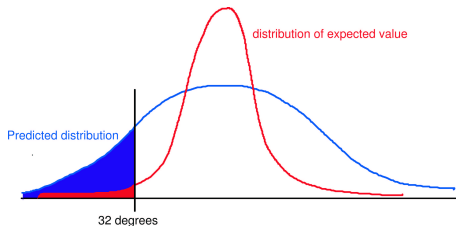
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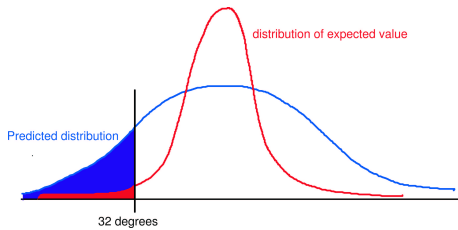


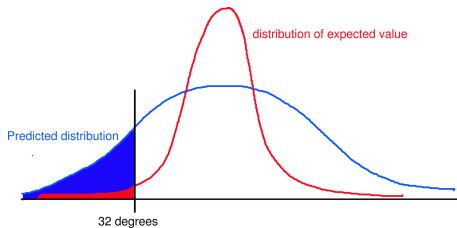
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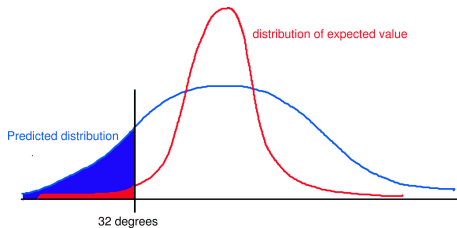


1. Predicted values: draws of Y that are or could be observed
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3. Predicted values: include estimation and fundamental uncertainty.
4. Expected values: average away fundamental uncertainty

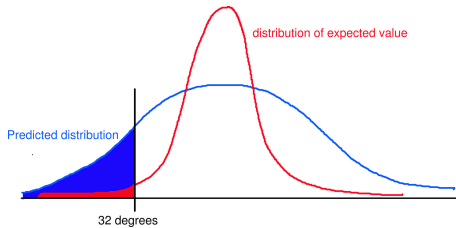




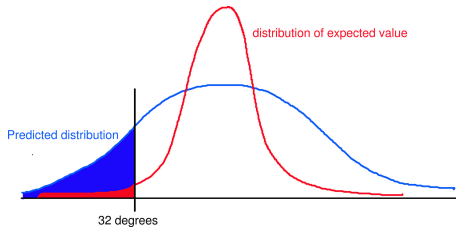
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8. Which to use for causal effects & first differences?

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5. Average over the fundamental uncertainty by calculating the mean of the **m simulations** to yield **one** simulated expected value
$$\tilde{E}(Y_c) = \sum_{k=1}^m \tilde{Y}_c^{(k)} / m.$$

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5. When $E(Y_c) = \theta_c$, we can skip the last two steps. E.g., in the logit model, once we simulate π_i , we don't need to draw Y and then average to get back to π_i . (If you're unsure, do it anyway!)

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4. (To save computation time, and improve approximation, use the same simulated β in each.)

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 - For $-1 \leq \rho \leq 1$, use $\rho = (e^{2\eta} - 1)/(e^{2\eta} + 1)$ (Fisher's Z transformation)

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 - For a probability, $\pi = [1 + e^{-\eta}]^{-1}$ (a logit transformation).
 - For $-1 \leq \rho \leq 1$, use $\rho = (e^{2\eta} - 1)/(e^{2\eta} + 1)$ (Fisher's Z transformation)

In all 3 cases, η is unbounded: estimate it, simulate from it, and reparameterize back to the scale you care about.

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5. **Clarify** does this all automatically in Stata. **Zelig** does the same and more in R.

