Section 1: Probability Review

Molly Roberts

January 30, 2013

Outline

1. Basic Probability
2. Random Variables & Probability Distributions
3. Simulation
   - Example 1: Finding a Mean
   - Example 2: Probability
   - Example 3: Integrating the Normal Density

Three Axioms of Probability

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3. If $A_1, A_2, \dots, A_n$ are mutually disjoint, then

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e.g. P(rolling a 3 or rolling a 2) $= \frac{1}{6} + \frac{1}{6} = \frac{1}{3}$

Conditional Probability

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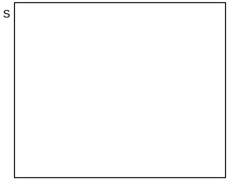

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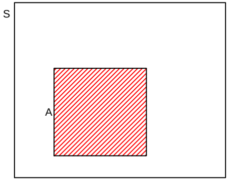

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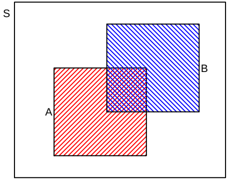

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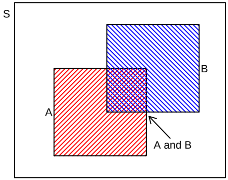

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Bayes' Rule:

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

Side Note: Bayes Rule

Bayes rule will be extremely important going forward.

Why?

- Often we want to know $P(\theta|\text{data})$.
- But what we *do* know is $P(\text{data}|\theta)$.
- We'll be able to *infer* θ by using a variant of Bayes rule. Stay tuned.

Bayes' Rule:

$$P(\theta|y) = \frac{P(y|\theta)P(\theta)}{P(y)}$$

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e.g. When rolling a two dice, we may be interested in whether or not the sum of the two dice is 7. Or we might be interested in the actual sum of the two dice.

Ex. Waiting for the Redline – How long will it take for the next T to get here?

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$$X = \begin{cases} 1 & \text{if the redline arrives within 1 minute} \\ 2 & \text{if } 1-2 \text{ minutes} \\ 3 & \text{if } 2-3 \text{ minutes} \\ 4 & \text{if } 3-4 \text{ minutes} \\ \vdots & \vdots \end{cases}$$

Ex. Waiting for the Redline Cont

Now, suppose the probability that the T comes in any given minute is a constant $\pi = .2$, and whether the T comes is independent of what has happened in previous periods.

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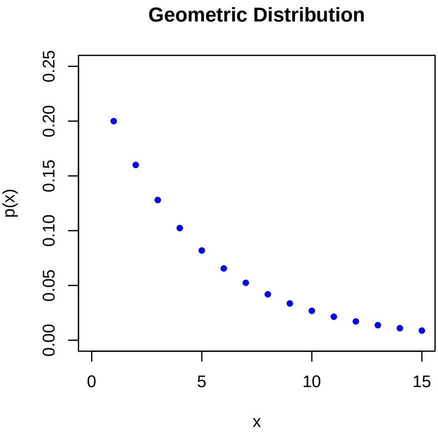
And generally...

$$Pr(X = x) = (1 - \pi)^{x-1}\pi = .8^{x-1} \cdot .2$$

A Pictorial Representation of the PMF

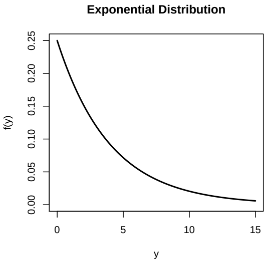


X is a Geometric random variable with parameter $\pi = .2$.

Probability Density Function

Ex. Waiting for the Redline: an alternative model where Y is exact time the T arrives.

$$f(y) = \lambda e^{-\lambda y} = .25e^{-.25y}$$



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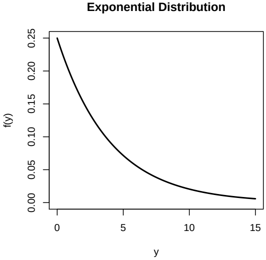

Y is an Exponential random variable with parameter $\lambda = .25$.

Ex. Calculating probabilities for continuous RVs

Recall that to identify probabilities for continuous random variables we have to use:

$$P(Y \in A) = \int_A f(y)dy.$$

$$Pr(2 \leq y \leq 10) \quad =$$

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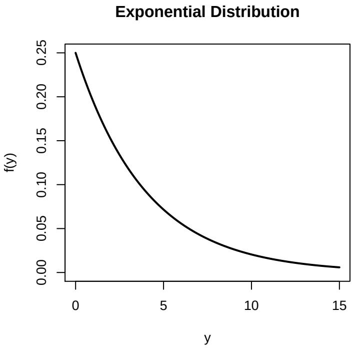

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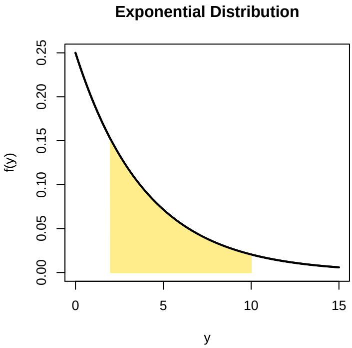

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[1] Here we also use the fact that the antiderivative of e^{bx} is $\frac{e^{bx}}{b}$

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Ex.:

1. $\int_0^\infty .25e^{-.25y}dy = -e^{-.25y}|_0^\infty = 0 + 1 = 1$
2. $.25e^{-.25y} \geq 0$ for all $y \in (0, \infty)$

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Continuous Time:

$$\begin{aligned} E(Y) &= \int_{-\infty}^{\infty} y f(y) dy \\ &= \int_0^{\infty} y \cdot .25 e^{-.25y} dy \\ &= 4 \end{aligned}$$

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> draws <- rexp(n = 100000, rate = .25)
> mean(draws)
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Neither of these are perfectly accurate but they become arbitrarily

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By the Strong Law of Large Numbers, our estimate $\hat{I}_M$ is a simulation consistent estimator of I as $M \to \infty$(our estimate gets better as we increase the number of simulations).

