# Advanced Quantitative Research Methodology, Lecture Notes: Matching Methods for Causal Inference[1]

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- Solution: Other methods do not share this problem
- (Coarsened Exact Matching is simple, easy, and powerful)
- $\rightsquigarrow$ Lots of insights revealed in the process

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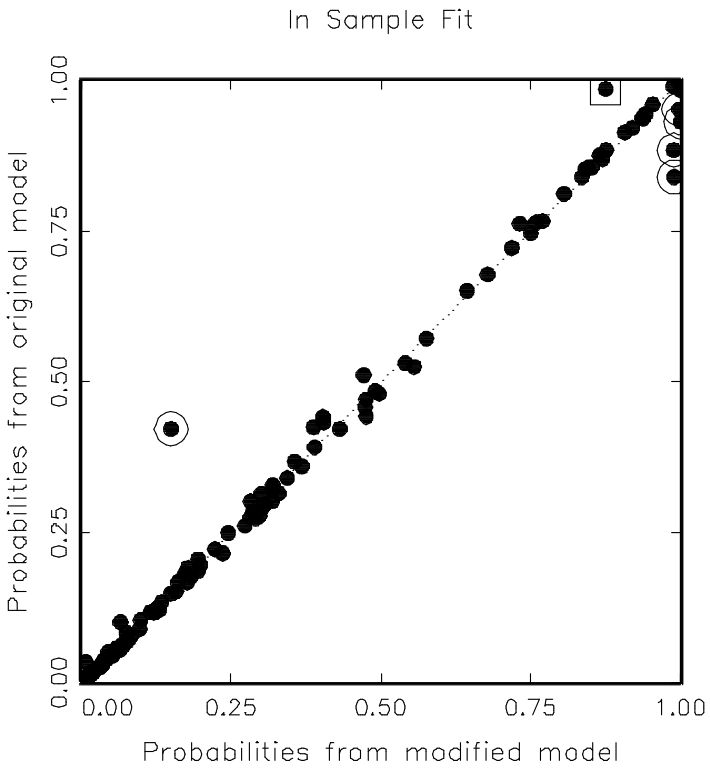
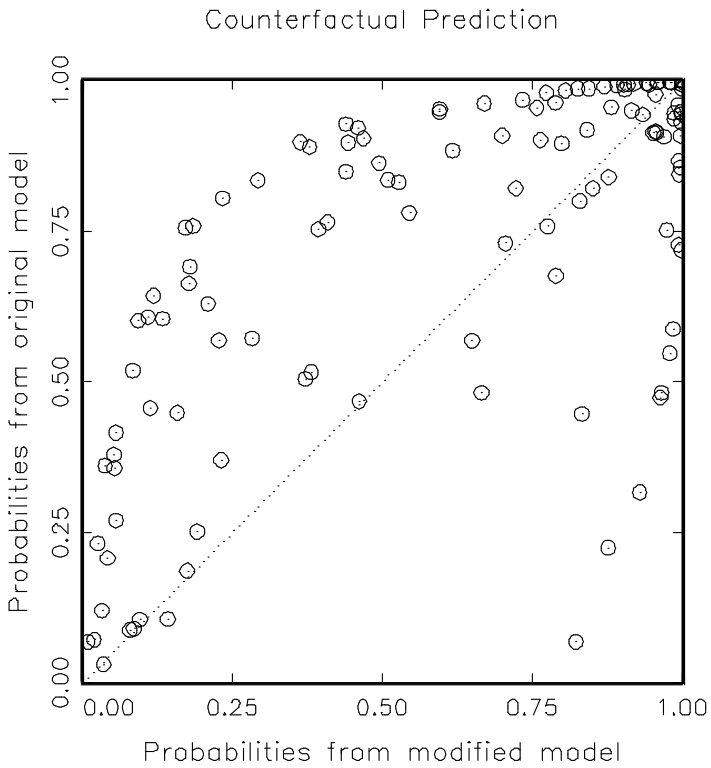
- Counterfactual question: UN intervention switched for each war
- Data analysis: Logit model
- The question: How *model dependent* are the results?

# Two Logit Models, Apparently Similar Results

| | Original "Interactive" Model | | | Modified Model | | |
|---|---|---|---|---|---|---|
| Variables | Coeff | SE | P-val | Coeff | SE | P-val |
| Wartype | −1.742 | .609 | .004 | −1.666 | .606 | .006 |
| Logdead | −.445 | .126 | .000 | −.437 | .125 | .000 |
| Wardur | .006 | .006 | .258 | .006 | .006 | .342 |
| Factnum | −1.259 | .703 | .073 | −1.045 | .899 | .245 |
| Factnum2 | .062 | .065 | .346 | .032 | .104 | .756 |
| Trnsfcap | .004 | .002 | .010 | .004 | .002 | .017 |
| Develop | .001 | .000 | .065 | .001 | .000 | .068 |
| Exp | −6.016 | 3.071 | .050 | −6.215 | 3.065 | .043 |
| Decade | −.299 | .169 | .077 | −0.284 | .169 | .093 |
| Treaty | 2.124 | .821 | .010 | 2.126 | .802 | .008 |
| UNOP4 | 3.135 | 1.091 | .004 | .262 | 1.392 | .851 |
| Wardur*UNOP4 | — | — | — | .037 | .011 | .001 |
| Constant | 8.609 | 2.157 | 0.000 | 7.978 | 2.350 | .000 |
| N | | 122 | | | 122 | |
| Log-likelihood | | -45.649 | | | -44.902 | |
| Pseudo $R^2$ | | .423 | | | .433 | |

# Doyle and Sambanis: Model Dependence



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  - If treated and control groups are better balanced than when you started, due to pruning, model dependence is reduced

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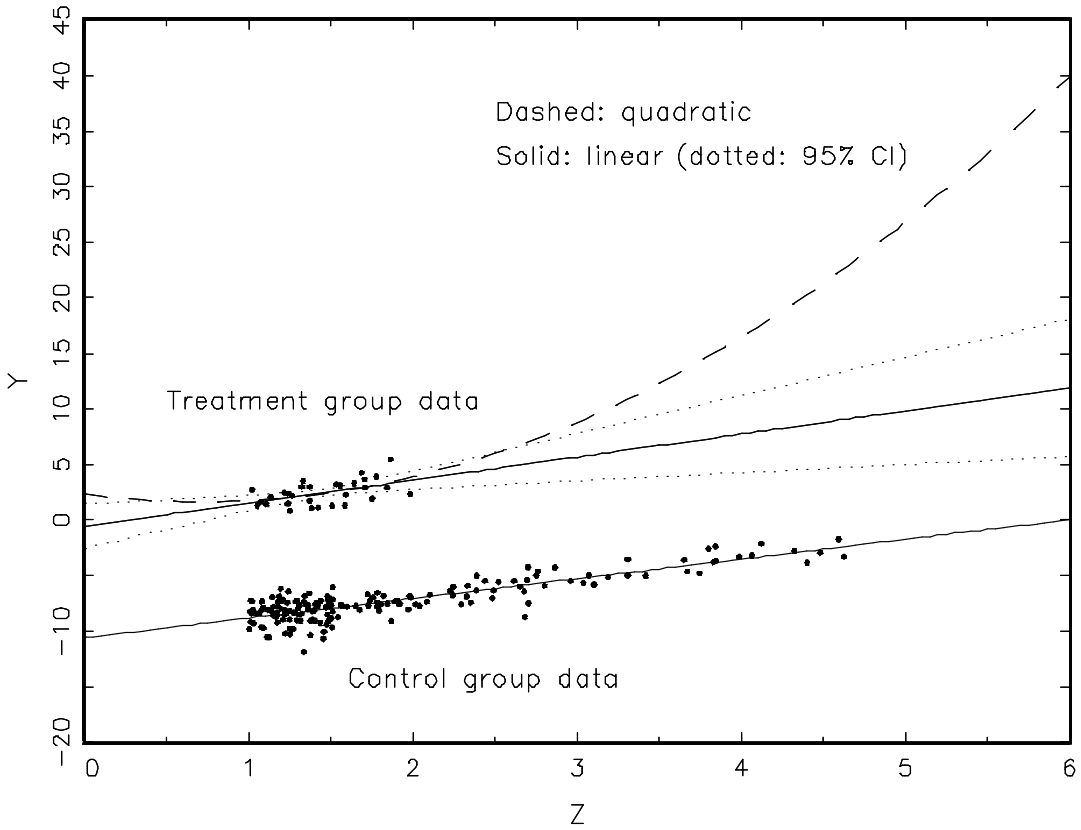

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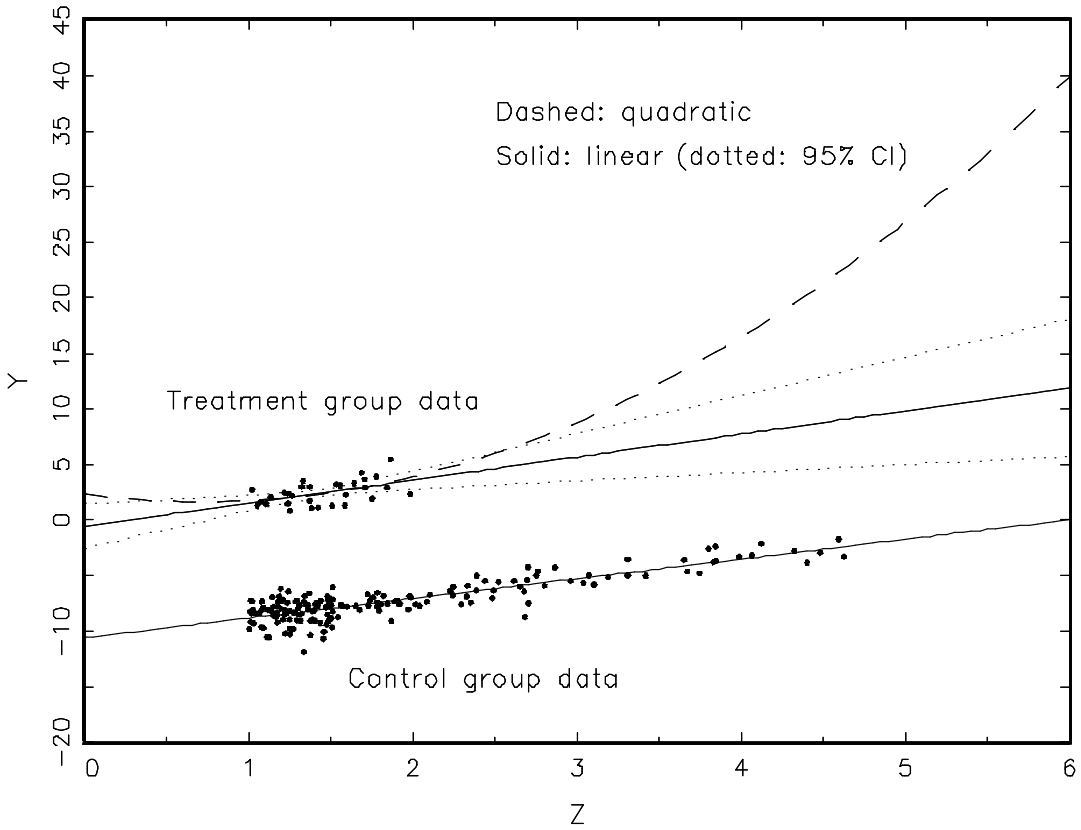


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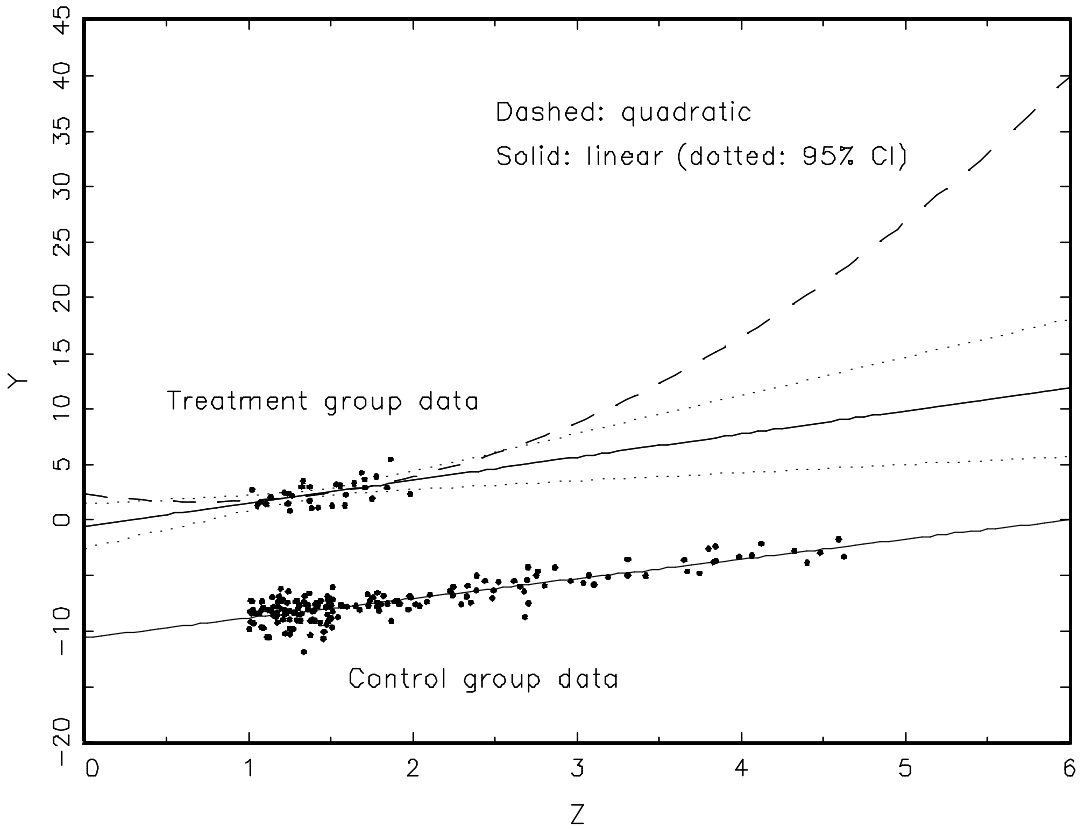


