

Robust and Clustered Standard Errors

Molly Roberts

March 6, 2013

Outline

1 An Introduction to Robust and Clustered Standard Errors

- Linear Regression with Non-constant Variance
- GLM's and Non-constant Variance
- Cluster-Robust Standard Errors

2 Replicating in R

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Residuals are the vertical distances between observations and the **estimated** regression function. Therefore, they are known.

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$$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \mathbf{u}$$

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Residuals represent the difference between the outcome and the estimated mean.

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Variance of $\hat{\beta}$ depends on the errors

$$\begin{aligned}\hat{\beta} &= (\mathbf{X}'\mathbf{X})^{-1} \mathbf{X}'\mathbf{y} \\ &= (\mathbf{X}'\mathbf{X})^{-1} \mathbf{X}'(\mathbf{X}\beta + \mathbf{u}) \\ &= \beta + (\mathbf{X}'\mathbf{X})^{-1} \mathbf{X}'\mathbf{u}\end{aligned}$$

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$$\begin{aligned}V[\hat{\beta}] &= V[\beta] + V[(\mathbf{X}'\mathbf{X})^{-1} \mathbf{X}'\mathbf{u}] \\&= \mathbf{0} + V[(\mathbf{X}'\mathbf{X})^{-1} \mathbf{X}'\mathbf{u}] \\&= E[(\mathbf{X}'\mathbf{X})^{-1} \mathbf{X}'\mathbf{u}\mathbf{u}'\mathbf{X} (\mathbf{X}'\mathbf{X})^{-1}] - E[(\mathbf{X}'\mathbf{X})^{-1} \mathbf{X}'\mathbf{u}]E[(\mathbf{X}'\mathbf{X})^{-1} \mathbf{X}'\mathbf{u}]' \\&= E[(\mathbf{X}'\mathbf{X})^{-1} \mathbf{X}'\mathbf{u}\mathbf{u}'\mathbf{X} (\mathbf{X}'\mathbf{X})^{-1}] - \mathbf{0}\end{aligned}$$

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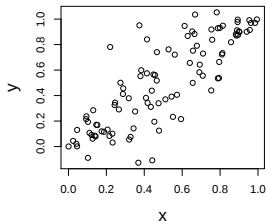
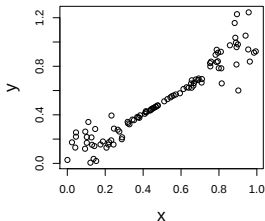
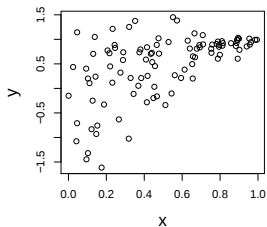
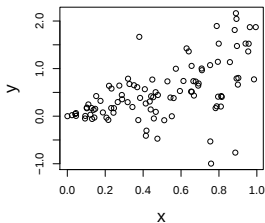
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What does this mean graphically for a CEF with one explanatory variable?

Evidence of Non-constant Error Variance (4 examples)



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- ▶ The degree of the problem depends on the amount of heteroskedasticity.
- ▶ $\hat{\beta}$ is still unbiased for β

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is a consistent estimator of $V[\hat{\beta}]$.

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2. Doesn't make $\hat{\beta}$ BLUE

3. What are you going to do with predicted probabilities?

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These are the robust standard errors that scholars now use for other glm's, and that happen to coincide with the linear case.

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- ▶ Heteroskedasticity in the latent variable formulation will completely change the functional form of $P(y = 1|x)$.
- ▶ What does this mean? The $P(y = 1|x) \neq \Phi(\mathbf{x}\beta)$. Your model is wrong.

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Remember, the Fisher information matrix is $-E_{\theta}[h_i(Y_i|\theta)]$.

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$$L(\theta) = L(\theta_0) + L'(\theta_0)(\theta - \theta_0) + \frac{1}{2}(\theta - \theta_0)^T L''(\theta_0)(\theta - \theta_0)$$

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$$Avar(\hat{\theta}) = [-L''(\theta_0)]^{-1} [Cov(L'(\theta_0))] [-L''(\theta_0)]^{-1}$$

RSEs for GLMs

It's the sandwich estimator.



$$\begin{aligned} Avar(\hat{\theta}) &= [-L''(\theta_0)]^{-1} [Cov(L'(\theta_0))] [-L''(\theta_0)]^{-1} \\ &= \left[-\sum_{i=1}^n h_i(Y_i|\hat{\theta}) \right]^{-1} \left[\sum_{i=1}^n g_i(Y_i|\hat{\theta})^T g_i(Y_i|\hat{\theta}) \right] \left[-\sum_{i=1}^n h_i(Y_i|\hat{\theta}) \right]^{-1} \end{aligned}$$

RSEs for GLMs

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$$= \left[- \sum_{i=1}^n h_i(Y_i|\hat{\theta}) \right]^{-1} \left[\sum_{i=1}^n g_i(Y_i|\hat{\theta})^T g_i(Y_i|\hat{\theta}) \right] \left[- \sum_{i=1}^n h_i(Y_i|\hat{\theta}) \right]^{-1}$$

Bread: $\left[- \sum_{i=1}^n h_i(Y_i|\hat{\theta}) \right]^{-1}$ Meat: $\left[\sum_{i=1}^n g_i(Y_i|\hat{\theta})^T g_i(Y_i|\hat{\theta}) \right]$

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- ▶ Sometimes it's difficult to figure out what is going on in Stata.
- ▶ But by really understanding what is going on in R, you will be able to replicate once you know the equation for Stata.

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Thanks to Michele Margolis and Dan Altman for their contributions to the library!

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```
fit <-glm(fmla, data=treaty1,  
  family=binomial(link="logit"))
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For the meat, we are going to use the estimating function to create the matrices first derivative:

```
est.fun <- estfun(fit)
```

Note: if `estfun` doesn't work for your `glm`, there is a way to do it using `numericGradient()`.

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And put them back in our table

The Sandwich

So we can create the sandwich

```
meat <- t(est.fun)*%est.fun  
sandwich <- bread*%meat*%bread
```

And put them back in our table

```
library(lm.test)  
coeftest(fit, sandwich)
```

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```
u <- estfun(fit)  
u.clust <- matrix(NA, nrow=m, ncol=k)  
for(j in 1:k){  
  u.clust[,j] <- tapply(u[,j], fc, sum)  
}
```

Last, we can make our cluster robust matrix:

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  %*% (u.clust)) %*% + vcov
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And test our coefficients

```
coeftest(fit, cl.vcov)
```

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