Advanced Quantitative Research Methodology, Lecture Notes: Single Equation Models[1]

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Binary Variable Regression Models

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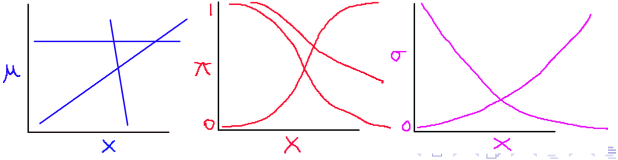
(The graph in the middle:)



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What do we do with this?

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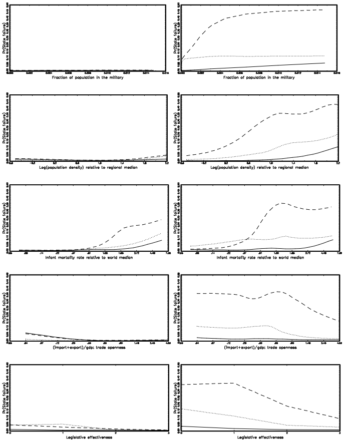
   (d) Average effects: compute effects for every observation and average



Example Marginal Effect Plot of a neural network model (about which more later). Full democracies (dotted), partial democracies (dashed), and autocracies (solid). From Gary King and Langche Zeng. "Improving Forecasts of State Failure," *World Politics*, Vol. 53, No. 4 (July, 2001): 623-58.

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For any quantity but a probability, always also include a measure of uncertainty (standard error, confidence interval, etc.)

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Variable	From		To	First Difference
Sex	Male	$\rightarrow$	Female	.05
Age	65	$\rightarrow$	75	$-.10$
Home	NYC	$\rightarrow$	Madison, WI	.26
Income	\$35,000	$\rightarrow$	\$75,000	.14

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   (b) Max value for probit $[\pi_i = \Phi(X_i\beta)]$ derivative: $\hat{\beta} \times 0.4$

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Since Y^* is unobserved anyway, define the threshold as $\tau = 0$. (Plus the same independence assumption, which from now on is assumed implicit.)

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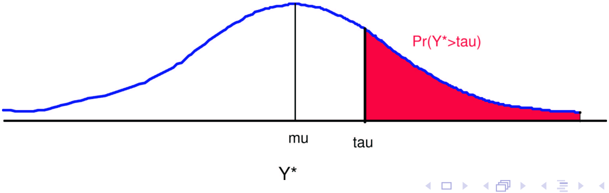

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5. $\Longrightarrow$ interpret β as regression coefficients of Y^* on X: $\hat{\beta}_1$ is what happens to Y^* on average (or μ_i) when X_1 goes up by one unit, holding constant the other explanatory variables (and conditional on the model). In probit, one unit of Y^* is one standard deviation.

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- Which would enable you to choose logit over probit?

How Not to Present Statistical Results

TABLE 1
Predicting Which Ethnic Group Conquered Most of Bosnia

Attention to Bosnia crisis	.609**
Age	.007**
Education	.289**
Family income	.151**
Race (non-White/White)	.695**
Gender (female/male)	.789**
Region (South/non-South)	.076
Network coverage	.000
Education × Time	–.003*
Time in months	.078**
Constant	–9.257**
Number	7,021
–2 log-likelihood	7,215.231
Goodness of fit	6,789.45
Cox & Snell R^2	.212
Nagelkerke R^2	.295
Overall correct classification (%)	73.96

SOURCE: *Times Mirror* polls from September 1992, January 1993, September 1993, January 1994, and June 1995.
NOTE: Unstandardized coefficients for logistic regression. Dependent variable is knowledge of which group conquered most of Bosnia.
$^*p \leq .05$, two-tailed. $^{**}p \leq .01$, two-tailed.

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1. This one is typical of current practice, not that unusual.

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How Not to Present Statistical Results

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Predicting Which Ethnic Group Conquered Most of Bosnia

Attention to Bosnia crisis	.609**
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Family income	.151**
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Gender (female/male)	.789**
Region (South/non-South)	.076
Network coverage	.000
Education × Time	–.003*
Time in months	.078**
Constant	–9.257**
Number	7,021
–2 log-likelihood	7,215.231
Goodness of fit	6,789.45
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4. Reading assignment: (at my web page) the handout on papers, and King, Tomz, Wittenberg, "Making the Most of Statistical Analyses: Improving Interpretation and Presentation" American Journal of Political Science, Vol. 44, No. 2 (March, 2000): 341-355.

