Gov 2001 Section 6: Dichotomous Outcomes

Konstantin Kashin[1]

March 6, 2013

[1]Thanks to Jen Pan, Brandon Stewart, Iain Osgood, and Patrick Lam for contributing to this material.

Outline

Replication Paper

- You should be in the process of replicating one (or more) articles!

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- Replication and preliminary write-up due Wednesday, March 27 at 7pm

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Problem Set

- Problem set 5 released today

Problem Set

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Problem Set

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- Due next Wednesday, March 13 at 7pm
- One assessment problem for which you will not be allowed to collaborate / talk to other people

Section Next Week

Section next week will be on Tuesday, March 12 from 7-8pm and 8-9pm in CGIS K354.

Outline

Generalized Linear Models

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Three elements of a GLM (equivalent to Gary's stochastic and systematic component):

- A distribution for Y (stochastic component)
- A linear predictor $X\beta$ (systematic component)
- A link function that relates the linear predictor to the mean of the distribution. (systematic component)

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Assume our data was generated from some distribution.

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Examples:

- Continuous and Unbounded: Normal
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- Duration: Exponential

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Examples:

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These distributions are all members of the exponential family.

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We are interested in allowing some parameter of the distribution θ to vary as a (linear) function of covariates. So we specify a linear predictor.

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$$X\beta = \beta_0 + x_1\beta_1 + x_2\beta_2 + \cdots + x_k\beta_k$$

3. Specify a link function

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This is the systematic component that we've been talking about all along.

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Use `optim` to estimate the parameters just like we have all along.

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1. Simulate parameters from multivariate normal.
2. Run $X\beta$ through inverse link function to get expected values.
3. Draw from distribution of Y for predicted values.

Note: this can also be calculated analytically via the Delta Method. We aren't going to cover this in section though...

Outline

Binary Dependent Variable

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Let our dependent variable be a binary random variable that can take on values of either 0 or 1.

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$$\begin{aligned}\ln L(\beta|y) &\propto \sum_{i=1}^{n} y_i \ln(\pi_i) + (1-y_i)\ln(1-\pi_i) \\ &= \sum_{i=1}^{n} y_i \ln(\Phi(X_i\beta)) + (1-y_i)\ln(1-\Phi(X_i\beta))\end{aligned}$$

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5. Simulate Quantities of Interest

Outline

The Data: Political Assassinations

Taken from Olken and Jones (2009), "Hit or Miss? The Effect of Assassinations on Institutions and War", *American Economic Journal: Macroeconomics*.

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Dataframe is called `as` and contains information on assassination attempts, success or failure, and various covariates.

```
> as[as$country == "United States" & as$year == "1975",]
      country year leadername age tenure attempt
United States 1975       Ford  62    510    TRUE
      survived result dem_score civil_war war
             1     24        10         0   0
           pop  energy solo weapon
        215973 2208506    1    gun
```

Observations are country-year-leaders, so some country-years have multiple observations.

The Probit Model

Let's try to predict assassination attempts with some of our covariates.

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"The question of which distribution to use is a natural one... There are practical reasons for favoring one or the other in some cases for mathematical convenience, but it is difficult to justify the choice of one distribution or another on theoretical grounds ...[A]s a general proposition, the question is unresolved. In most applications, the choice between these two seems not to make much difference."
-*Econometric Analysis*, Greene. pg. 774.

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The model:

1. $Y_i \sim f_{\text{bern}}(y_i|\pi_i)$.
2. $\pi_i = \Phi(X_i\beta)$ where Φ is the CDF of the standard normal distribution.
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3. Y_i and Y_j are independent for all $i \neq j$.

