

# Advanced Quantitative Research Methodology, Lecture

## Notes: Robust Standard Errors<sup>1</sup>

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- **Are:** A good test for misspecification

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- A better use for RSEs: as a test for misspecification

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Variance matrix unconstrained (includes covariances)

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Still allows for heteroskedasticity

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Standard linear-normal regression model assumptions

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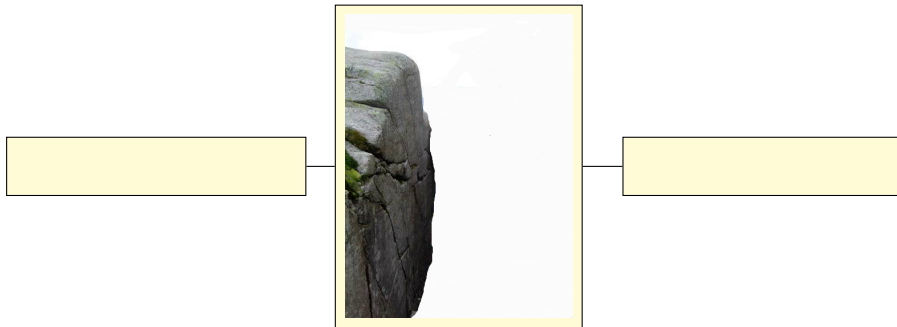
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  - The idea generalizes to any MLE model

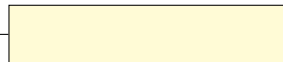
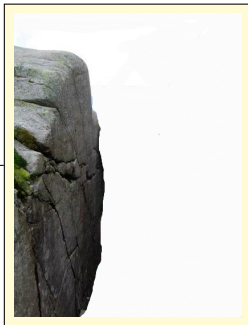
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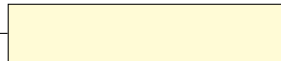
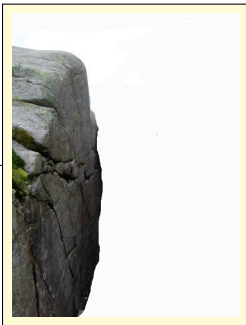
Model Misspecified



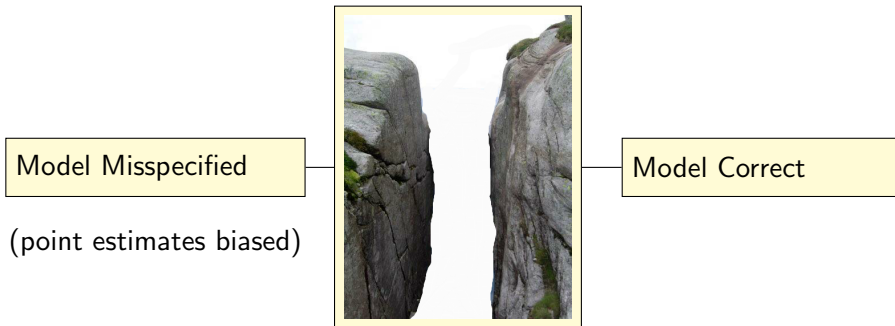
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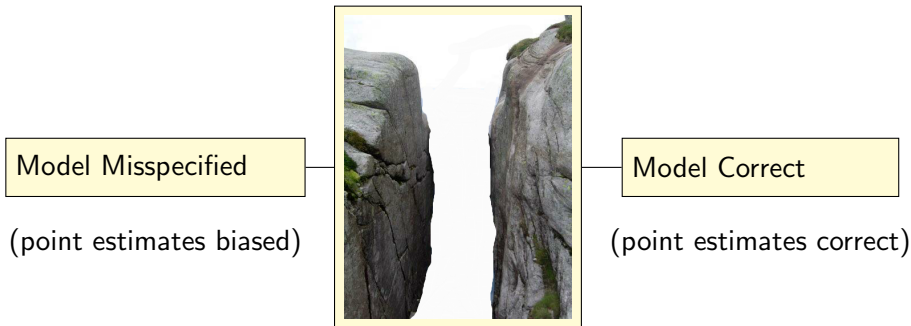
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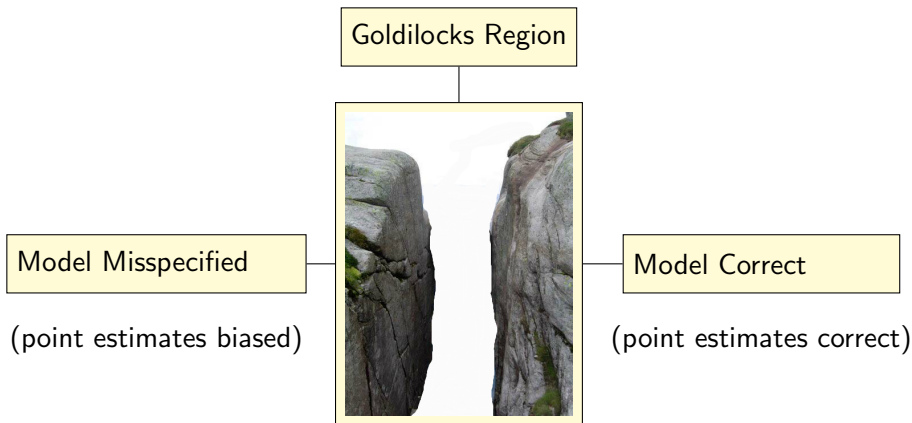
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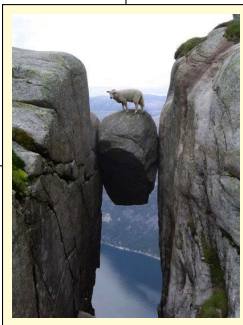


