Multiple Equation Models & Amelia

Outline

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- The number of presidential vetoes and the number of Congressional overrides in any given year
- The number of hostile acts directed toward Israel and the number of hostile acts directed toward Palestinians
- The monthly unemployment rate in the United States and the monthly inflation rate.

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- Less restrictive in the assumptions it requires.
- May result in better estimates.

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- If not, you are probably ok using single equation models.

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- The vector of y's are jointly determined by the probability density function.

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- If you have stochastic and parametric independence, great. Just use a single equation model.
- But if you don't, then think about using a multiple equation model.

A Closer Look: Parametric vs. Stochastic Dependence

The model:

$$(Y_{1i}, Y_{2i}) \sim N(y_{1i}, y_{2i} | \mu_{1i}, \mu_{2i}, \sigma_1, \sigma_2, \sigma_{12})$$

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Are Y_{1i} and Y_{2i} either parametrically or stochastically dependent, or both?

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What would we have to restrict for these to be estimated as two separate regression?

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This is a "Seemingly Unrelated Regression Model"

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- Why use a multiple equations model?

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X_{i1}

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1170.6	97.8
2015.8	104.4
2803.3	118.0
2039.7	156.2
2256.2	172.6
2132.2	186.6

X_{i2}

Fw	Cw
191.5	1.8
516.0	0.8
729.0	7.4
560.4	18.1
519.9	23.5
628.5	26.5

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  - $\vec{Y_i}$ and $\vec{\mu_i}$ are $J \times 1$
  - Σ is $J \times J$
  - Σ is symmetric and full; Why? Stochastic Dependence
- Systematic Component

$$\mu_{i1} = X_{i1}\vec{\beta}$$
$$\mu_{i2} = X_{i2}\vec{\gamma}$$

  where
  - β: Estimates that predict GE mean

Multiple Equation Example

Let's analyze the data using the Seemingly Unrelated Regression Model (SURM). Here, our data are multivariate normal:

- Stochastic Component

$$\vec{Y}_i \sim N(\vec{y}_i | \vec{\mu}_i, \Sigma)$$

  where
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$$\mu_{i1} = X_{i1}\vec{\beta}$$
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  where
  - β: Estimates that predict GE mean
  - γ: Estimates that predict Westinghouse mean

Multiple Equation Example

- The Likelihood

Multiple Equation Example

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$$L(\mu, \Sigma) = \prod_{i=1}^{n} N(\vec{y}_i | \vec{\mu}_i, \Sigma)$$

Multiple Equation Example

- The Likelihood

$$
\begin{aligned}
L(\mu, \Sigma) &= \prod_{i=1}^{n} N(\vec{y_i} | \vec{\mu_i}, \Sigma) \\
&= \prod_{i=1}^{n} (2\pi)^{-\frac{J}{2}} |\Sigma|^{-\frac{1}{2}} exp\left[-\frac{1}{2}(\vec{y_i} - \vec{\mu_i})' \Sigma^{-1} (\vec{y_i} - \vec{\mu_i}) \right]
\end{aligned}
$$

Multiple Equation Example

- The Likelihood

$$\begin{aligned} L(\mu, \Sigma) &= \prod_{i=1}^{n} N(\vec{y_i}|\vec{\mu_i}, \Sigma) \\ &= \prod_{i=1}^{n} (2\pi)^{-\frac{J}{2}} |\Sigma|^{-\frac{1}{2}} exp\left[-\frac{1}{2}(\vec{y_i} - \vec{\mu_i})' \Sigma^{-1} (\vec{y_i} - \vec{\mu_i})\right] \end{aligned}$$

- What if Σ had off-diagonal elements that were zero? Stochastic Independence (so using independent models ok)

Multiple Equation Example

We can operationalize this model in `Zelig`

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```
data(grunfeld)
formula <- list(mu1 = Ige ~ Fge + Cge,
                mu2 = Iw ~ Fw + Cw)
```

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```
data(grunfeld)
formula <- list(mu1 = Ige ~ Fge + Cge,
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z.out <- zelig(formula = formula,
            model = "sur", data = grunfeld)
```

Multiple Equation Example

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```
z.out <- zelig(formula = formula,
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z.out

systemfit results
method: SUR

Coefficients:
mu1_(Intercept)         mu1_Fge         mu1_Cge mu2_(Intercept)         mu
    -27.7193171       0.0383102       0.1390363      -1.2519882       0.057
```

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mu1_(Intercept)         mu1_Fge         mu1_Cge mu2_(Intercept)         mu
    -27.7193171       0.0383102       0.1390363      -1.2519882       0.057
```

and then we can use the usual tricks to interpret this

Multiple Equation Example

Are the μ's correlated?