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2. *Fundamental uncertainty*: Represented by the stochastic component. Exists no matter what the researcher does.

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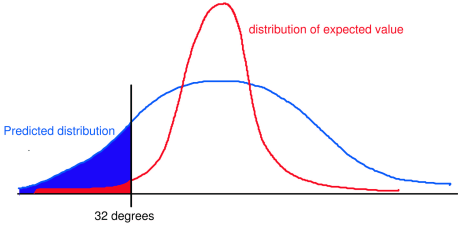
Repeat algorithm say $M = 1000$ times, to produce 1000 predicted values. Use these to compute a histogram for the full posterior, the average, variance, percentile values, or others.

How the Distribution of Expected and Predicted Values Differ



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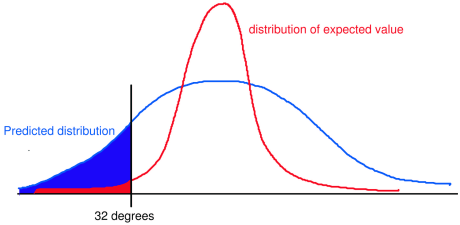

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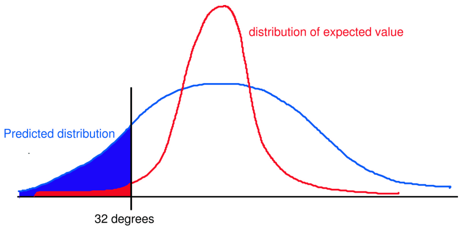

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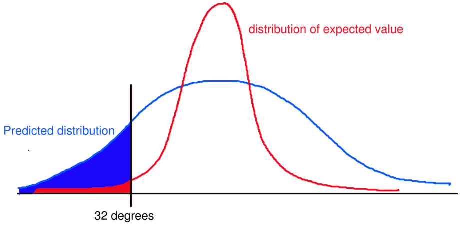

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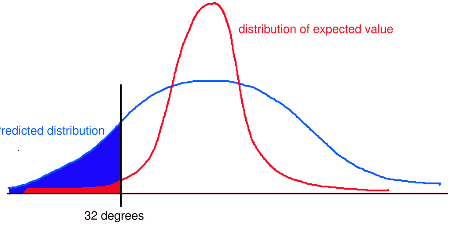

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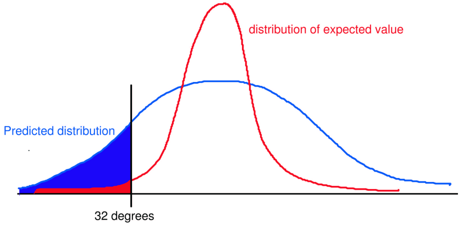
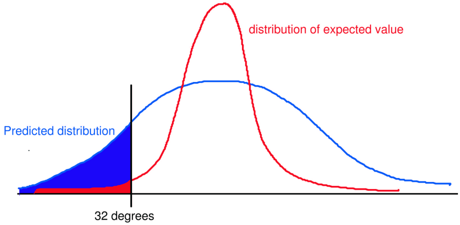

distribution of expected value
Predicted distribution
32 degrees



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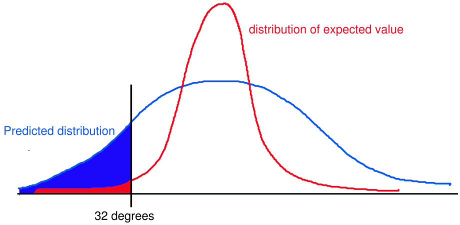

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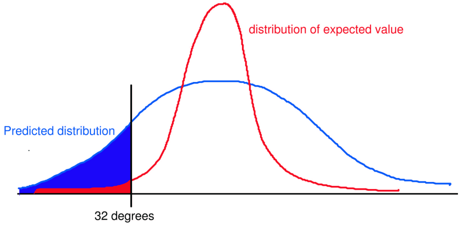