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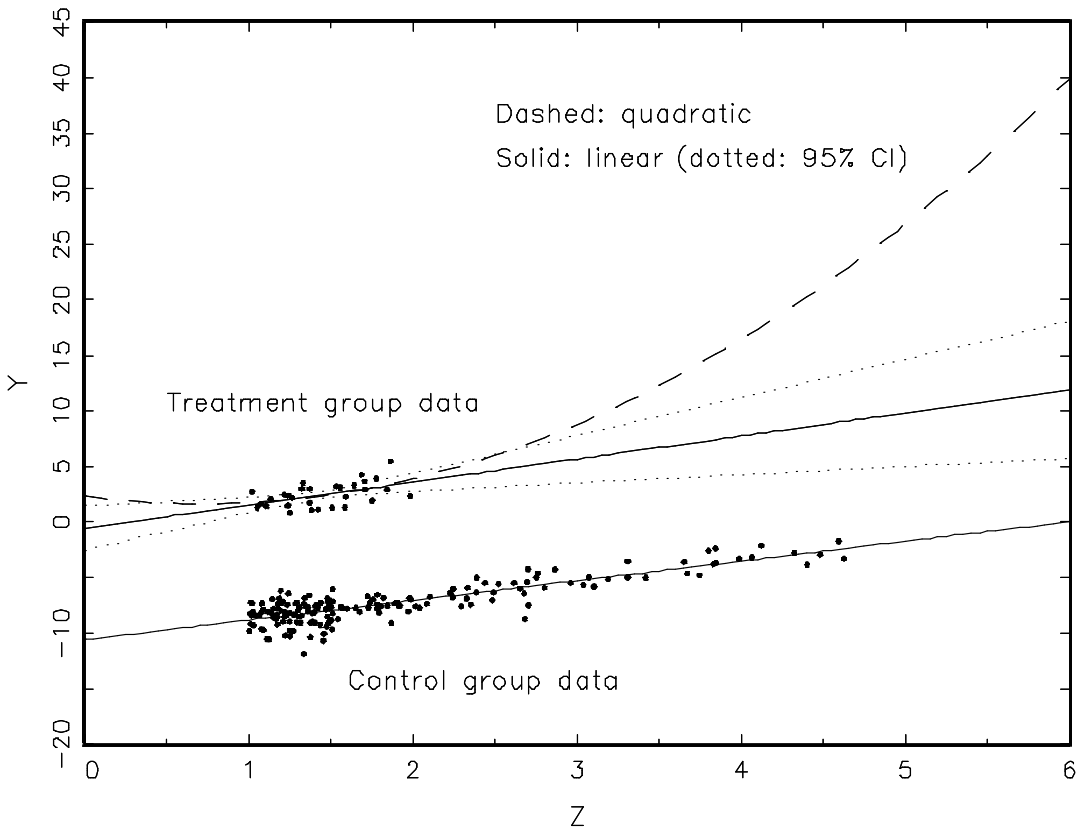


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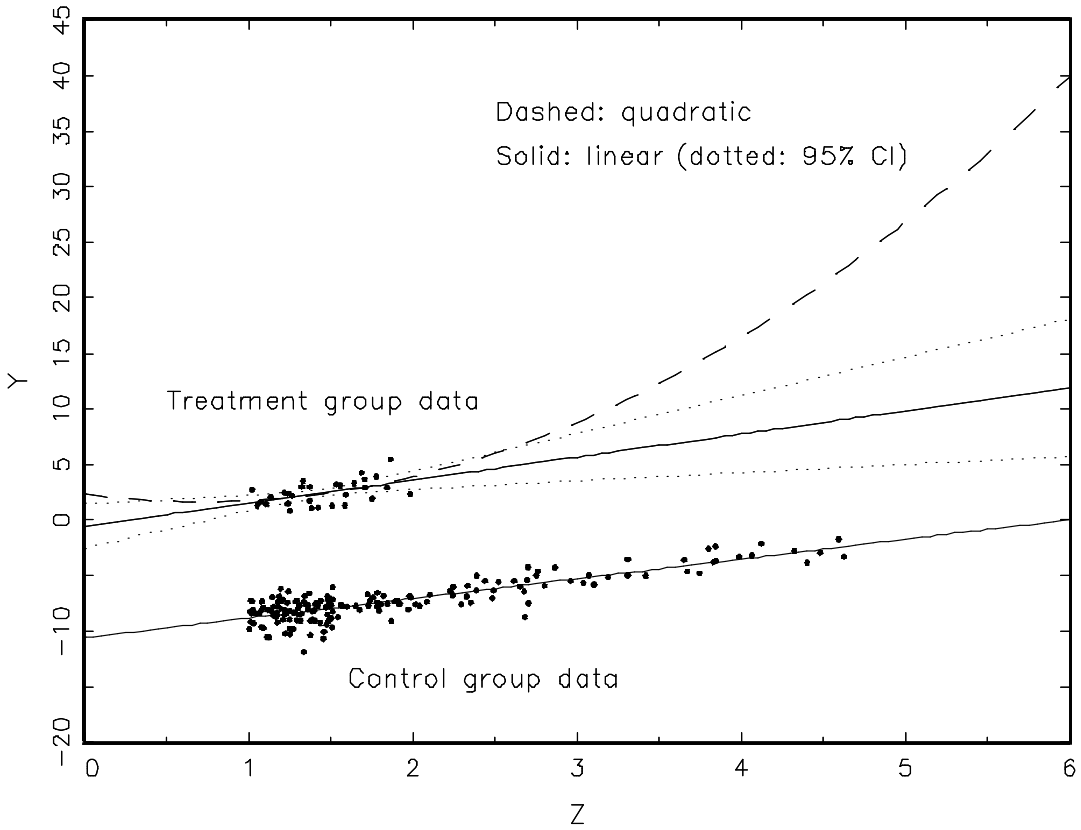


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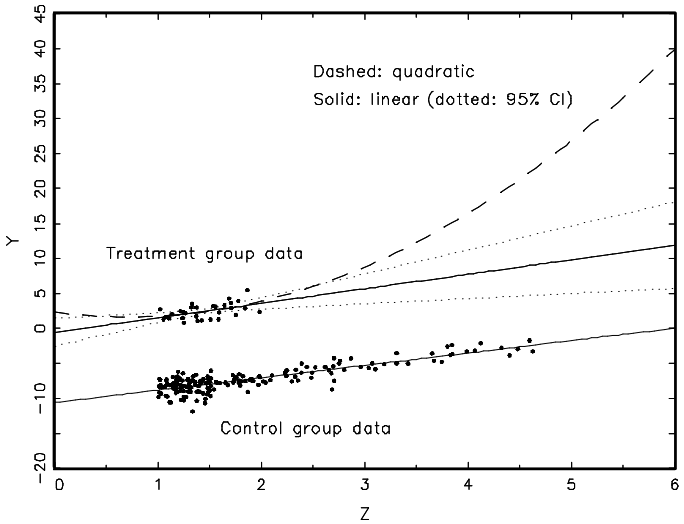

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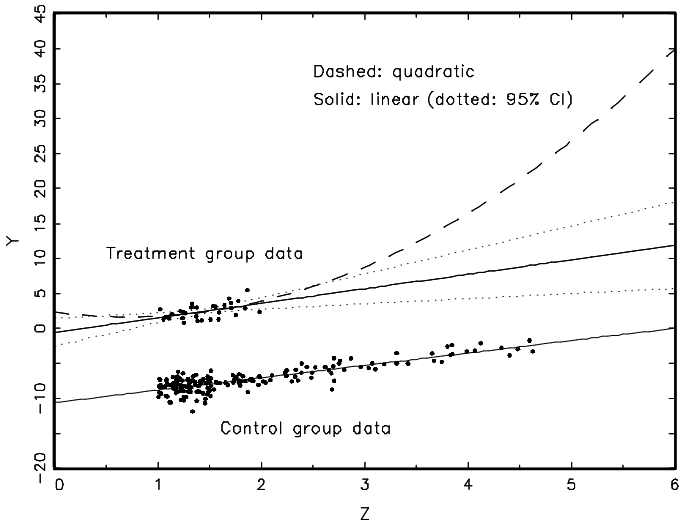


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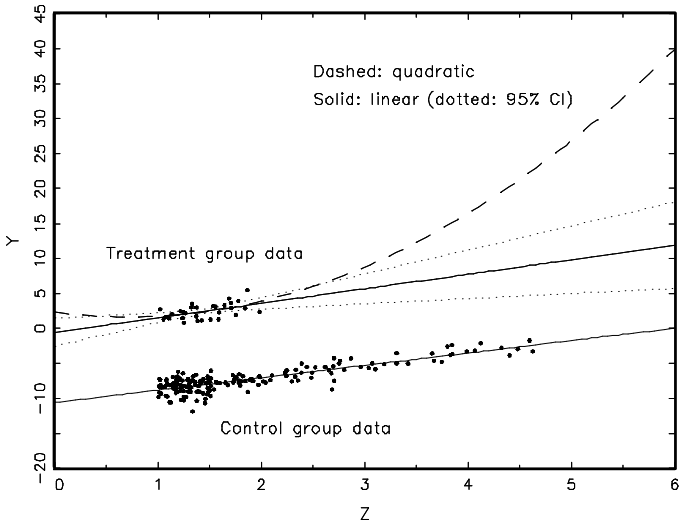


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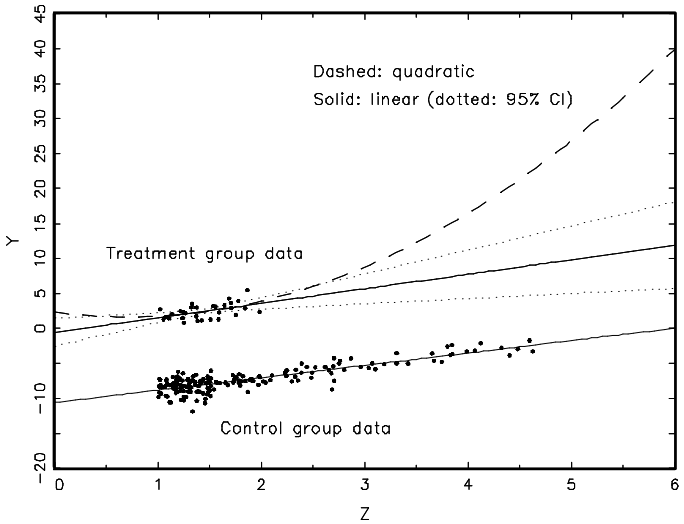


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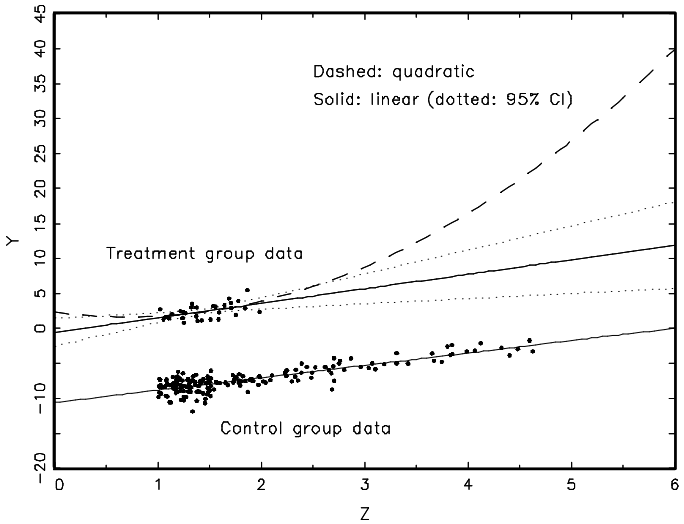


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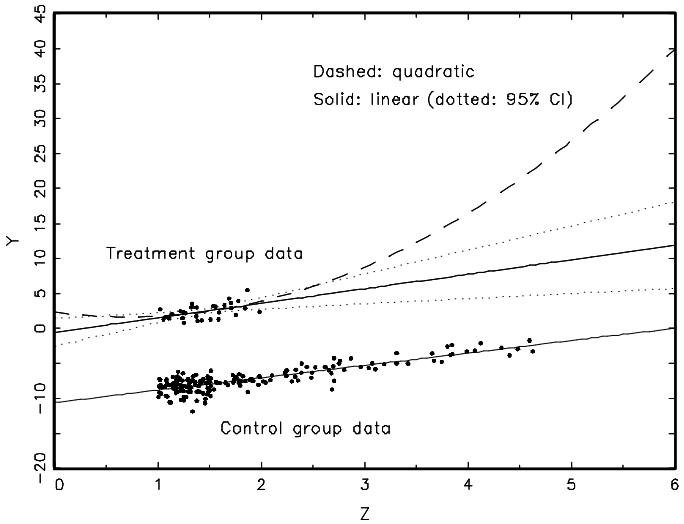


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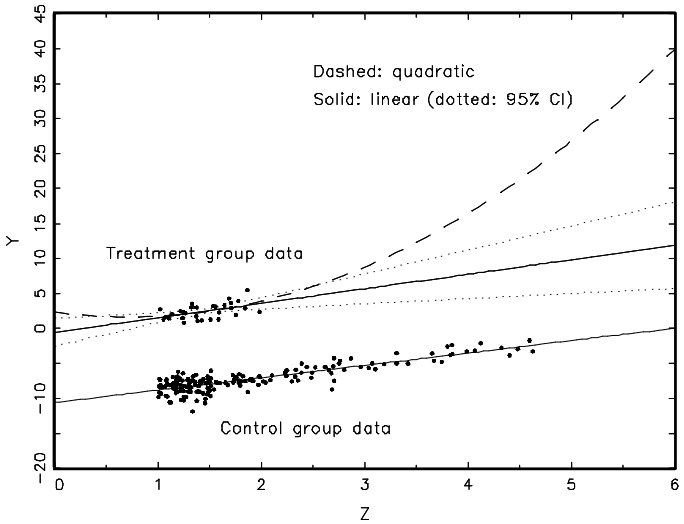