Multiple Equation Example

Are the μ's correlated?

```
summary(z.out)
```

⋮

```
The correlations of the residuals
         mu1      mu2
mu1 1.000000 0.765043
mu2 0.765043 1.000000
```

⋮

Multiple Equation Example

```
x.out <- setx(z.out, x=list(Fge=0, Fw = 800))
```

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```
x.out <- setx(z.out, x=list(Fge=0, Fw = 800))

s.out <- sim(z.out, x=x.out)
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  Model: sur
  Number of simulations: 1000

Values of X
  (Intercept)      Fge    Cge    Fw  Cw
1           1 1941.325 400.16 191.5 1.8

Expected Values: E(Y|X)
       mean        sd       2.5%     97.5%
1 -54611.349 55970.197 -158049.19 50144.240
2  -5388.513  5521.624  -15592.78  4945.616

Predicted Values: Y|X
       mean        sd       2.5%     97.5%
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2  -5388.513  5521.624  -15592.78  4945.616
```

Multiple Equation Example

```
plot(s.out)
```

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```
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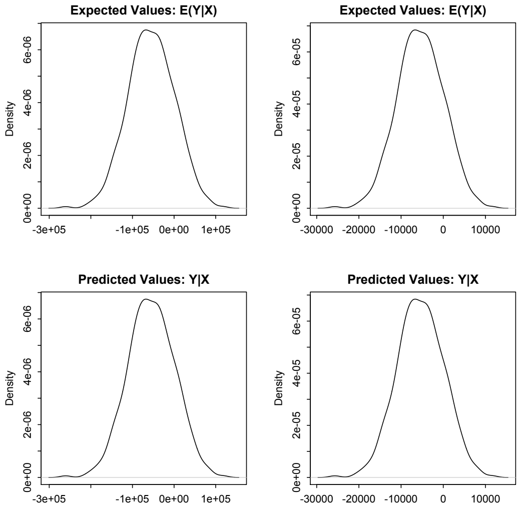
```



Outline

Multinomial models

Multinomial models

Suppose you have a choice among different, non-ordered things:

Multinomial models

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- in what to wear – a blue, yellow, or green sweater

Multinomial models

Suppose you have a choice among different, non-ordered things:

- Democrat, Republican, Independent
- Republican candidates for president
- at a restaurant between beef, chicken, or vegetarian
- in what to wear – a blue, yellow, or green sweater

The choice sets are not ordered. But we can use generalizations of ordered logit and ordered probit to analyze.

Multinomial models

We'll use a running example of vote choice between three candidates.

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Why is this a Multiple Equation Model?

Multinomial Logit

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In our example, $\vec{\pi}_i = (\pi_{i1}, \pi_{i2}, \pi_{i3})$. We can write the systematic component as

$$\pi_{ij} = \frac{exp(X_i\beta_j)}{\sum_{k=1}^{3} exp(X_i\beta_k)}$$

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We can operationalize this in `R` using `Zelig` or the `mlogit` package.

Mutlinomial Logit

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```
z.out <- zelig(as.factor(vote88) ~ age + female,
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```

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```

```
z.out
Coefficients:
(Intercept):1 (Intercept):2         age:1         age:2      female:1
  0.104832885  -0.500933247   0.016842559   0.008101289   0.265989231    0.2
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```

As usual, this is a bit difficult to interpret...

Multinomial Logit

...so we can look to first differences

```
x.young <- setx(z.out, age = min(mexico$age))
```

Multinomial Logit

...so we can look to first differences

```
x.young <- setx(z.out, age = min(mexico$age))

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x.old<- setx(z.out, age = max(mexico$age))

s.out <- sim(z.out, x1 = x.old, x = x.young)

summary(s.out)
First Differences: Pr(Y=k|X1) - Pr(Y=k|X)
              mean         sd        2.5%       97.5%
Pr(Y=1)  0.1831760 0.06264429  0.0618293  0.30277899
Pr(Y=2) -0.0406195 0.05047512 -0.1350517  0.06689080
Pr(Y=3) -0.1425565 0.04747605 -0.2322232 -0.04139632
```
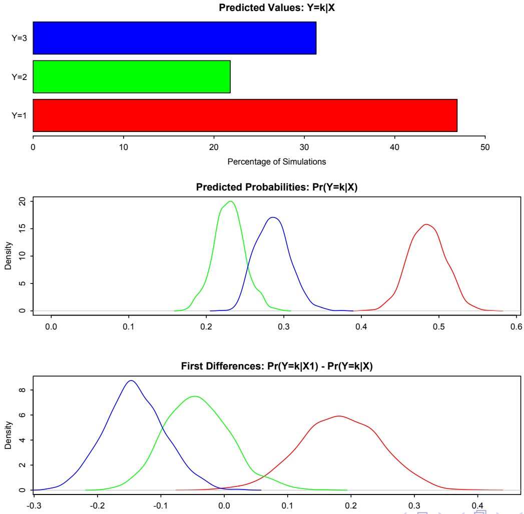

Multinomial Logit