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Goldilocks Region



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(point estimates biased)

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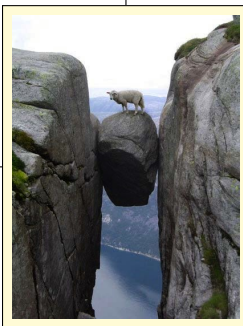
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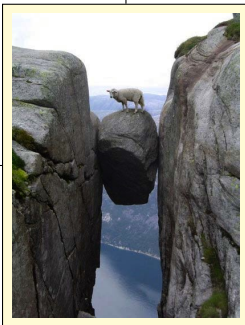
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- If  $SE \neq RSE$ : find misspecification, fix model, rerun until  $SE = RSE$

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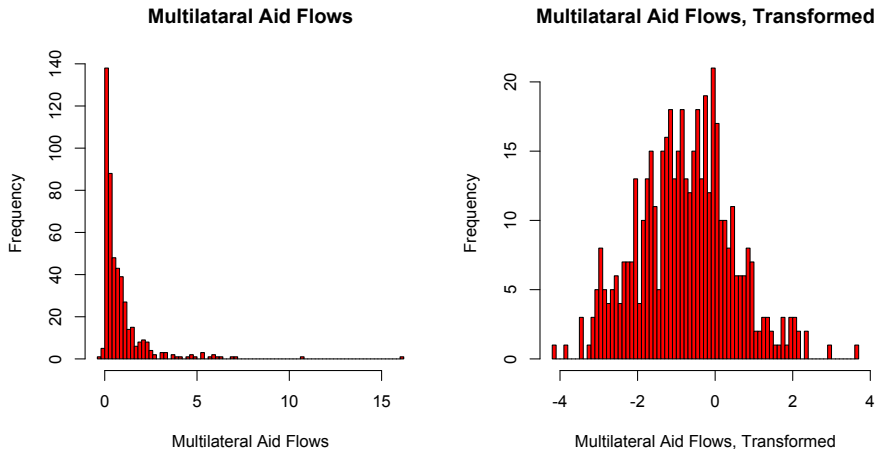
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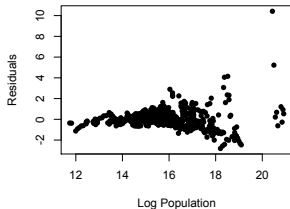
# Transformation of $Y$ : Becomes Normal



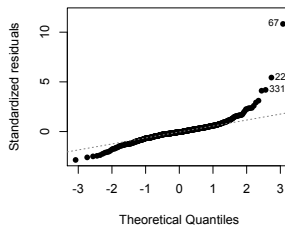
**Figure:** Distribution of the dependent variable before (left) and after (right) the Box-Cox transformation.

# Transformation of $Y$ : Removes Heteroskedasticity

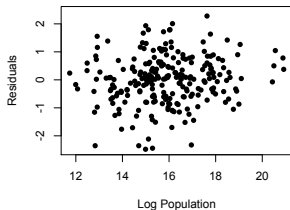
Population vs Residuals, Author's Model



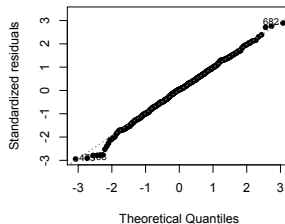
Q-Q Plot, Author's Model



Population vs Residuals, Altered Model



Q-Q Plot, Altered Model



# Results: Transformed Model, Opposite Results

The misspecification  $\rightsquigarrow$  bias, not inefficiency