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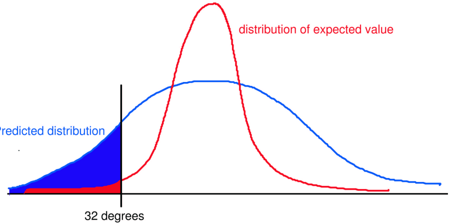

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5. Average over the fundamental uncertainty by calculating the mean of the m simulations to yield one simulated expected value $\tilde{E}(Y_c) = \sum_{k=1}^{m} \tilde{Y}_c^{(k)}/m$.

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5. When $E(Y_c) = \theta_c$, we can skip the last two steps. E.g., in the logit model, once we simulate π_i, we don't need to draw Y and then average to get back to π_i.

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Tricks for Simulating Parameters

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   In all 3 cases, η is unbounded, so estimate it, simulate from it, and reparameterize back to the scale you care about.

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5. Clarify does this all automatically in Stata. Zelig does the same and more in R.

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3. Use $M = 1000$ and compute 99% CI:

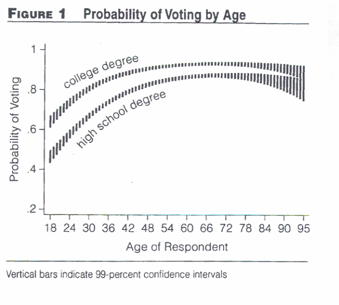
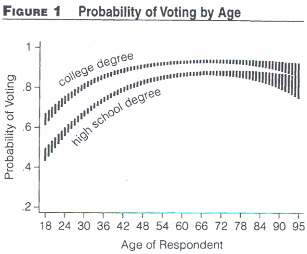


FIGURE 1 Probability of Voting by Age
college degree
high school degree
Probability of Voting
1
.8
.6
.4
.2
18 24 30 36 42 48 54 60 66 72 78 84 90 95
Age of Respondent
Vertical bars indicate 99-percent confidence intervals



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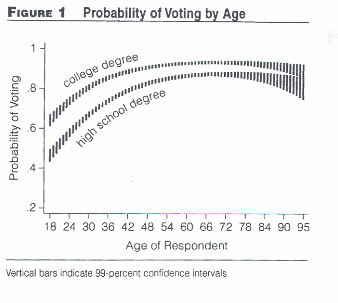
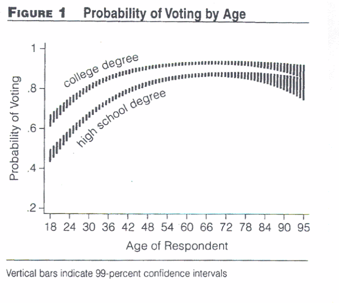
To create this graph, simulate:



To create this graph, simulate:

1. Set age=24, education=high school, income=average, Race=white



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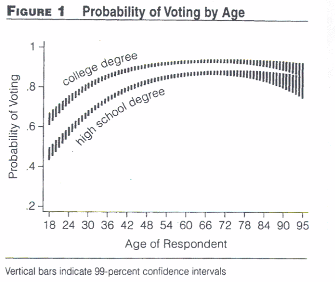
2. Run logistic regression



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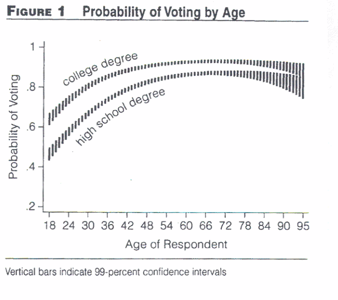

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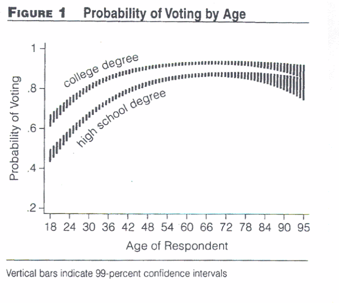

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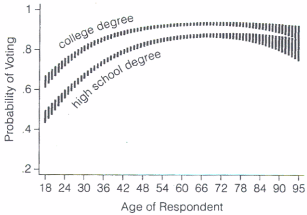