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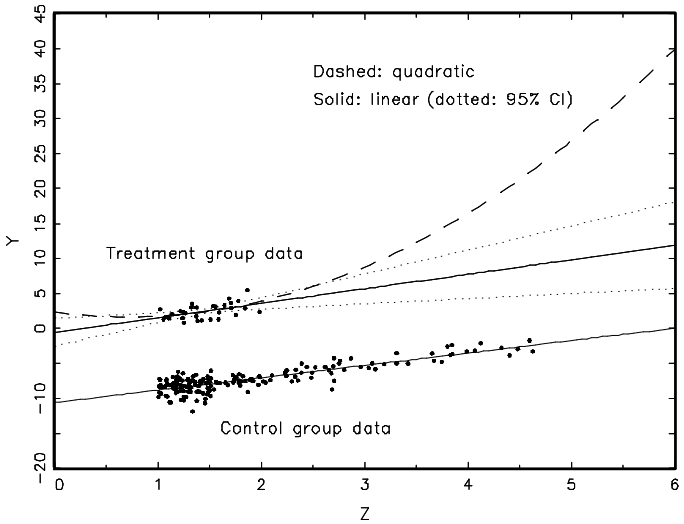


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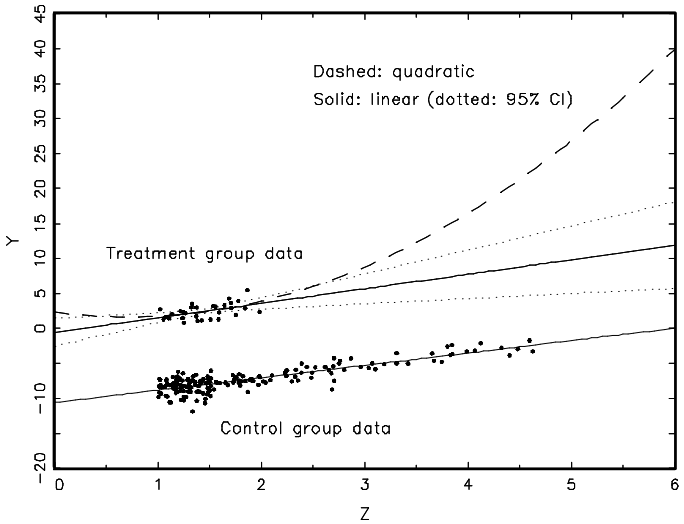


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  4. Easiest: Coarsened Exact Matching, no separate step needed

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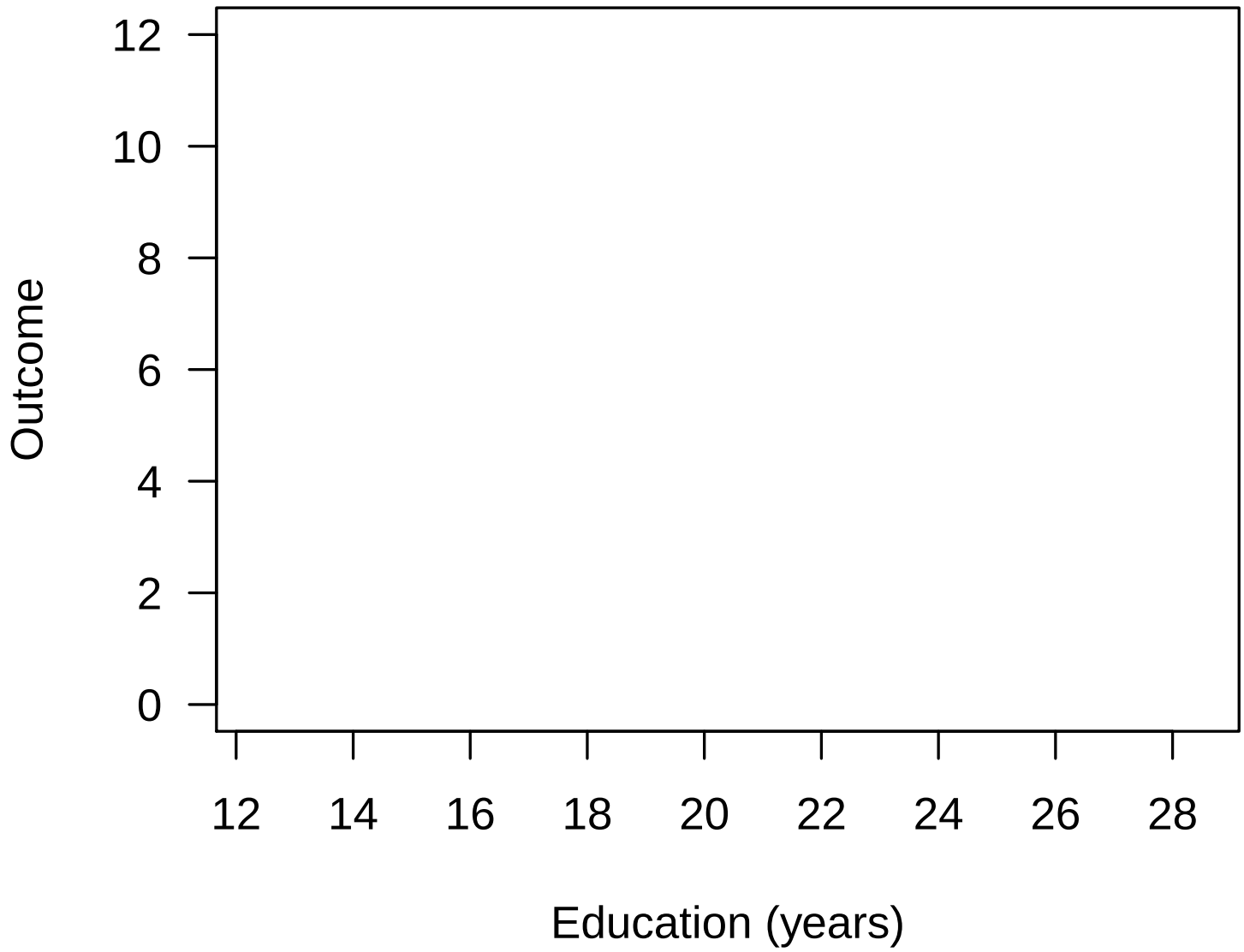
(Ho, Imai, King, Stuart, 2007: fig.1, *Political Analysis*)



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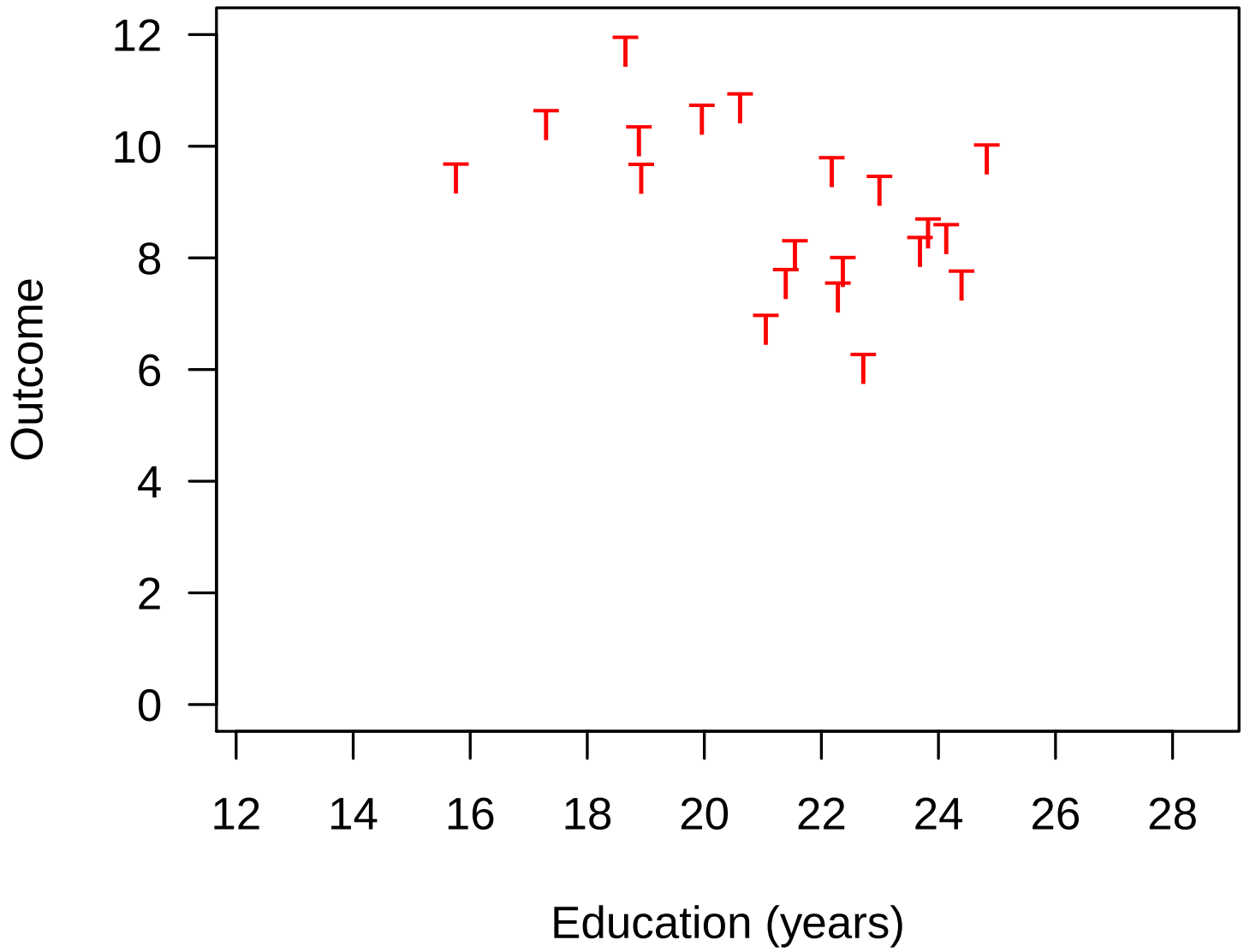

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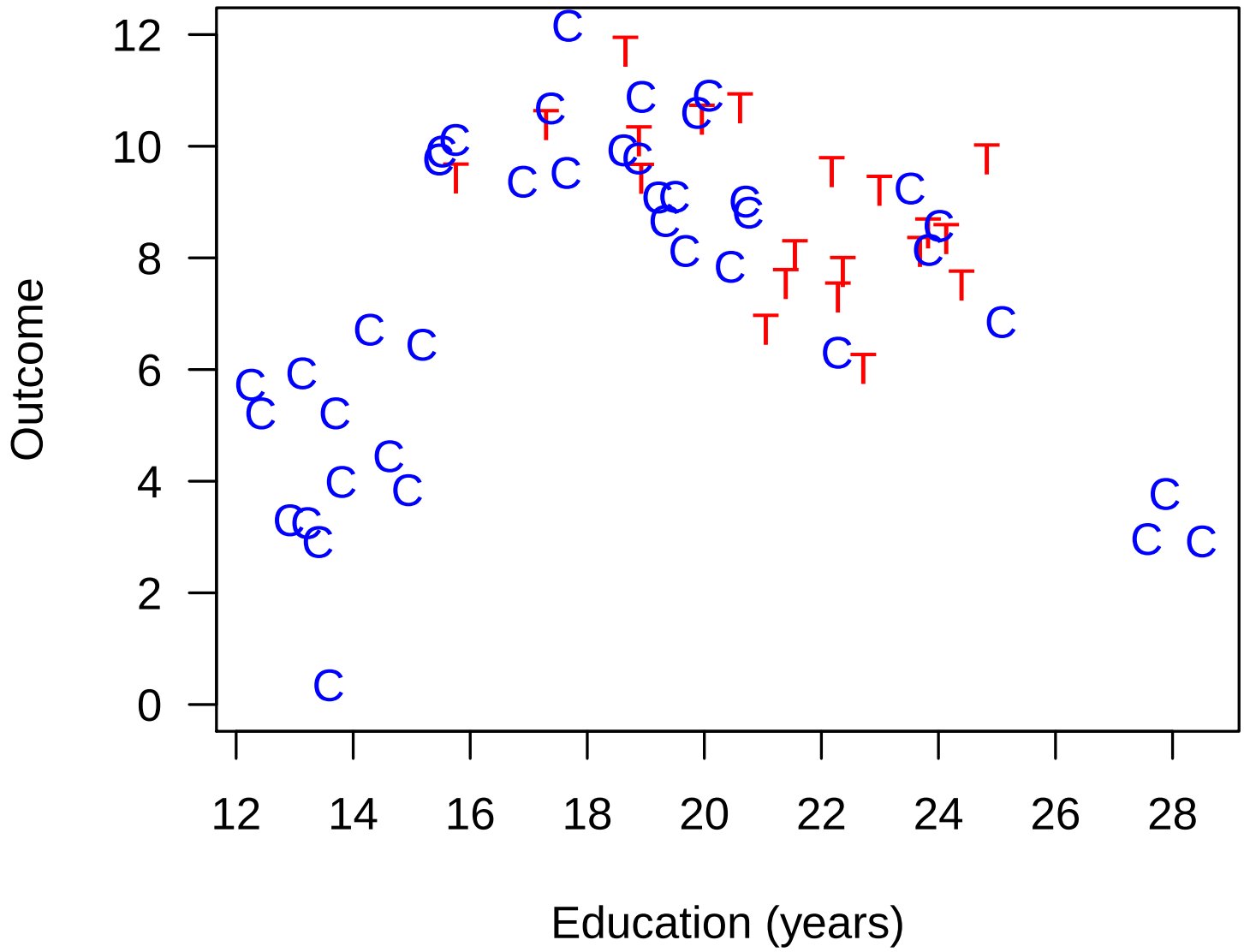