Replication of Rosenstone and Hansen from King, Tomz and Wittenberg (2000)

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1. Logit of reported turnout on Age, Age², Education, Income, and Race

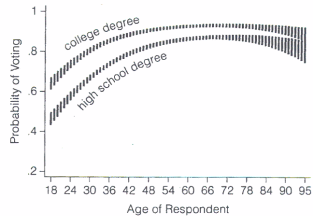
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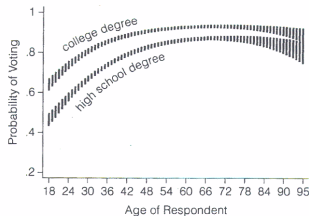
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3. Use $M = 1000$ and compute 99% CI:

FIGURE 1 Probability of Voting by Age



Vertical bars indicate 99-percent confidence intervals

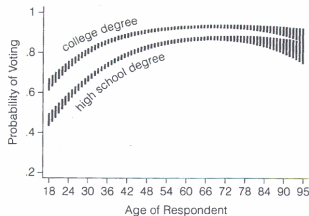
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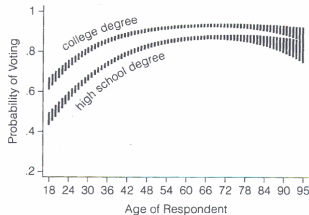


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To create this graph, simulate:

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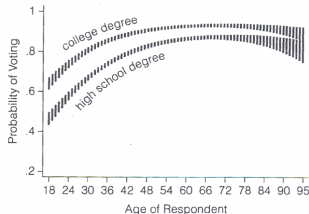


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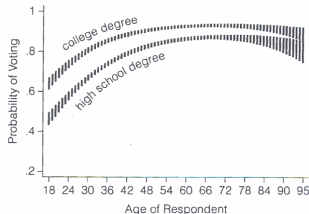


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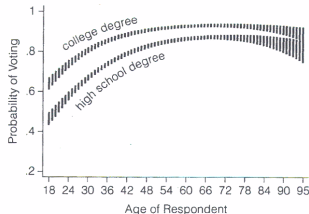


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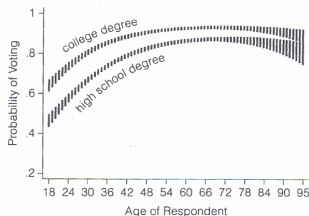


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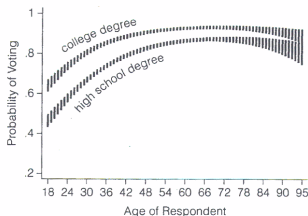


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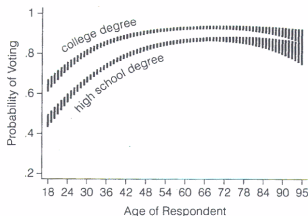


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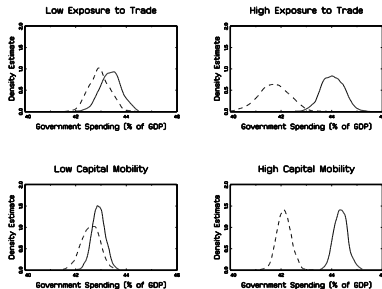
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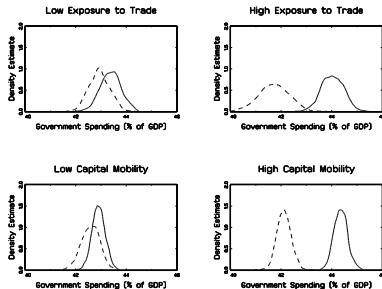
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8. Repeat for other ages and for college degree.