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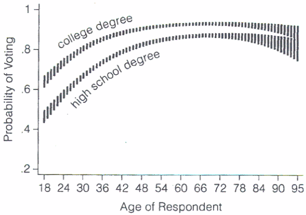

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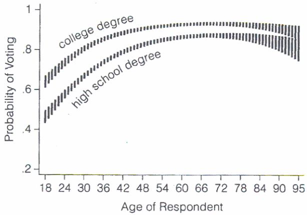

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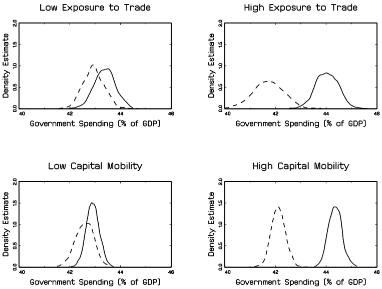

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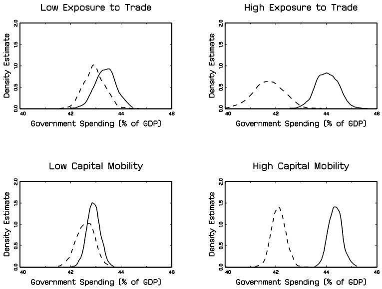

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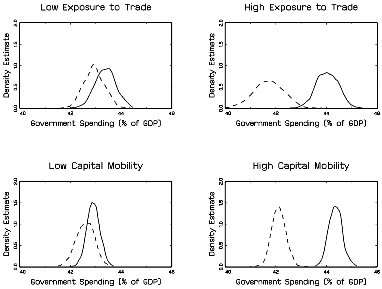

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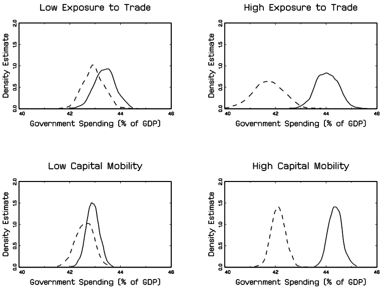

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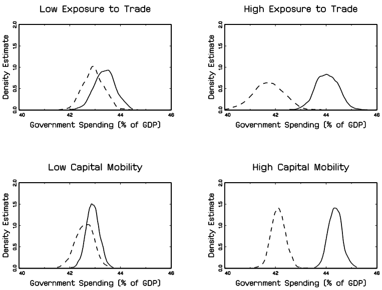

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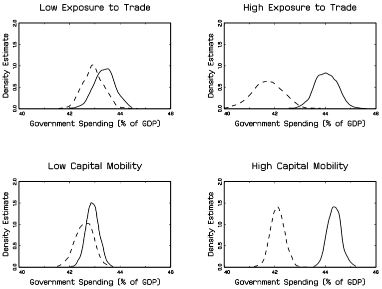

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- How could we summarize this information with less real estate?

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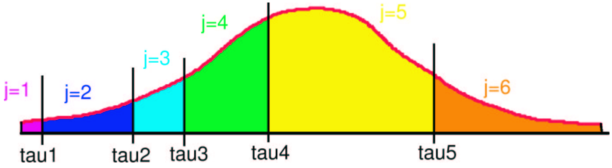

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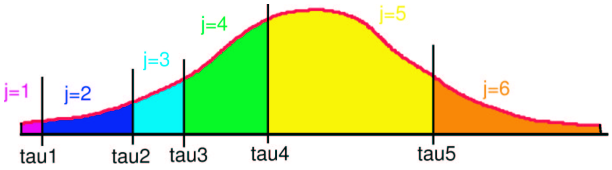

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Bracketed portion has only one factor for each i.