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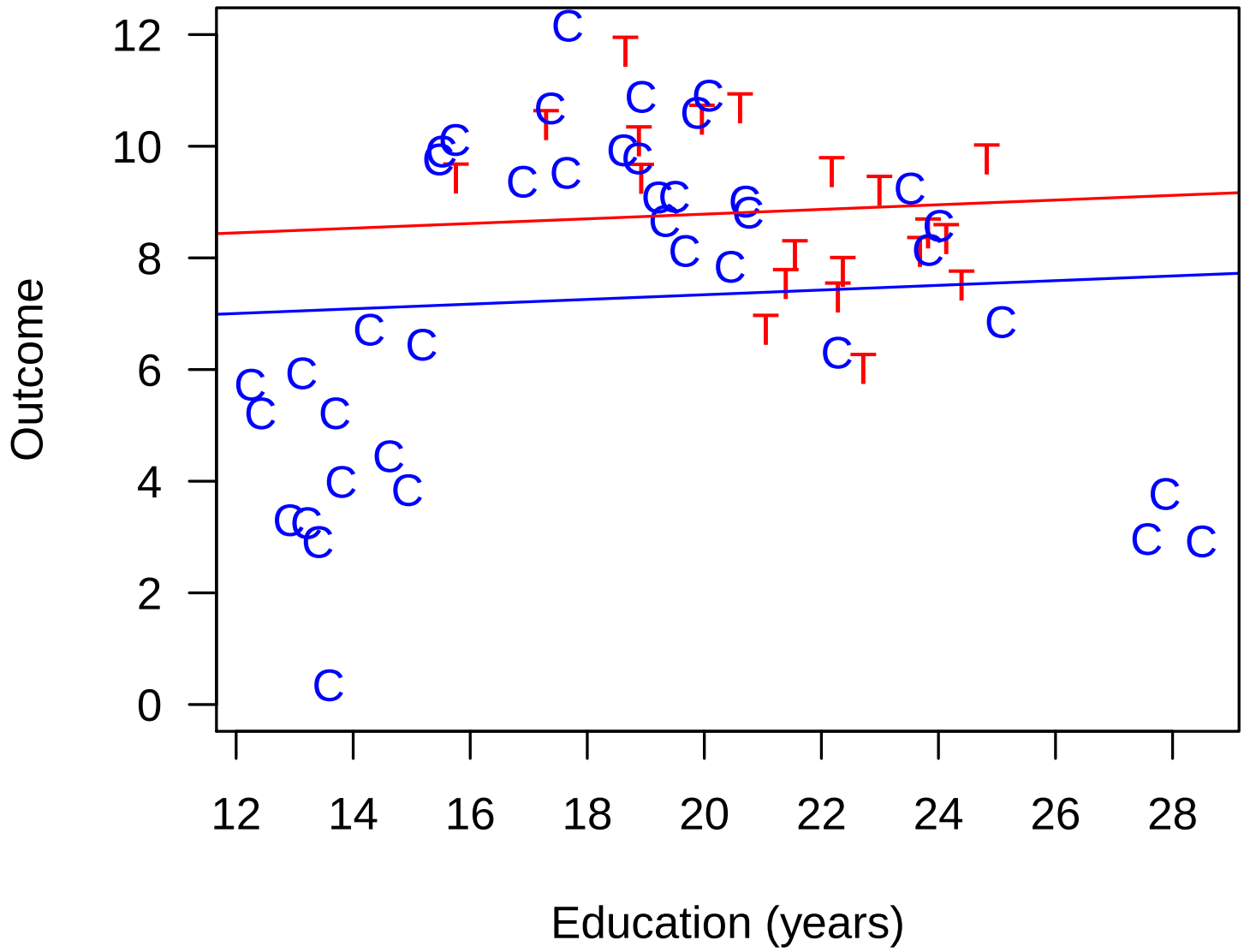

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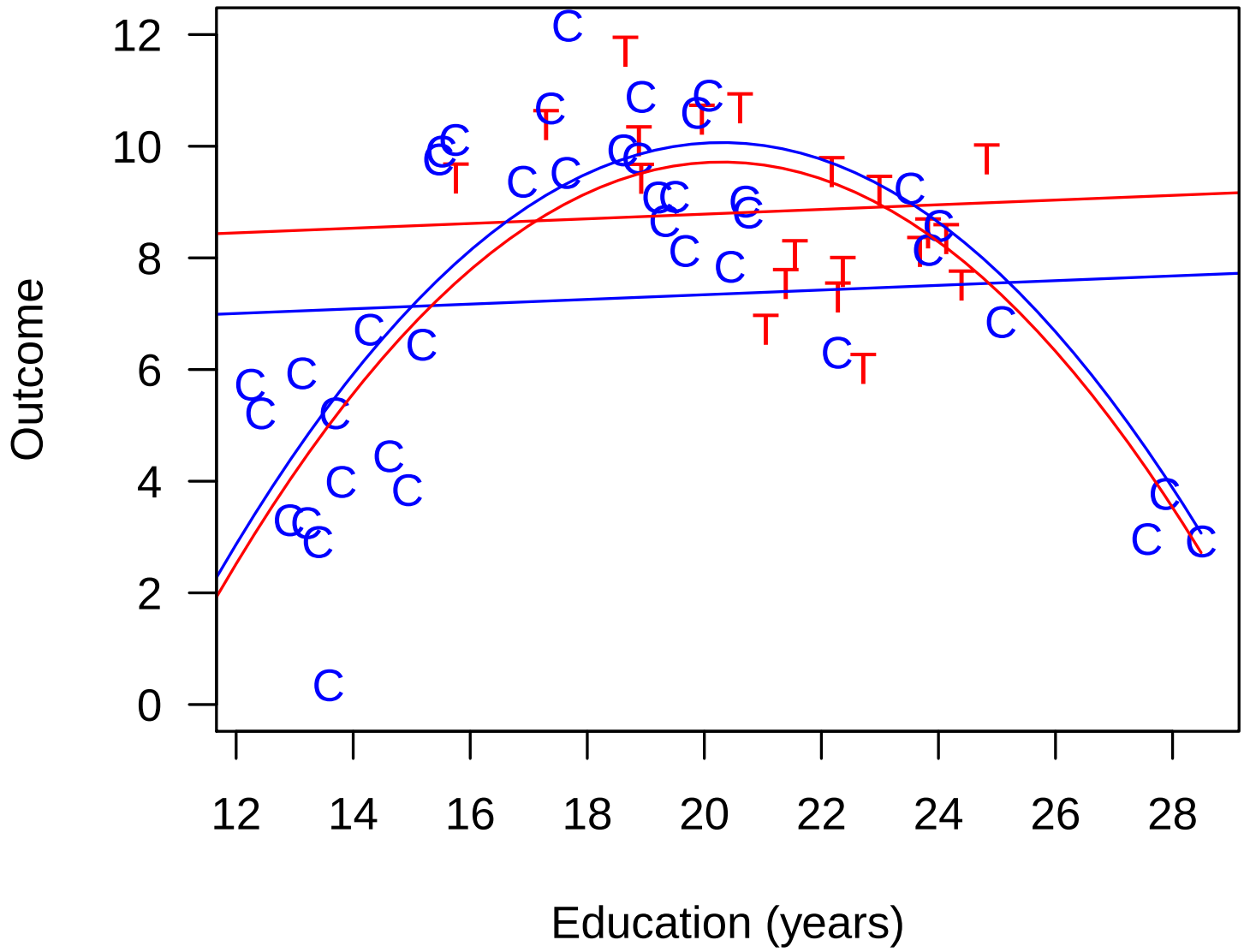

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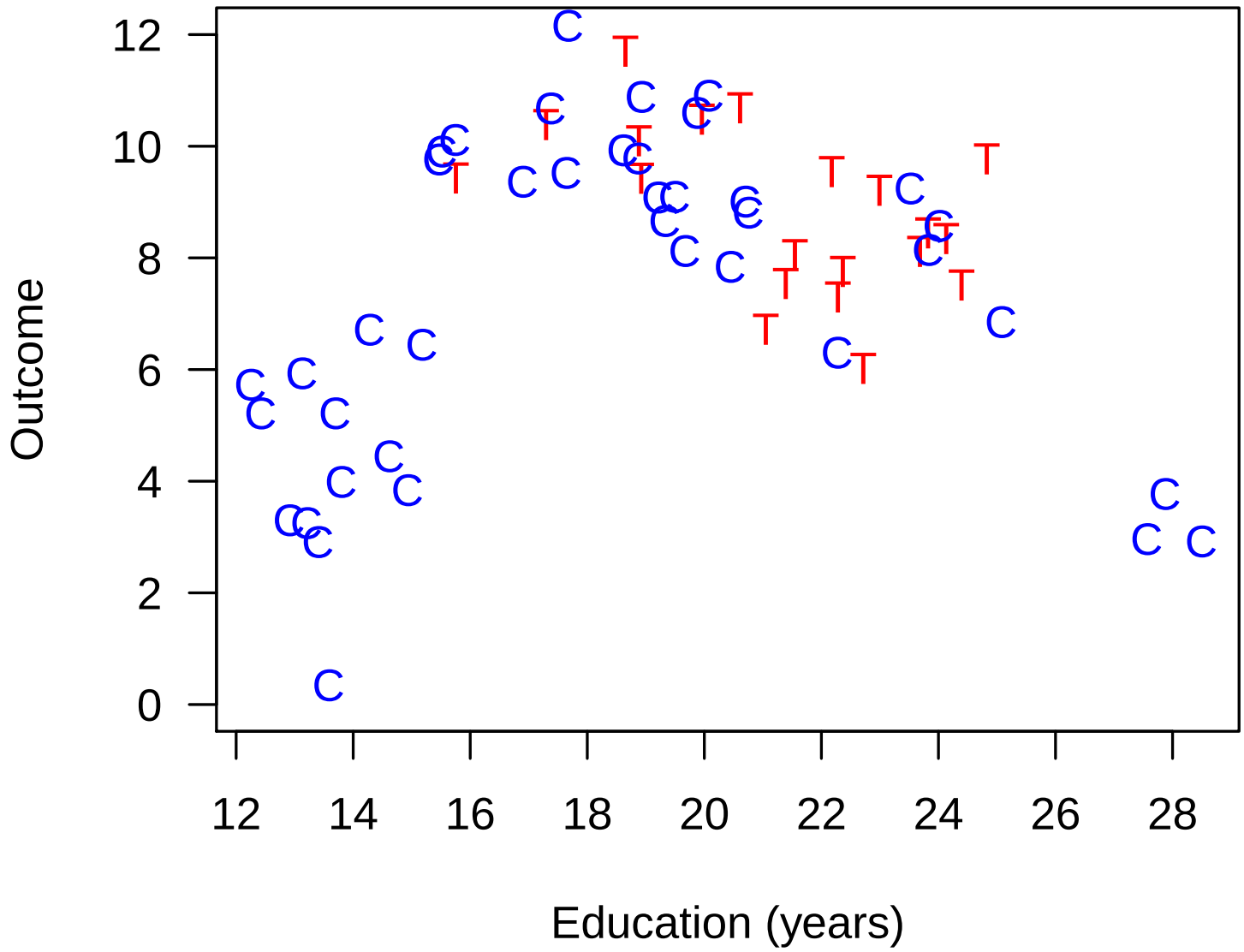

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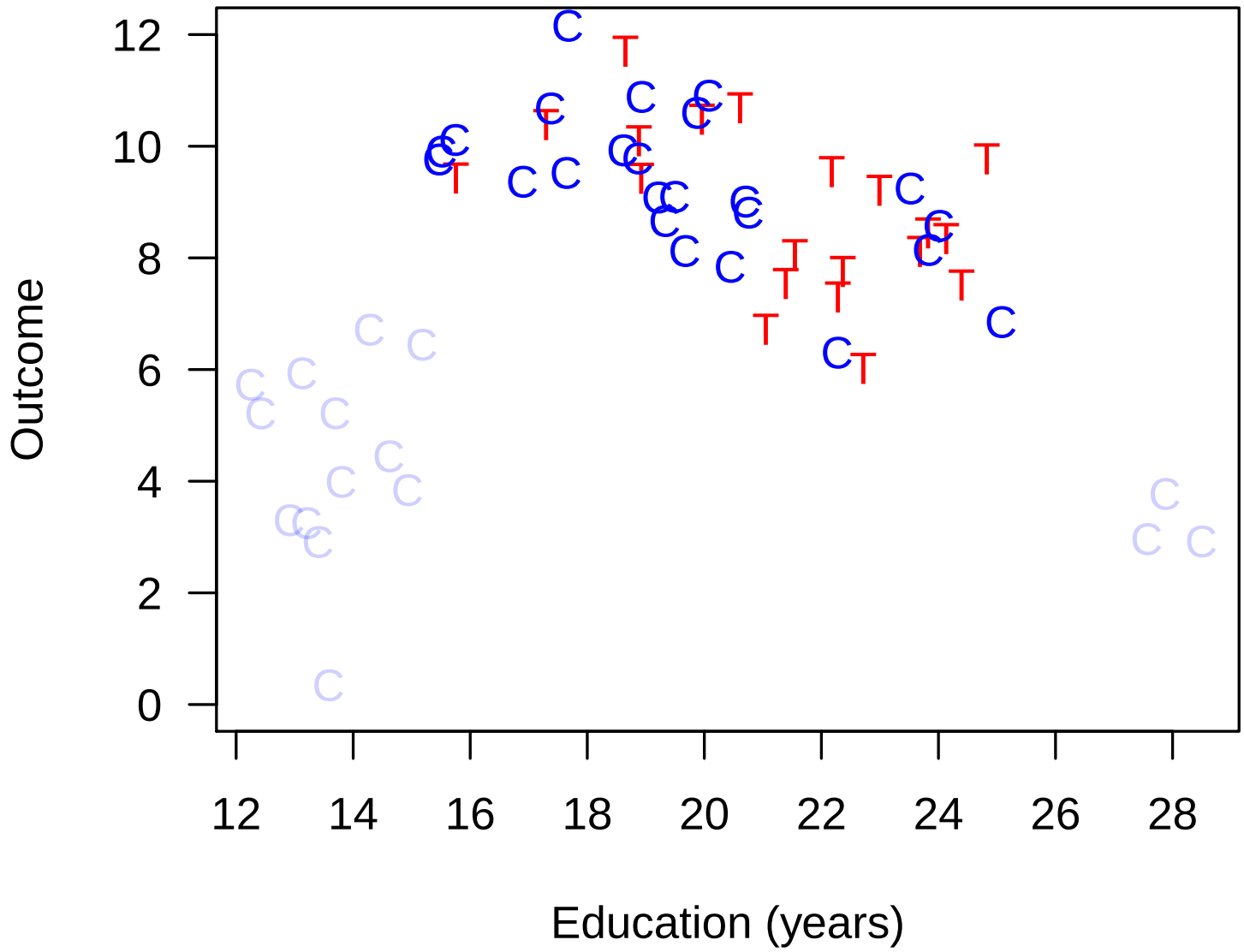