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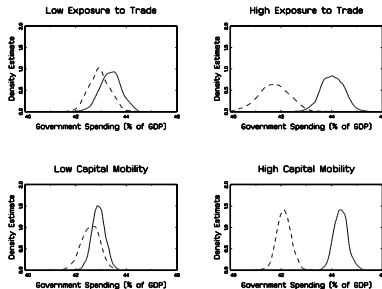


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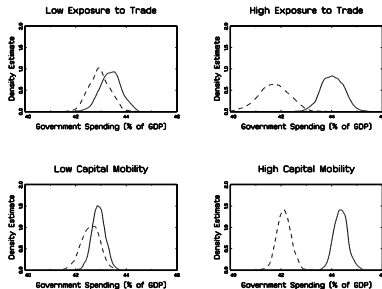
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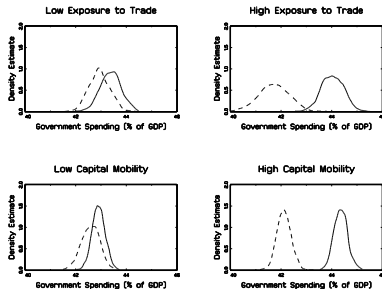
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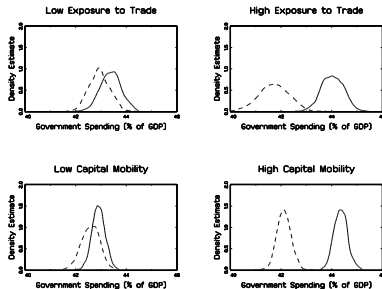
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- How could we summarize this information with less real estate?

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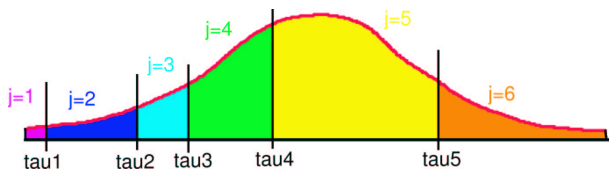
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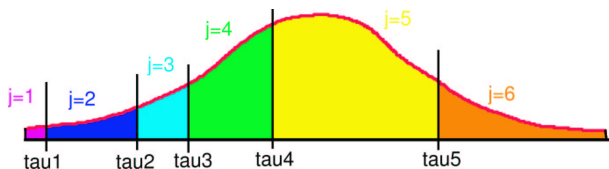
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Bracketed portion has only one factor for each i .

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(Careful of constraints during optimization: $\tau_{j-1} < \tau_j, \forall j$)

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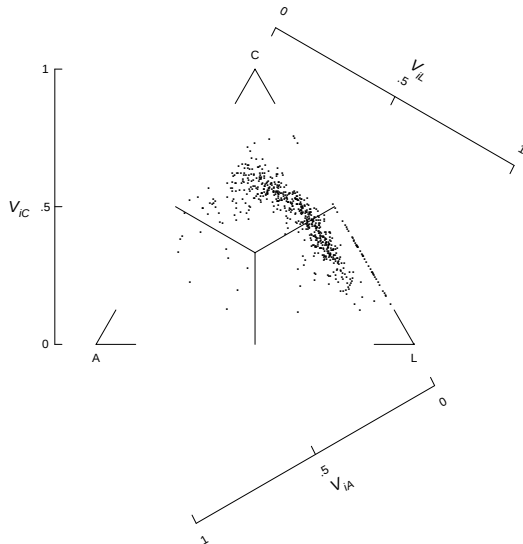
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5. Can use ternary diagrams if $J = 3$

Representing 3 variables, with $Y_j \in [0, 1]$ and $\sum_{j=1}^3 Y_j = 1$



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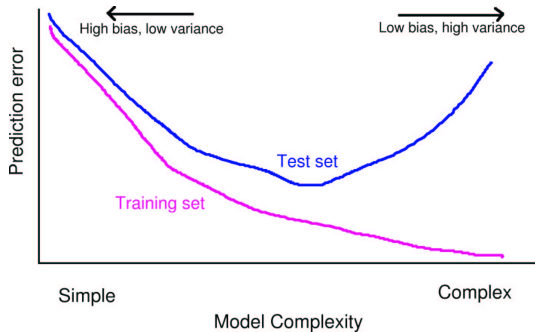
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(See Trevor Hastie et al. 2001. *The Elements of Statistical Learning*, Springer, Chapter 7: Fig 7.1.)