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5. Can use ternary diagrams if $J = 3$

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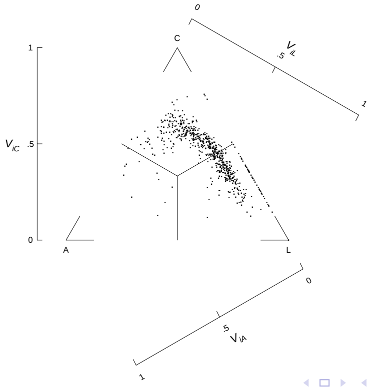

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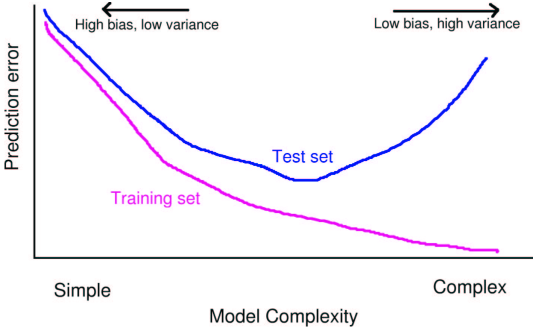
   (h) If the world changes between the time the training and test sets are observed, even an otherwise good model will fail. But this is still the right test.



See Trevor Hastie, Robert Tibshirani, and Jerome Friedman. 2001. *The Elements of Statistical Learning: Data Mining, Inference, and Prediction*, Springer, Chapter 7: Fig 7.1.

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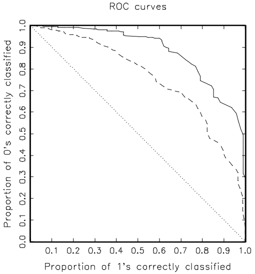
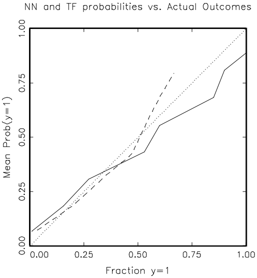
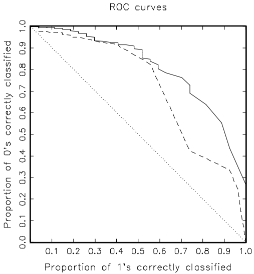
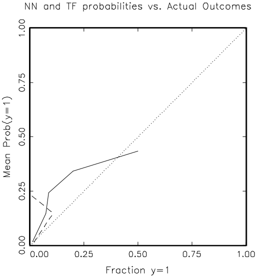
   (v) Otherwise, one model is better than the other in only given specificed ranges of C (i.e., for only some normative perspectives).





In sample ROC, on left (from Gary King and Langche Zeng. "Improving Forecasts of State Failure," World Politics, Vol. 53, No. 4 (July, 2001): 623-58)





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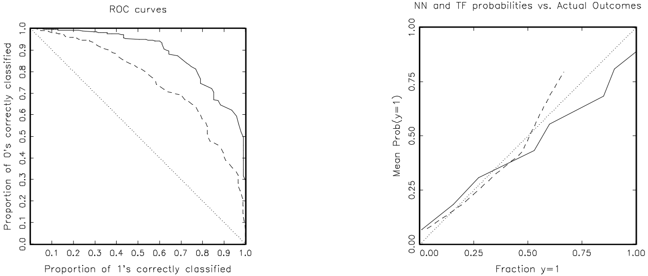
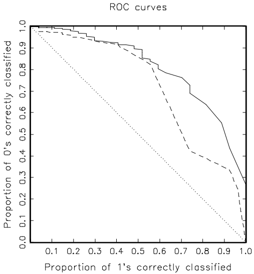
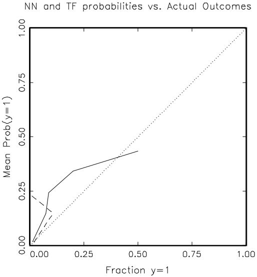


In sample calibration graph on right (from Gary King and Langche Zeng. "Improving Forecasts of State Failure," World Politics, Vol. 53, No. 4 (July, 2001): 623-58)





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which implies

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and a likelihood of

$$\begin{aligned} L(\pi|y) &\propto \prod_{i=1}^{n} \text{Binomial}(y_i|\pi_i) \\ &= \prod_{i=1}^{n} \binom{N_i}{y_i} \pi_i^{y_i}(1-\pi_i)^{N_i-y_i} \end{aligned}$$

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7. Some event count datasets go over time (count per year), some across areas (count per state), and some both.

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so we could use $\bar{y}\beta$ for an approximate linearized effect.