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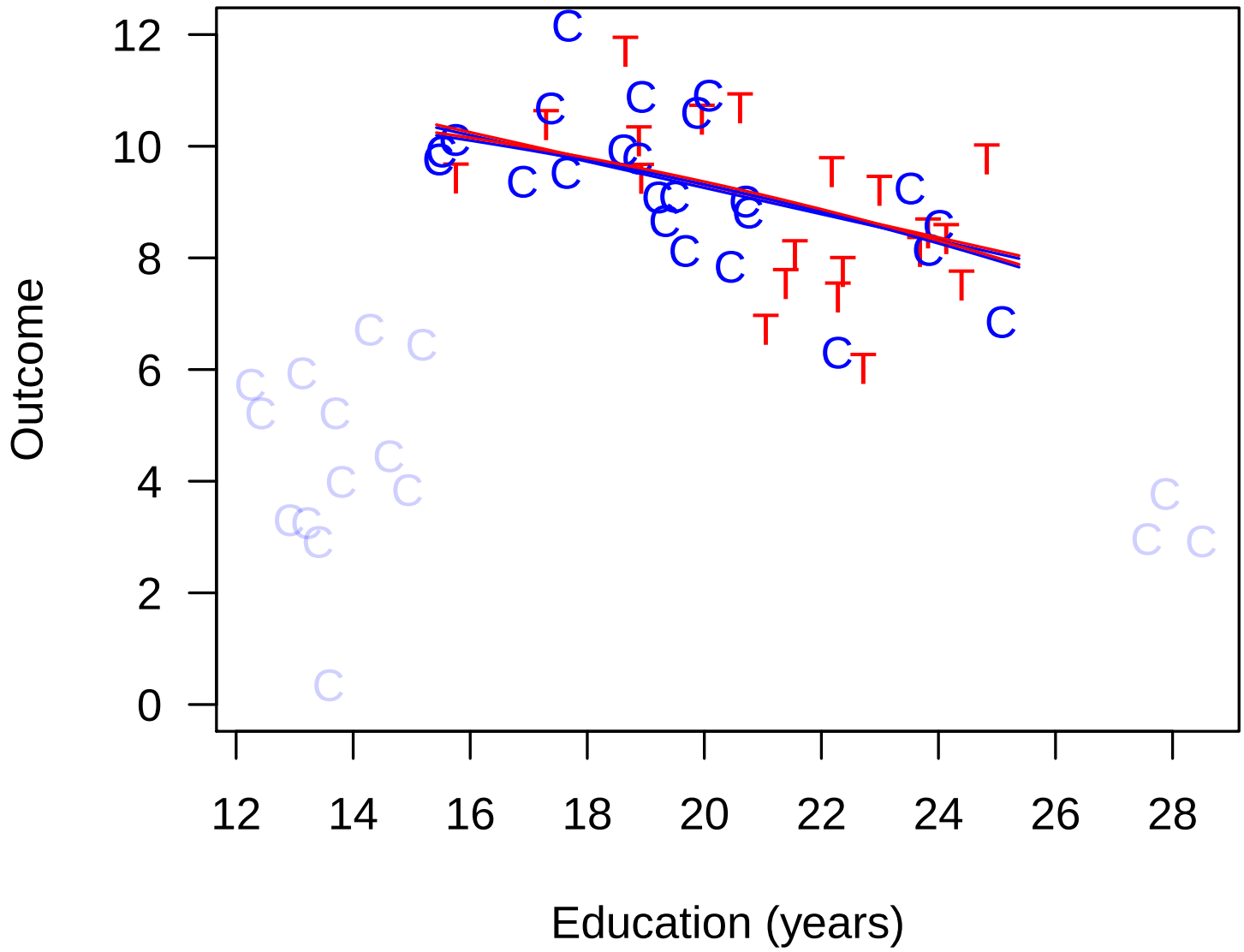

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Matching reduces model dependence, bias, and variance

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- 18 control variables (clinical factors, firm characteristics, media variables, etc.)

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- Look at *variability* in ATE estimate across specifications.

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- run 262,143 possible specifications and calculates ATE for each.
- Look at *variability* in ATE estimate across specifications.
- (Normal applications would only do one or a small number of specifications.)

# Reducing Model Dependence

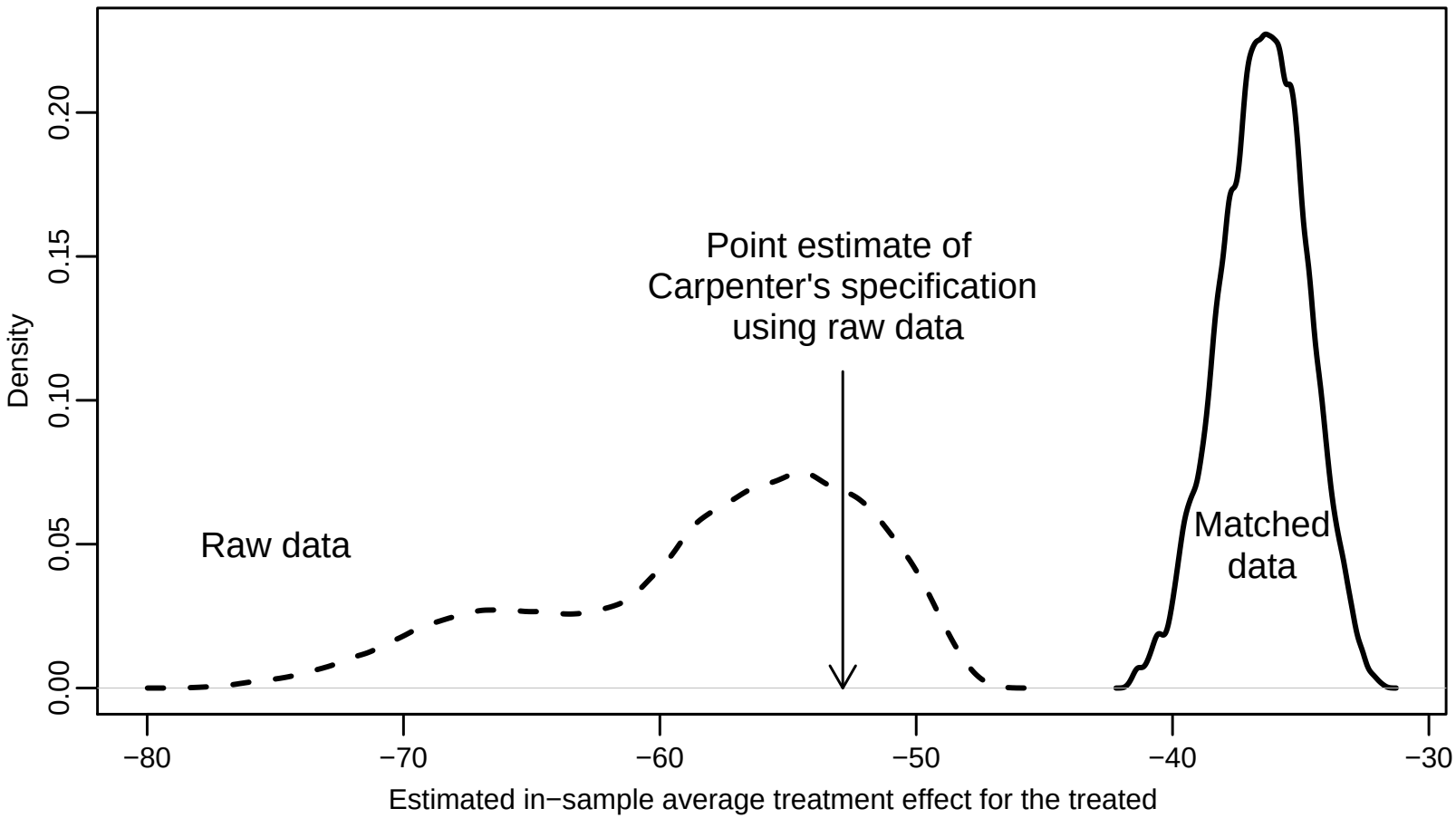


Figure: Histogram of estimated in-sample average treatment effect for the treated (ATT) of the Democratic Senate majority on FDA drug approval time across 262,143 specifications.

# Another Example: Jeffrey Koch, AJPS, 2002

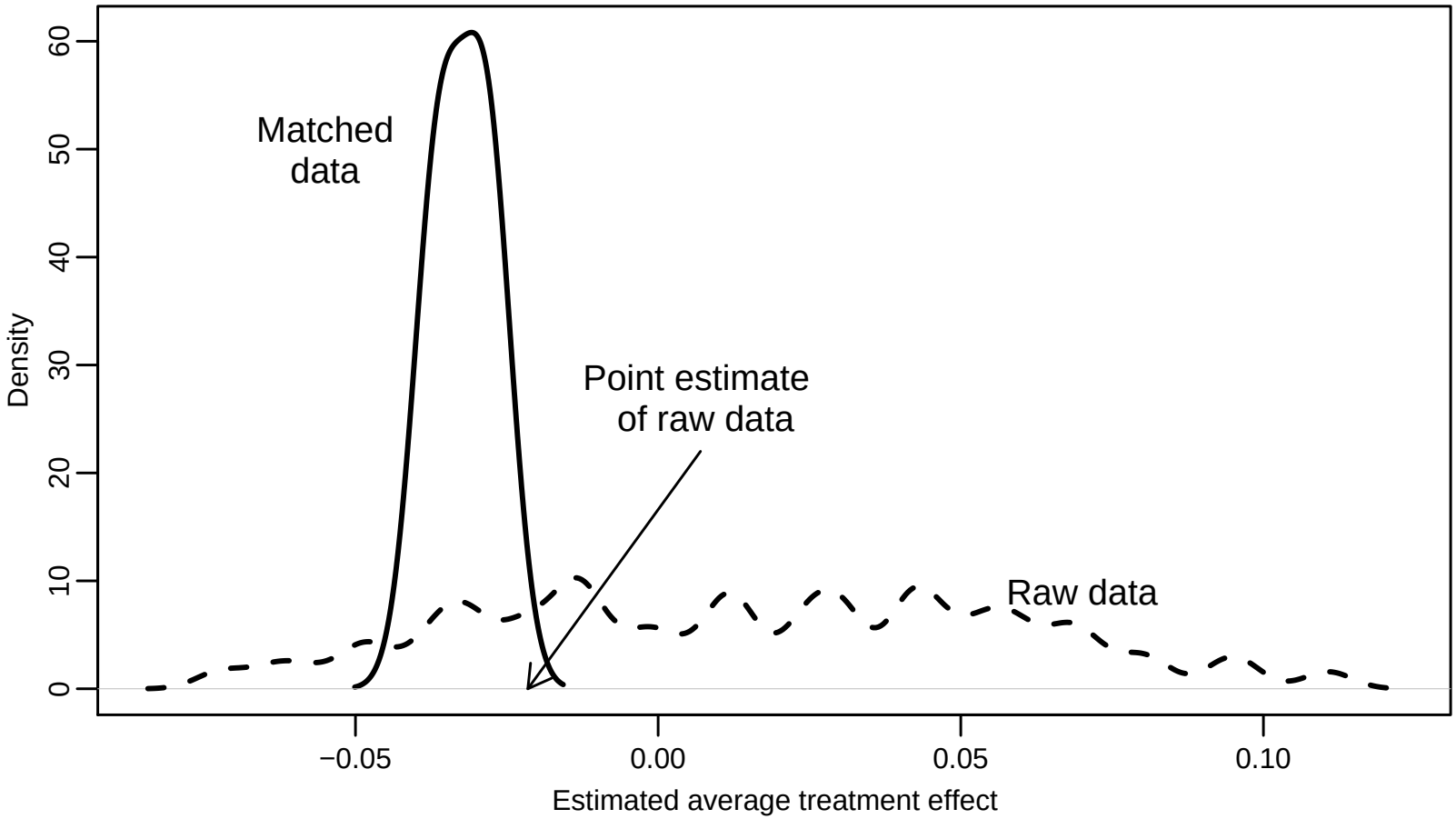


Figure: Estimated effects of being a highly visible female Republican candidate across 63 possible specifications with the Koch data.