```
plot(s.out)
```



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Multinomial Probit

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- If you think you may be violating IIA, then use a multinomial probit, which relaxes the IIA assumption.

Multinomial Probit

- If you think you may be violating IIA, then use a multinomial probit, which relaxes the IIA assumption.
- Usually more difficult to compute, though, and no `Zelig` function :(

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where Σ is the 3×3 var-cov matrix.

The voter will vote for whoever gives her the highest utility, so we have the following observation mechanism

$$V_{ij} = \begin{cases} 1 & \text{if } U_{ij}^* > U_{ij'}^* \forall j \neq j' \\ 0 & \text{otherwise.} \end{cases}$$

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And now we can use the same stochastic component

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$$\sum_{j=1}^{3} \pi_{ik} = 1$$

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$$Pr(U^*_{i3} > U^*_{ij} \; \forall j \neq 3) = \int_{-\infty}^{\infty} \int_{-\infty}^{U_{i3}} \int_{-\infty}^{U_{i3}} f(U_1, U_2, U_3) dU_1 dU_2 dU_3.$$

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- We no longer need to assume IIA because we have included other options directly in this integral.

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- We no longer need to assume IIA because we have included other options directly in this integral.
- However, this is hard to identify because of potential correlations between unobserved latent dimensions.
- Note: Imai and Van Dyck's `MNP` package will calculate multinomial probit using Bayesian methods.

Outline

Multiple Equation Models and Missing Data: A Preview

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This could be our data matrix:

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Observation	*Age*	*Education*	*Income*
1	13	4	\$0
2	56	32	\$100,000
3	24	15	\$ 30,000
4	100	12	\$ 20,000
⋮	⋮	⋮	⋮

Multiple Equation Models and Missing Data: A Preview

Wait, it's a multiple equation model!

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$\vdots$	$\vdots$	$\vdots$	$\vdots$

We could model this as:

$$\mu_{i,Age} \qquad \mu_{i,Education} \qquad \mu_{i,Income}$$

Multiple Equation Models and Missing Data: A Preview

Wait, it's a multiple equation model!

Observation	*Age*	*Education*	*Income*
$Y_{1j} =$	(13	4	\$0)
$Y_{2j} =$	(56	32	\$100,000)
$Y_{3j} =$	(24	15	\$ 30,000)
$Y_{4j} =$	(100	12	\$ 20,000)
$\vdots$	$\vdots$	$\vdots$	$\vdots$

We could model this as:

$$\mu_{i,Age} \qquad \mu_{i,Education} \qquad \mu_{i,Income}$$

$$\Rightarrow \vec{\mu_i},\ \Sigma$$

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- As usual, we can make μ a function of some covariates (what predicts our data?).

Multiple Equation Models and Missing Data: A Preview

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$$
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L(\mu, \Sigma | D_{obs}) &\propto \prod_{i=1}^{n} \int N(D_i | \mu, \Sigma) dD_{mis} \\
&= \prod_{i=1}^{n} N(D_{i,obs} | \mu_{obs}, \Sigma_{obs})
\end{aligned}
$$

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- Then we can maximize the likelihood to find the parameters of interest.
- From there we'll simulate, just like we've done before to estimate the missing data.

Outline

Basic Syntax

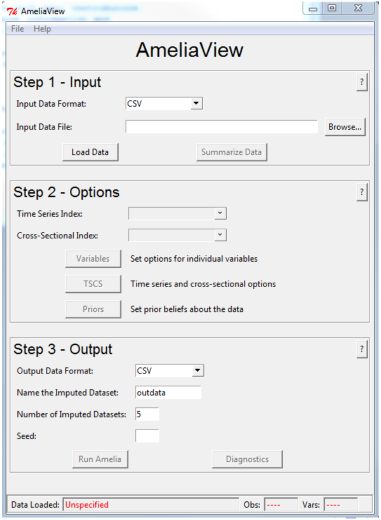
```
library(Amelia)
data(africa)

> africa[1,]
  year      country gdp_pc  infl trade civlib population
1 1972 Burkina Faso    377 -2.92 29.69    0.5    5848380

a.out <- amelia(x = africa, cs = ‘‘country’’, ts = ‘‘year’’,
  logs = ‘‘gdp_pc’’, m = 5)

write.amelia(obj=a.out, file.stem = ‘‘a.out’’)
```

`a.out` is a list of 5 imputed datasets, each of which can be accessed using `a.out$imputations[[i]]`.

GUI

You can use the GUI by typing:

```
AmeliaView()
```



Basic Syntax

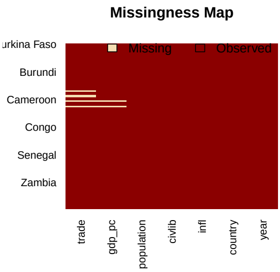
```
z.out.imp <- zelig(trade ~ log(population) + log(gdp_pc) + infl
  + civlib, data = a.out$imputations, model = "ls")
summary(z.out.imp)

Coefficients:
                   Value Std. Error t-stat   p-value
(Intercept)     112.4097   45.47779  2.472 1.345e-02
log(population) -17.8513    2.36646 -7.543 4.595e-14
log(gdp_pc)      31.3875    2.31108 13.581 1.834e-41
infl              0.2605    0.05836  4.463 8.075e-06
civlib           27.2104    6.67210  4.078 4.59$e-05
```

`Zelig` will automatically combine the results of the different models, but if a model you are using isn't programmed in `Zelig`, it isn't hard to combine your estimates using the equations I showed you before.

Diagnostics

The missingness map gives an overall sense of the shape and extent of the missingness.

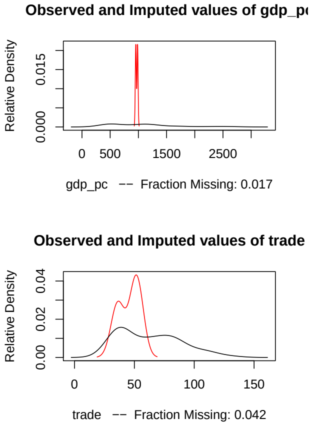
```
missmap(a.out)
```



Diagnostics

Plotting the Amelia object contrasts empirical and imputed densities.

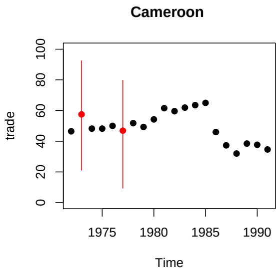
```
plot(a.out)
```



Diagnostics

Plotting Time-series cross-sectional plots

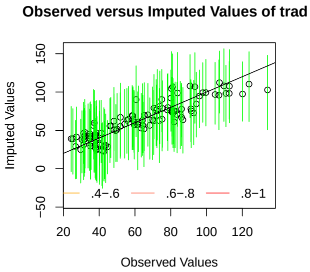
```
tscsPlot(a.out, var = "trade", cs = "Burundi")
```



Diagnostics

Overimputation for a specific variable tests the imputation model by imagining that each observation is missing and generating some imputations to check performance.

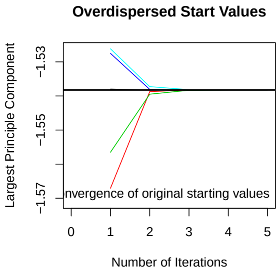
```
overimpute(a.out, var = ``trade'')
```



Diagnostics

The disperse function starts the algorithm at some unlikely values to check that AMELIA II hasn't found a local rather a global maximum for the likelihood of the complete data.

```
disperse(a.out, dims = 1, m = 5)
```



Considerations: Transformations

Recall that our model assumes multivariate normal data. This suggests some issues:

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Recall that our model assumes multivariate normal data. This suggests some issues:

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2. Nominal (unordered) variables: AMELIA II automatically converts into factors and imputes accordingly if a variable is passed to the `noms` argument.
3. Various transformations (logarithmic, root) are pre-programmed to make skewed distributions more normal.

Considerations: Time Series/Panel Data

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Considerations: Time Series/Panel Data

1. Use the `ts` and `cs` arguments to tell AMELIA II the panel structure. It uses this information for constructing time serieses correctly.
2. `polytime` can be used to add in polynomial time trends (up to a cubic) and `intercs = TRUE` creates a separate trend for each panel.
3. Add in lags and leads if you think it will help predict the missing data. Use `lags` and `leads` with some variable.

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3. Don't impute things that don't make sense.
4. Check diagnostics (and think carefully about applicability).
5. Remember transformations, polynomials and data structure.

Gary's Papers

King, Gary; James Honaker; Ann Joseph; Kenneth Scheve. 2001. "Analyzing Incomplete Political Science Data: An Alternative Algorithm for Multiple Imputation," *APSR* 95, 1 (March, 2001): 49-69.

James Honaker and Gary King. "What to do about Missing Values in Time Series Cross-Section Data," *AJPS* 54, 2 (April, 2010): 561-581

Amelia II: A Program for Missing Data