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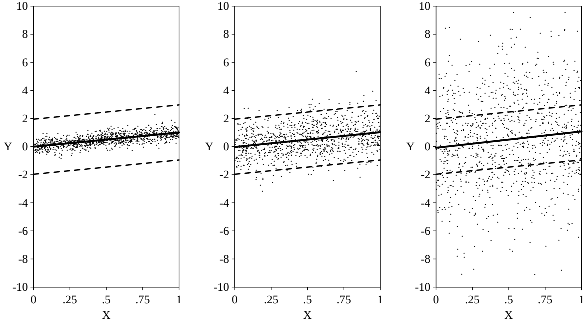
What happens without an extra parameter? Stylized Normal.



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$E(Y|X)$ and 95% CI.

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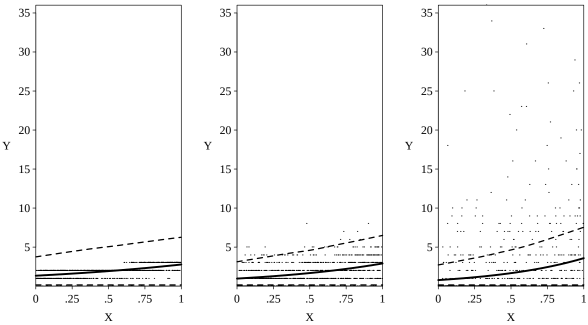

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Negative Binomial Event Count Model

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3. So a test of Poisson vs NegBin can be based on σ^2, even though a likelihood ratio doesn't work since Poisson doesn't exactly nest within Negbin.

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3. $\sigma^2 > 1$, and so we estimate γ, where $\sigma^2 = e^\gamma + 1$.

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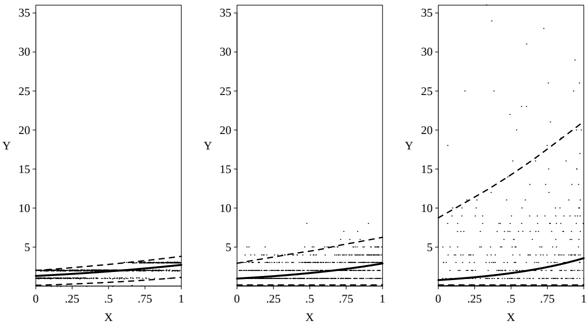

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King, Gary and Curtis S. Signorino. "The Generalization in the Generalized Event Count Model" in *Political Analysis*, 6 (1996): 225-252.

King, Gary. "Variance Specification in Event Count Models: From Restrictive Assumptions to a Generalized Estimator," *American Journal of Political Science*, 33, 3 (August, 1989): 762-784.

Duration Models and Censoring: The Exponential Model

King, Gary; James Alt; Nancy Burns; and Michael Laver. "A Unified Model of Cabinet Dissolution in Parliamentary Democracies," American Journal of Political Science, Vol. 34, No. 3 (August, 1990): Pp. 846-871; Errata Vol. 34, No. 4 (November, 1990): P. 1168. (replication dataset: ICPSR s1115).

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5. We will model these observations explicitly as censored.

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$$y_i = \begin{cases} y_i^* & \text{if } y_i^* < C \\ y_i^C & \text{if } y_i^* \geq C \end{cases}$$

The likelihood function for censored observations. All we know is:

$$Y_i^* \sim \text{expon}(y_i^*|\lambda_i)$$
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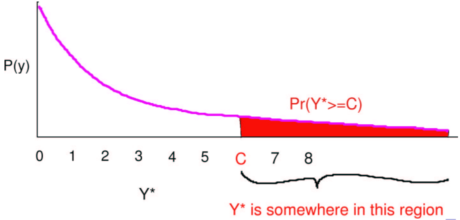

Censoring

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&= \int_{y_i^c}^{\infty} \lambda_i e^{-\lambda_i y_i} dy_i
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$$

Thus, the full likelihood:

$$
L(\beta|y) = \left[\prod_{y_i^* < y_i^c} \text{expon}(y_i|\lambda_i)\right] \left[\prod_{y_i^* \geq y_i^c} \Pr(Y_i^* \geq y_i^c)\right]
$$