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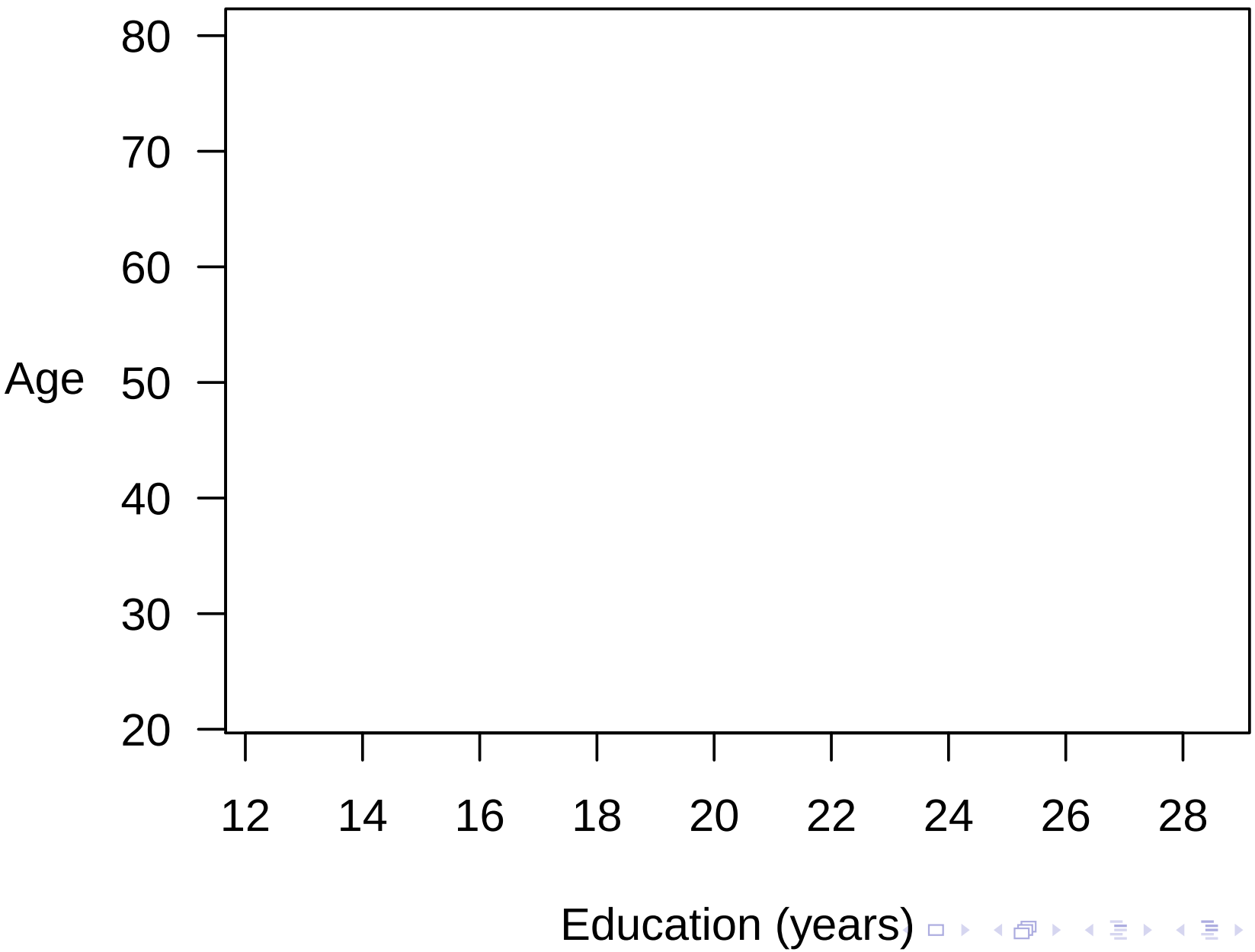

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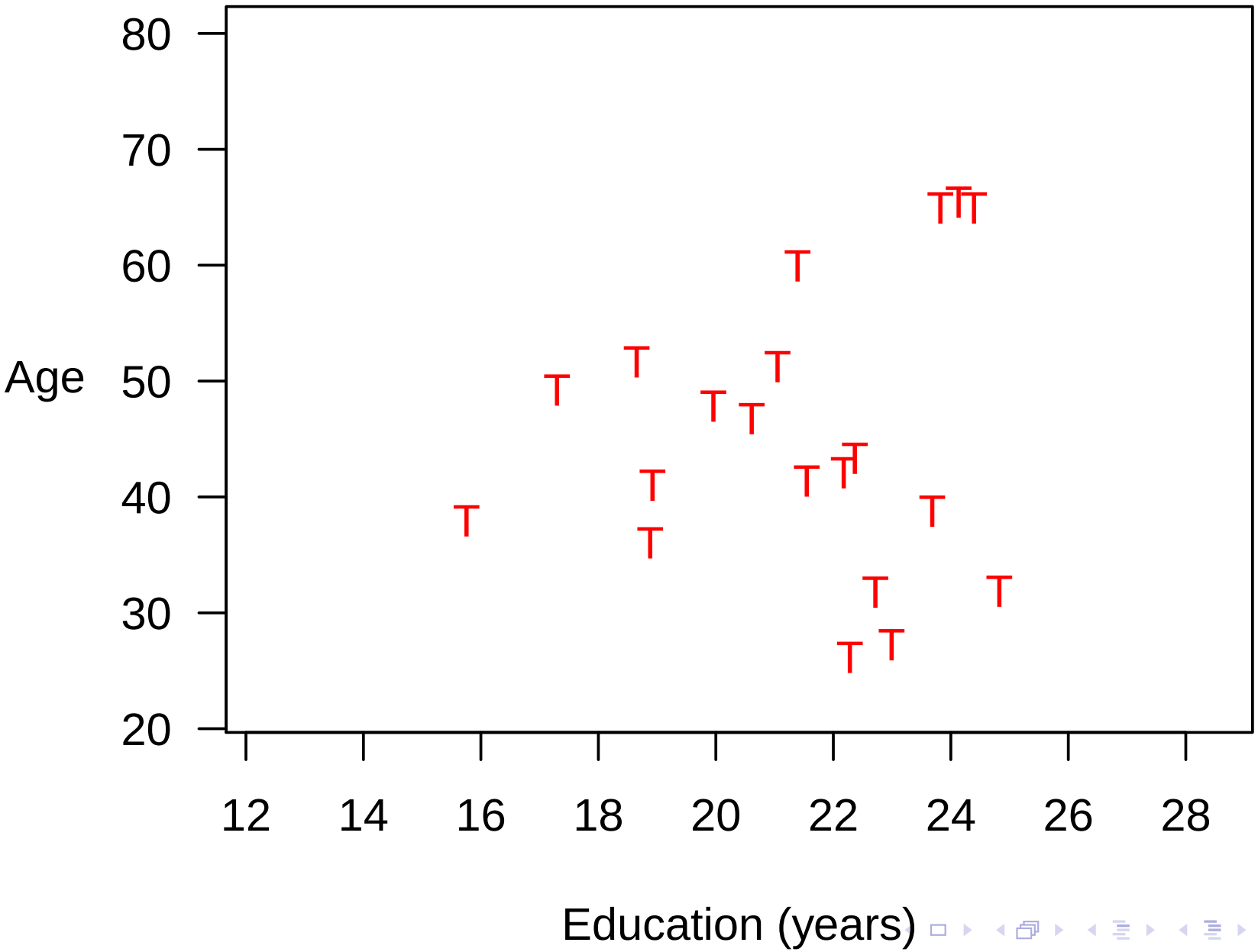

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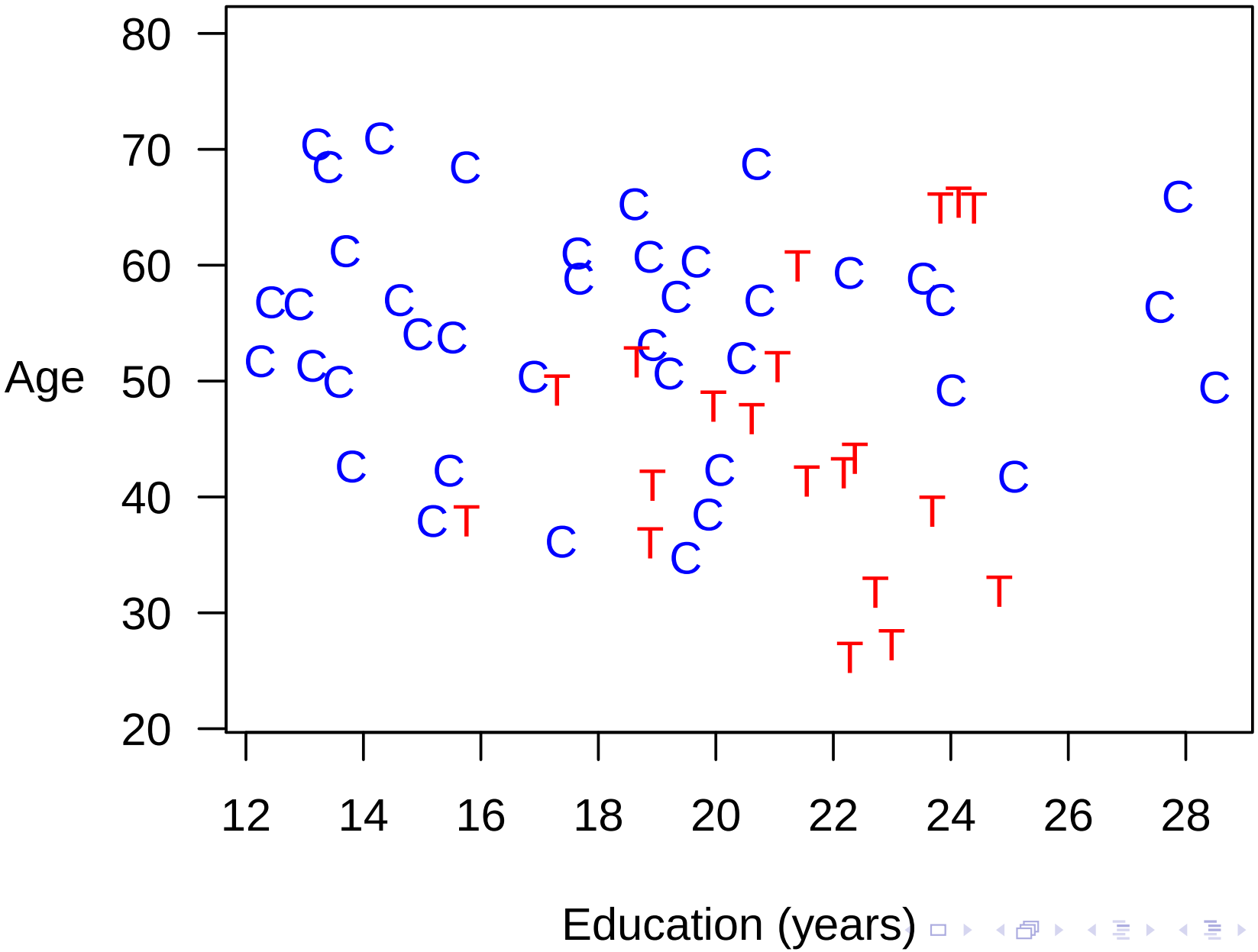

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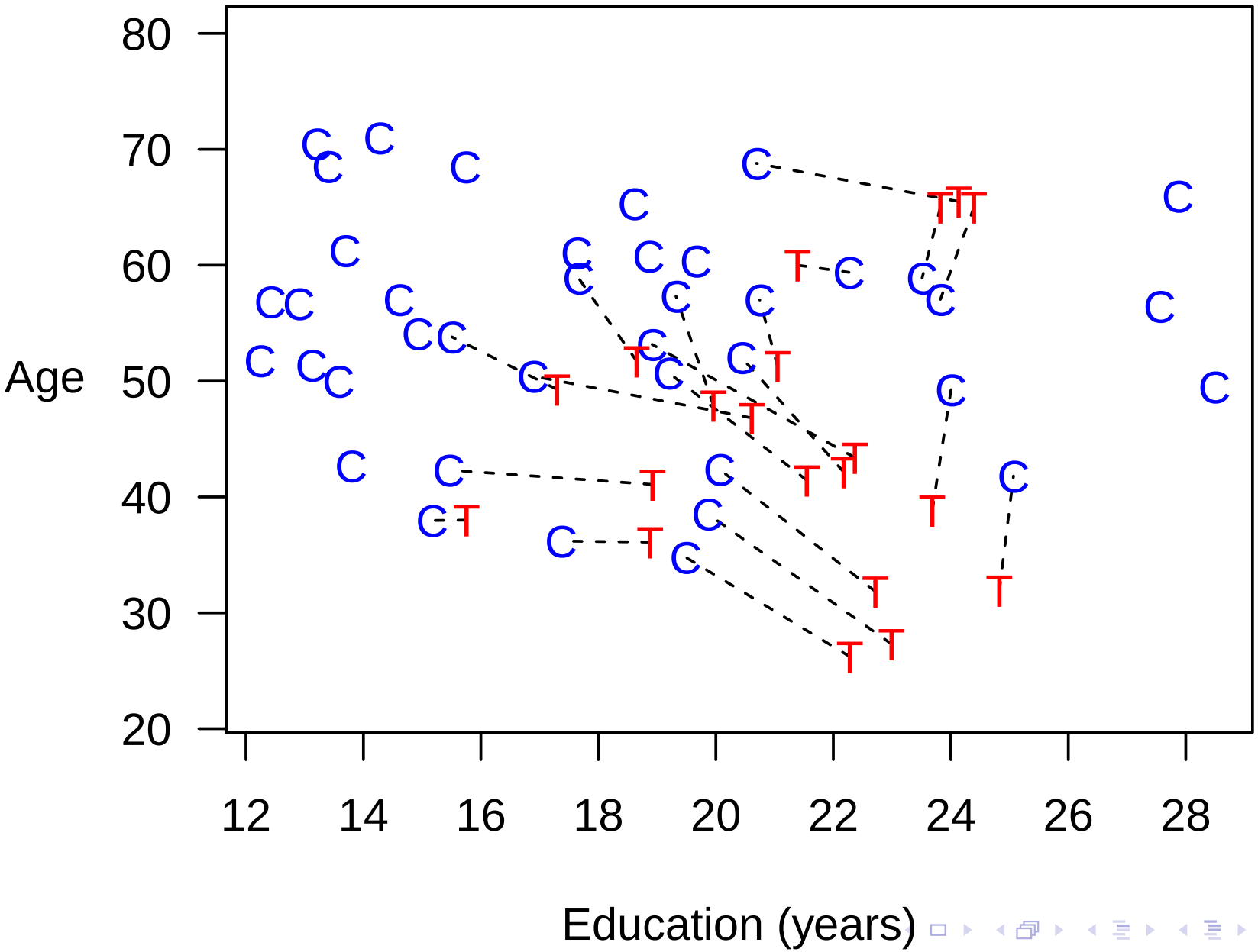