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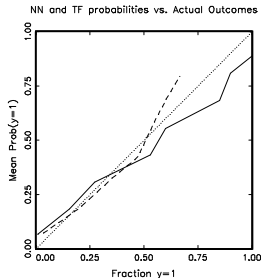
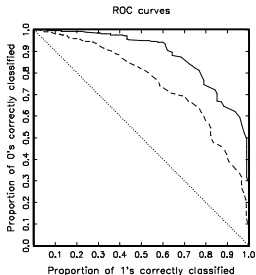
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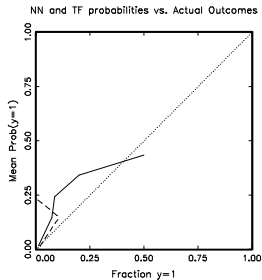
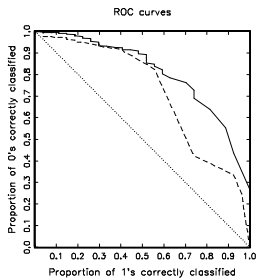
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 - Otherwise, one model is better than the other in only given specified ranges of C (i.e., for only some normative perspectives).



In sample ROC, on left (from Gary King and Langche Zeng. “Improving Forecasts of State Failure,” *World Politics*, Vol. 53, No. 4 (July, 2001): 623-58)



Out of sample ROC on left.

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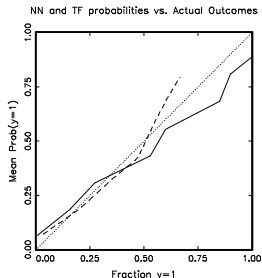
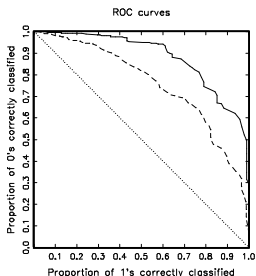
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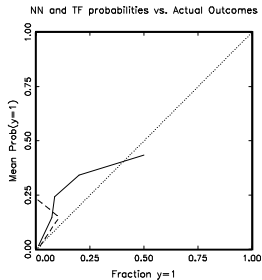
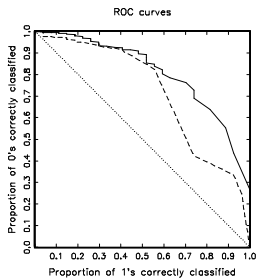
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In sample calibration **graph on right** (from Gary King and Langche Zeng. "Improving Forecasts of State Failure," World Politics, Vol. 53, No. 4 (July, 2001): 623-58)



Out of sample calibration graph on right.