Back to our examples

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> mean(g.draws)
[1] 2.312
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> draws <- rexp(n = 100000, rate = .25)
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> mean(g.draws)
[1] 2.096
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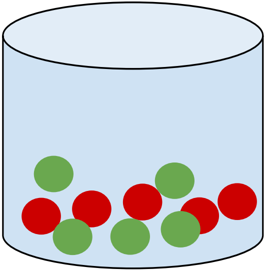
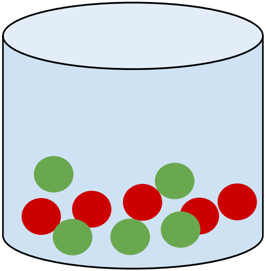
Simulation for Probability Problems



I have an urn composed of 5 red balls and 5 green balls. If I sample 4 balls without replacement from the urn, what is the probability of drawing 4 balls all of the same color?



Here are some steps:

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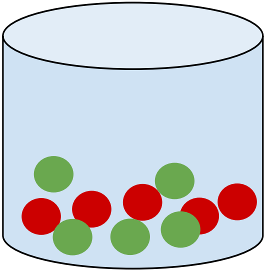

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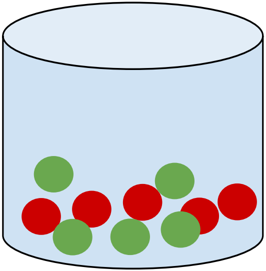

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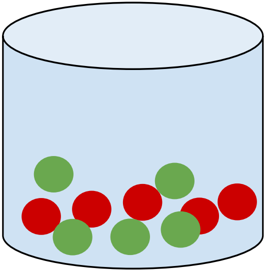

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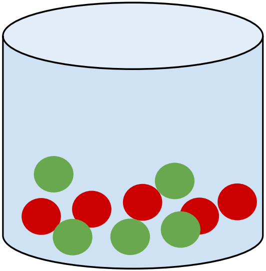
3. Figure out a programming rule for determining whether our condition was met i.e. we drew "4 red balls or 4 green balls".
4. Throw a for loop around it and sample repeatedly.



Here are some steps:

1. Construct our population aka our urn.
2. Figure out how to take one sample from it.
3. Figure out a programming rule for determining whether our condition was met i.e. we drew "4 red balls or 4 green balls".
4. Throw a for loop around it and sample repeatedly.
5. Determine what proportion of times our condition was successful.

1. Construct our population

Here are some possibilities:

```
> urn <- c("G","G","G","G","G","R","R","R","R","R")
> urn
 [1] "G" "G" "G" "G" "G" "R" "R" "R" "R" "R"
```

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```

```
> urn <- c(rep("red",5),rep("green",5))
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 [1] "red"   "red"   "red"   "red"   "red"
  "green" "green" "green" "green"  "green"
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```
> urn <- c(rep(1,5),rep(0,5))
> urn
 [1] 1 1 1 1 1 0 0 0 0 0
```

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```

I'll use this last one because numbers will be easier to use later on.

2. Figure out how to take one sample

We need to use the `sample()` function with the `replace = FALSE` argument:

```
> sample(x = urn, size = 4, replace = FALSE)
[1] 1 1 1 0
```

3. Determine if a success or failure

Because we used numeric signifiers for red and green, there is an easy test for whether or not we have drawn balls of all one color. If the numbers sum up to either 0 or 4, then we have a 'success'.

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> draw <- sample(x = urn, size = 4, replace = FALSE)
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[1] FALSE
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[1] TRUE
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[1] 0 0 0 0

> sum(draw) == 4
[1] FALSE
> sum(draw) == 0
[1] TRUE
```

But we can combined these test using '|' which means 'or'

```
> (sum(draw) == 4) | (sum(draw) == 0)
[1] TRUE
```

4. Throw a for-loop around it

Here's our guts so far:

```
draw <- sample(x = urn, size = 4, replace = FALSE)
success <- sum(draw) == 4 | sum(draw) == 0
```

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Here's our guts so far:

```
draw <- sample(x = urn, size = 4, replace = FALSE)
success <- sum(draw) == 4 | sum(draw) == 0
```

We can repeat this over and over:

```
sims <- 100000
success <- NULL
for(i in 1:sims){
  draw <- sample(x = urn, size = 4, replace = FALSE)
  success[i] <- sum(draw) == 4 | sum(draw) == 0
}
```

(4. Throw replicate around it)

for-loops in R are slow. To get fancy:

```
eval <- function(draw){
    success <- sum(draw) == 4 | sum(draw) == 0
    return(success)
}
```

(4. Throw replicate around it)

for-loops in R are slow. To get fancy:

```
eval <- function(draw){
    success <- sum(draw) == 4 | sum(draw) == 0
    return(success)
}
```

We can use replicate to make the loop faster:

```
sims <- 100000
success <- replicate(sims, eval(sample(urn, 4, replace=F)))
```

5. Determine proportion of success

Here are two equivalent approaches.

```
> sum(success)/sims
[1] 0.047
> mean(success)
[1] 0.047
```

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  Which is the standard normal density.
- How could we solve this?
  1. Sample 1000 points, $X_1, \ldots, X_{1000}$, uniformly distributed over the interval $(0, 1)$.
  2. Evaluate the function at each of these points and take the mean.

$$\frac{1}{1000}\left(\frac{1}{\sqrt{2\pi}}\right)\sum_{i=1}^{1000} e^{-\frac{x_i^2}{2}}$$