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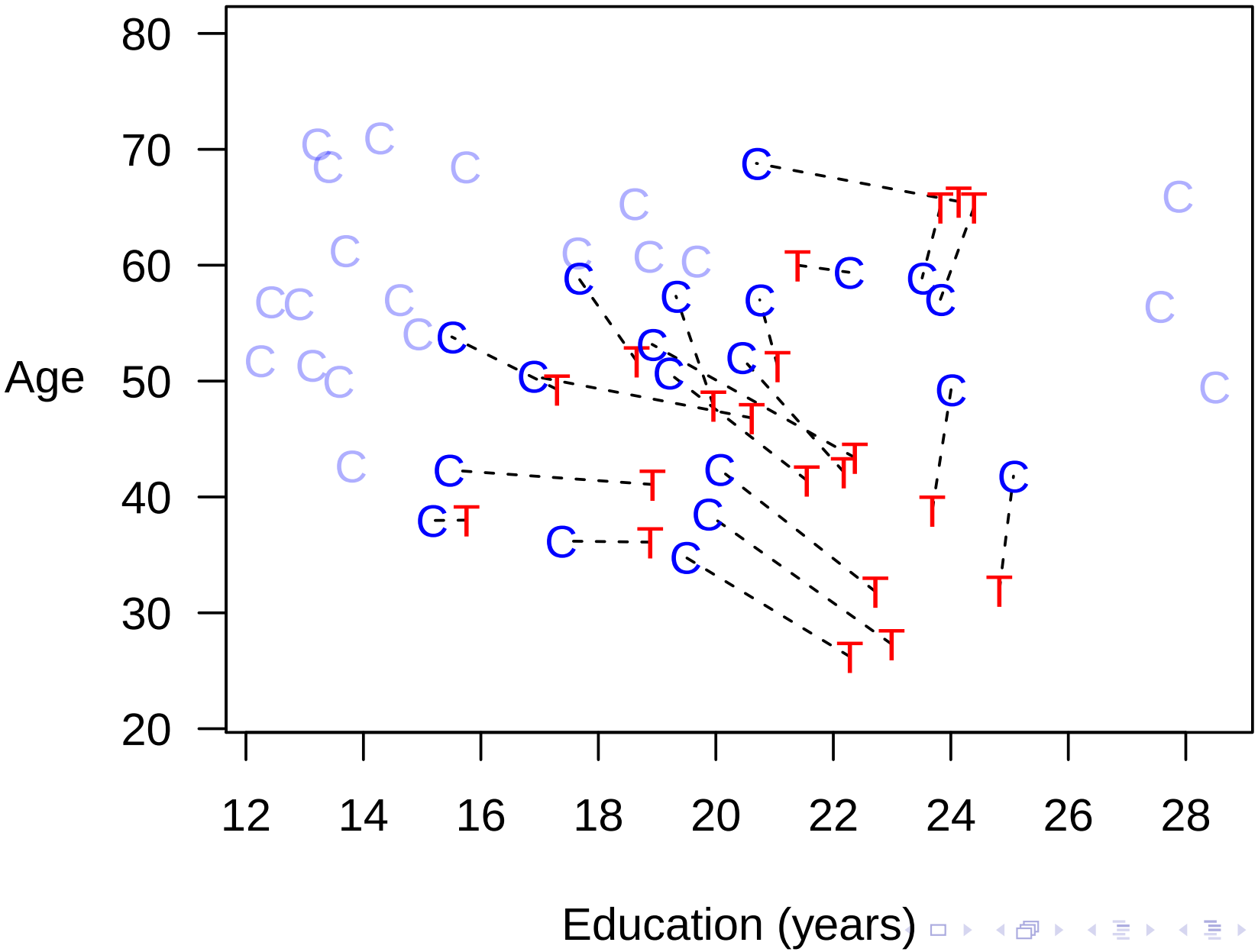

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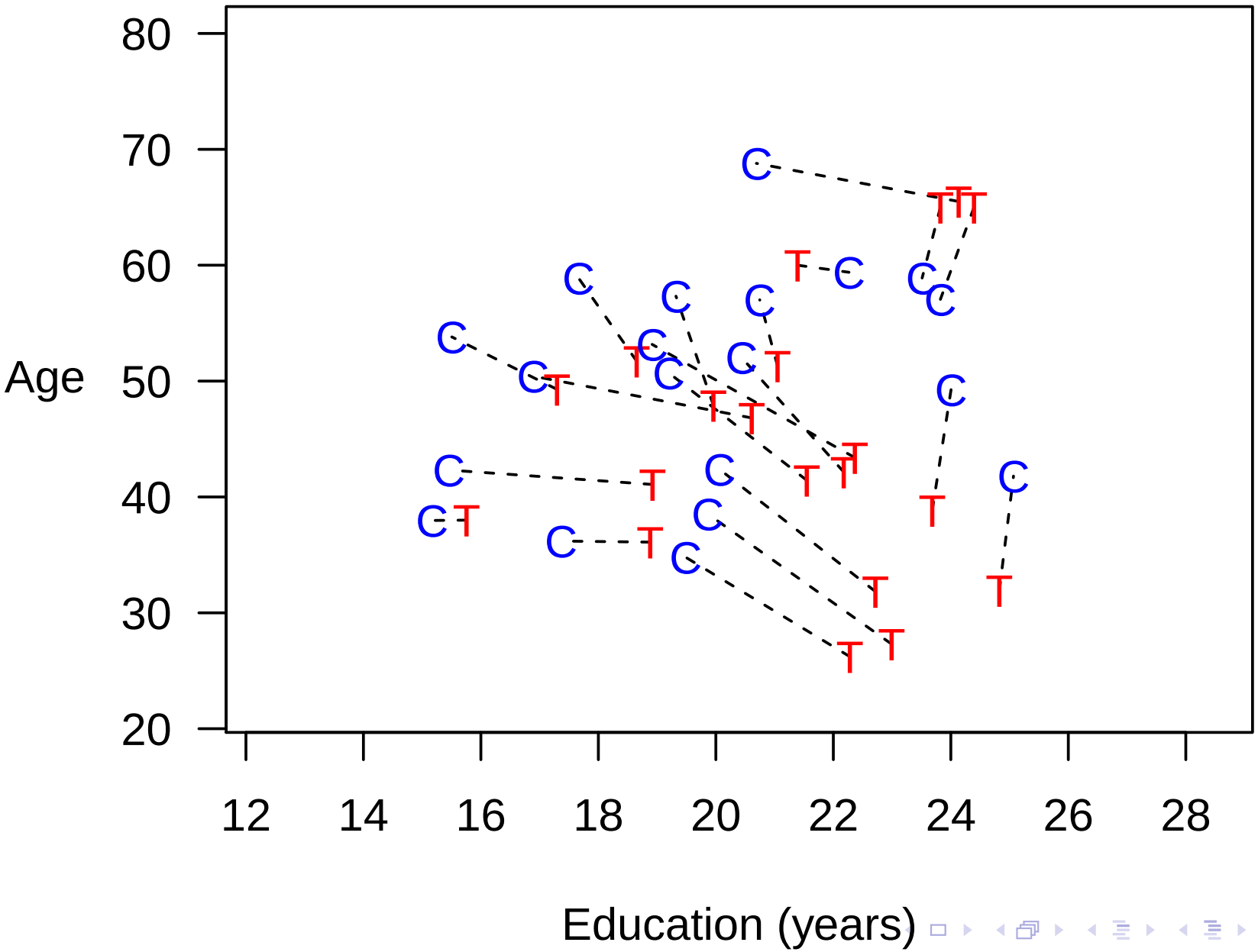

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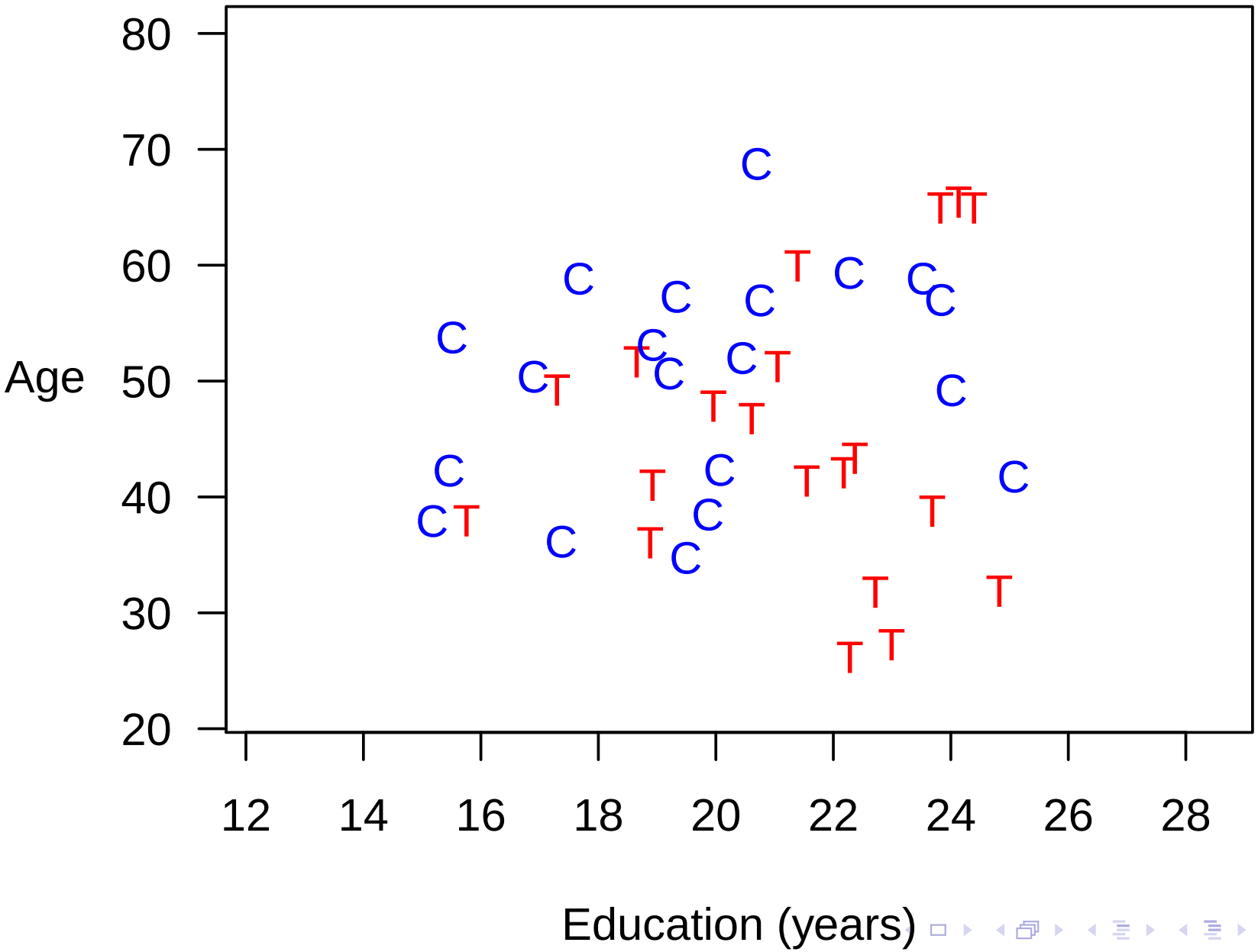