Like all CDF's, Φ has range 0 to 1, so it puts our π_i on the right scale.

$$\Phi(z) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} \exp(\frac{z^2}{2}) dz.$$

Visualizing the Probit Link

$X_i\beta$

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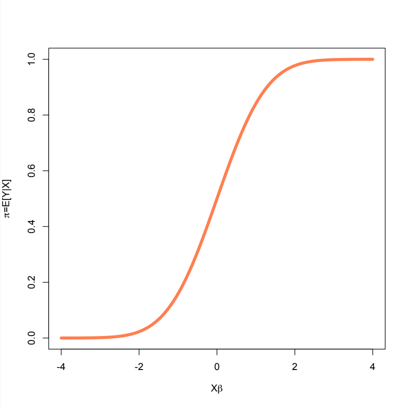
$X_i\beta$ $\longrightarrow$



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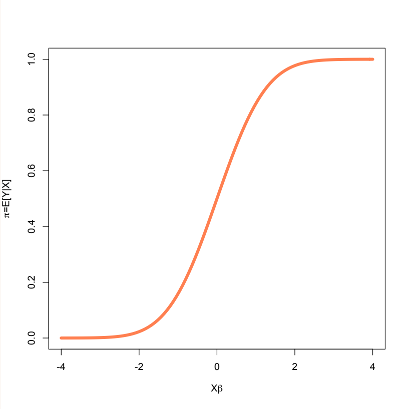
$X_i\beta \longrightarrow$



$\longrightarrow \pi_i = \mathrm{E}[Y_i]$

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\end{aligned}
$$

Therefore:

$$
\begin{aligned}
\ln L(\beta|y) &\propto \sum_{i=1}^{n} y_i \ln(\pi_i) + (1-y_i)\ln(1-\pi_i) \\
&= \sum_{i=1}^{n} y_i \ln(\Phi(X_i\beta)) + (1-y_i)\ln(1-\Phi(X_i\beta))
\end{aligned}
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ll.probit <- function(beta, y=y, X=X){
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2. If `lower.tail = FALSE` gives $Pr(Z \geq z)$.
3. Uses a logical test to check that an intercept column has been added

What's in our model?

We wish to predict assassination attempts for country-year-leaders.

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- `tenure`: number of days in office
- `age`: age of leader, in years
- `dem_score`: polity score, -10 to 10
- `civil_war`: is there currently a civil war?
- `war`: is country in an international conflict?
- `pop`: the country's population, in thousands
- `energy`: energy usage

Maximization

```
y <- as$attempt
X <- as[,c("tenure","age","dem_score","civil_war",
           "war","pop","energy")]

yX <- na.omit(cbind(y,X))
y <- yX[,1]; X <- yX[,-1]

X$tenure <- (X$tenure-mean(X$tenure))/sd(X$tenure)
X$pop <- (X$pop-mean(X$pop))/sd(X$pop)
X$energy <- (X$energy-mean(X$energy))/sd(X$energy)

opt <- optim(par = rep(0,8), fn = ll.probit, y=y, X=X,
       method = "BFGS", control = list(fnscale = -1,
       maxit = 1000), hessian = TRUE)
```

The Fit Model

	Coef	SE	Z-stat
tenure	0.0291	0.0294	0.9922
age	-0.0058	0.0025	-2.2960
dem score	-0.0124	0.0045	-2.7921
civil war	0.0663	0.0890	0.7445
war	0.3175	0.1014	3.1298
pop	0.0409	0.0237	1.7264
energy	0.0269	0.0243	1.1034
intercept	-1.8362	0.1399	-13.1262

Outline

Quantities of Interest

General considerations:

1. Incorporating estimation uncertainty.
2. Incorporating fundamental uncertainty when making predictions.
3. Establishing appropriate baseline values for QOI, and considering plausible changes in those values.

Expected Values

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```
attempt.vec <- y == 1   # y is attempt without NAs
highrisk <- apply(X = X[attempt.vec,],
                  MARGIN = 2, FUN = median)
```

Predictor:	tenure	age	dem score	civil war	war	pop.	energy
Value:	1392	54	-3	0	0	12980000	3828

Table: Median values for year-states with an assassination attempt.

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```
library(MASS)
beta.draws <- mvrnorm(10000, mu = opt$par, Sigma
              = solve(-opt$hessian))

p.ests <- c()
for(i in 1:10000){
  p.ass.att <- pnorm(highrisk%*%beta.draws[i,])
  outcomes <- rbinom(10000, 1, p.ass.att)
  p.ests[i] <- mean(outcomes)
}

> mean(p.ests)
[1] 0.0166266
> quantile(p.ests, .025); quantile(p.ests, .975)
 2.5%    97.5%
0.0134  0.0201
```

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6. return to step 3.