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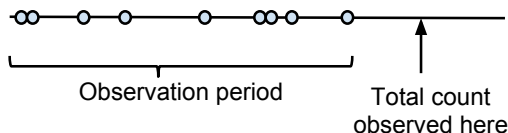
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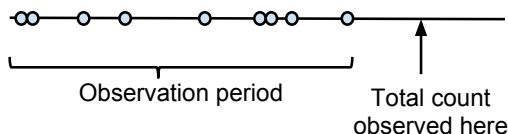
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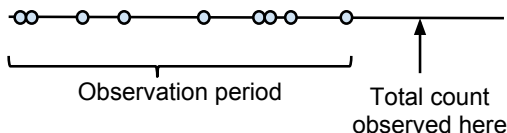
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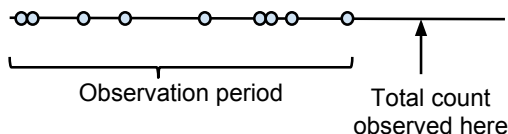
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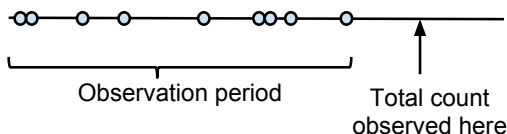
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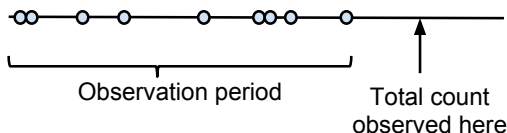
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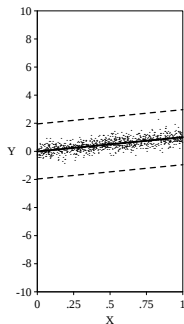
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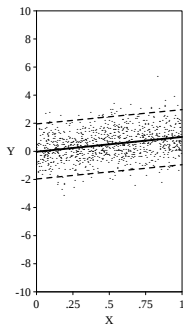
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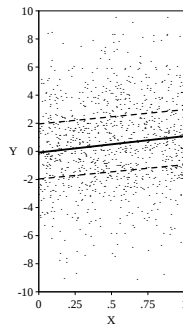
What happens without an extra parameter? Stylized Normal.



(a)



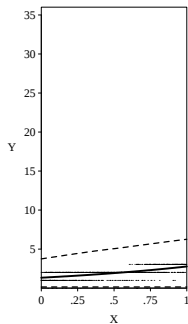
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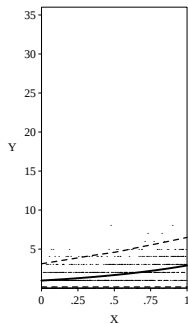
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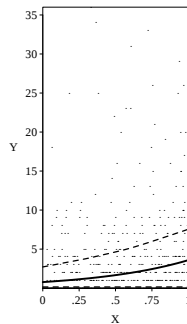
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(b)



(c)

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4. Careful of off-the-shelf programs: maybe $V(Y|X) = \phi(1 + \sigma^2\phi)$

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Computational Issues

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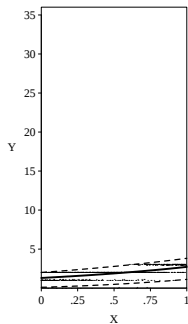
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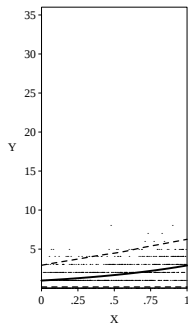
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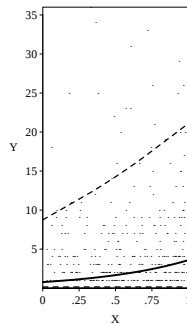
What happens with the extra parameter? GEC



(a)



(b)



(c)

$E(Y|X)$ and 95% CI.

King, Gary and Curtis S. Signorino. "The Generalization in the Generalized Event Count Model" in *Political Analysis*, 6 (1996): 225-252.

King, Gary. "Variance Specification in Event Count Models: From Restrictive Assumptions to a Generalized Estimator," *American Journal of Political Science*, 33, 3 (August, 1989): 762-784.

Duration Models and Censoring: The Exponential Model

King, Gary; James Alt; Nancy Burns; and Michael Laver. "A Unified Model of Cabinet Dissolution in Parliamentary Democracies," *American Journal of Political Science*, Vol. 34, No. 3 (August, 1990): Pp. 846-871; Errata Vol. 34, No. 4 (November, 1990): P. 1168. (replication dataset: ICPSR s1115).

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 - **Include censoring information in the likelihood**

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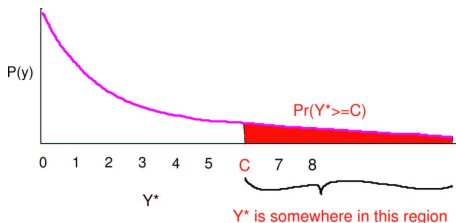
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