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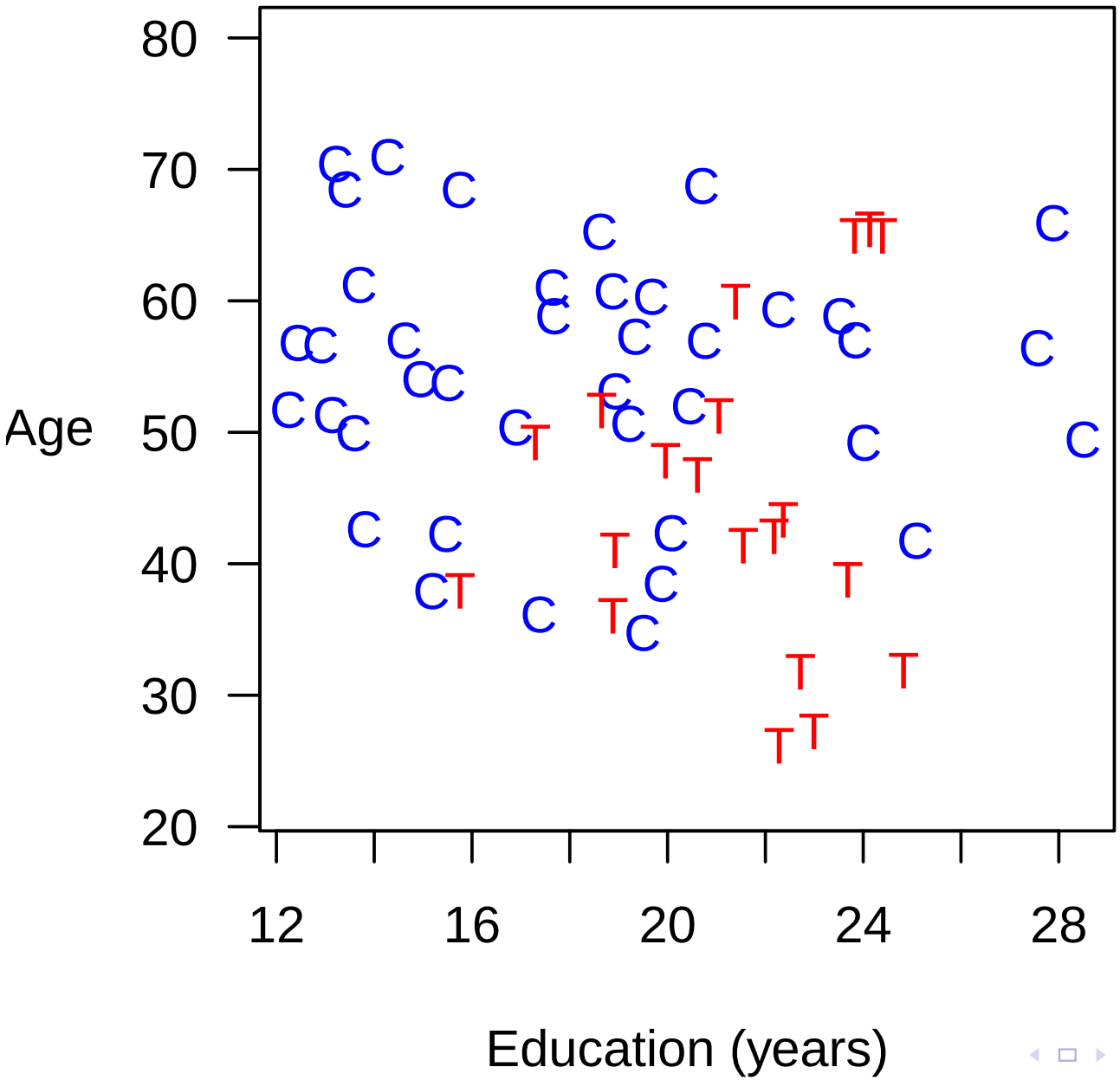

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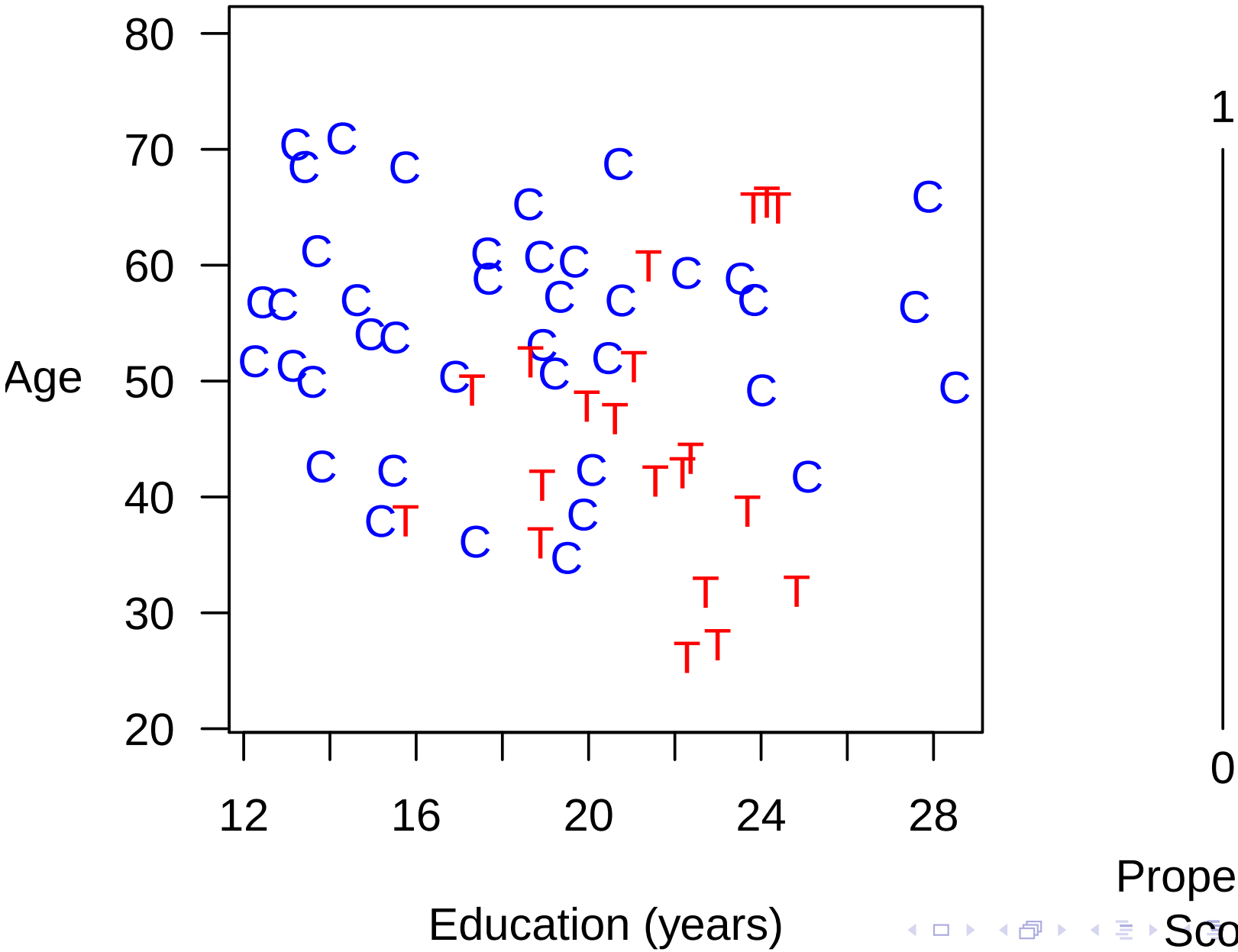

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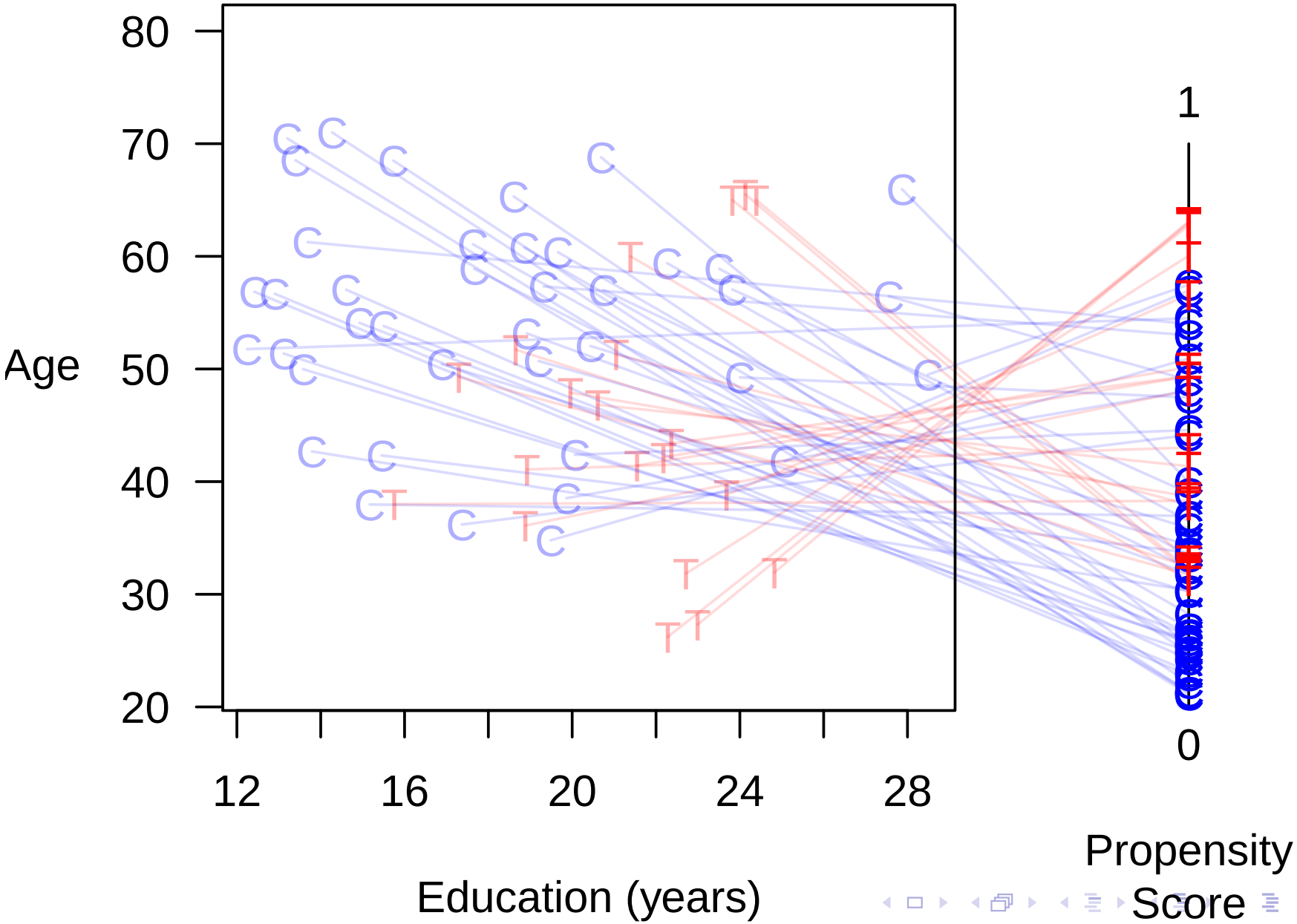

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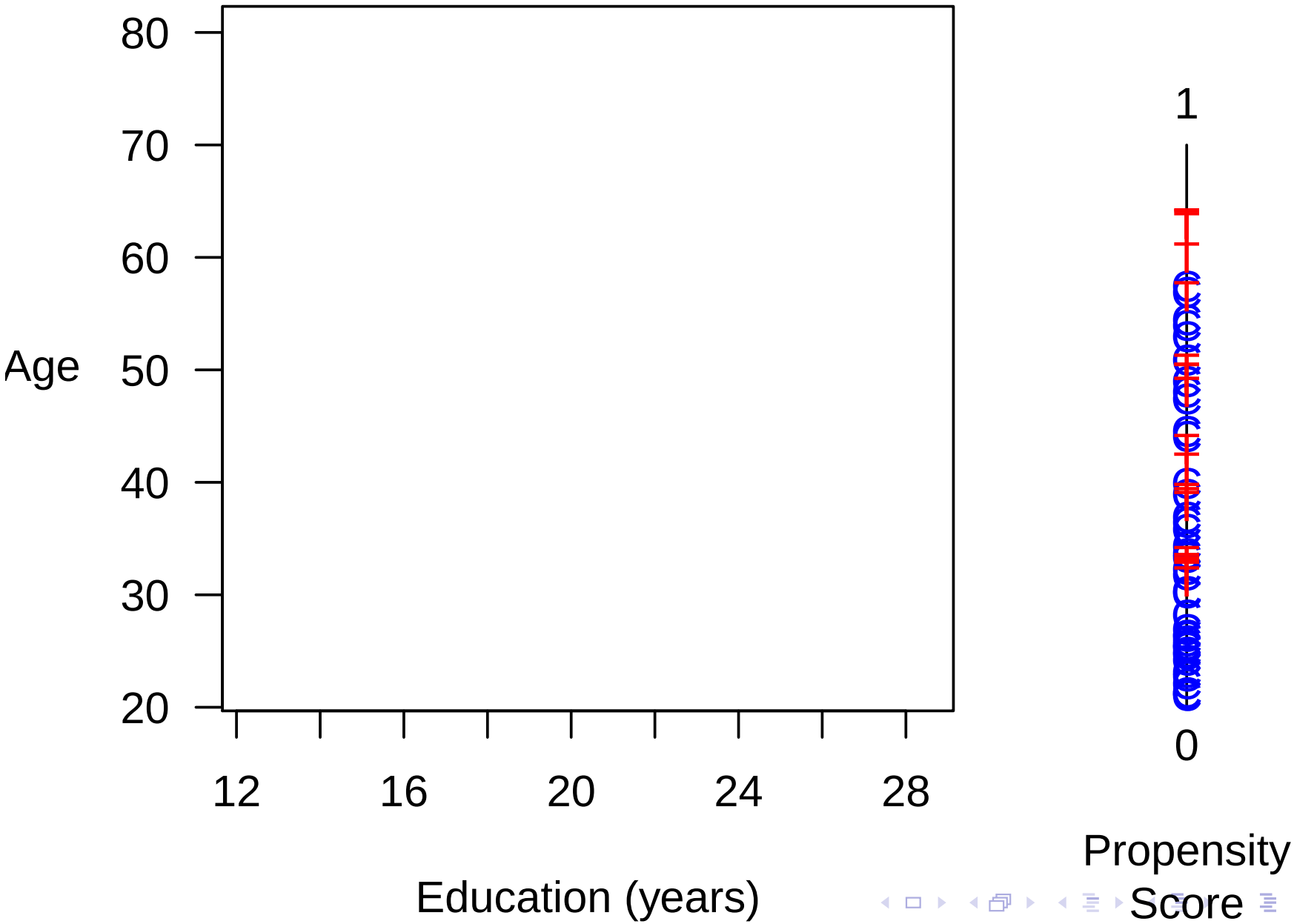

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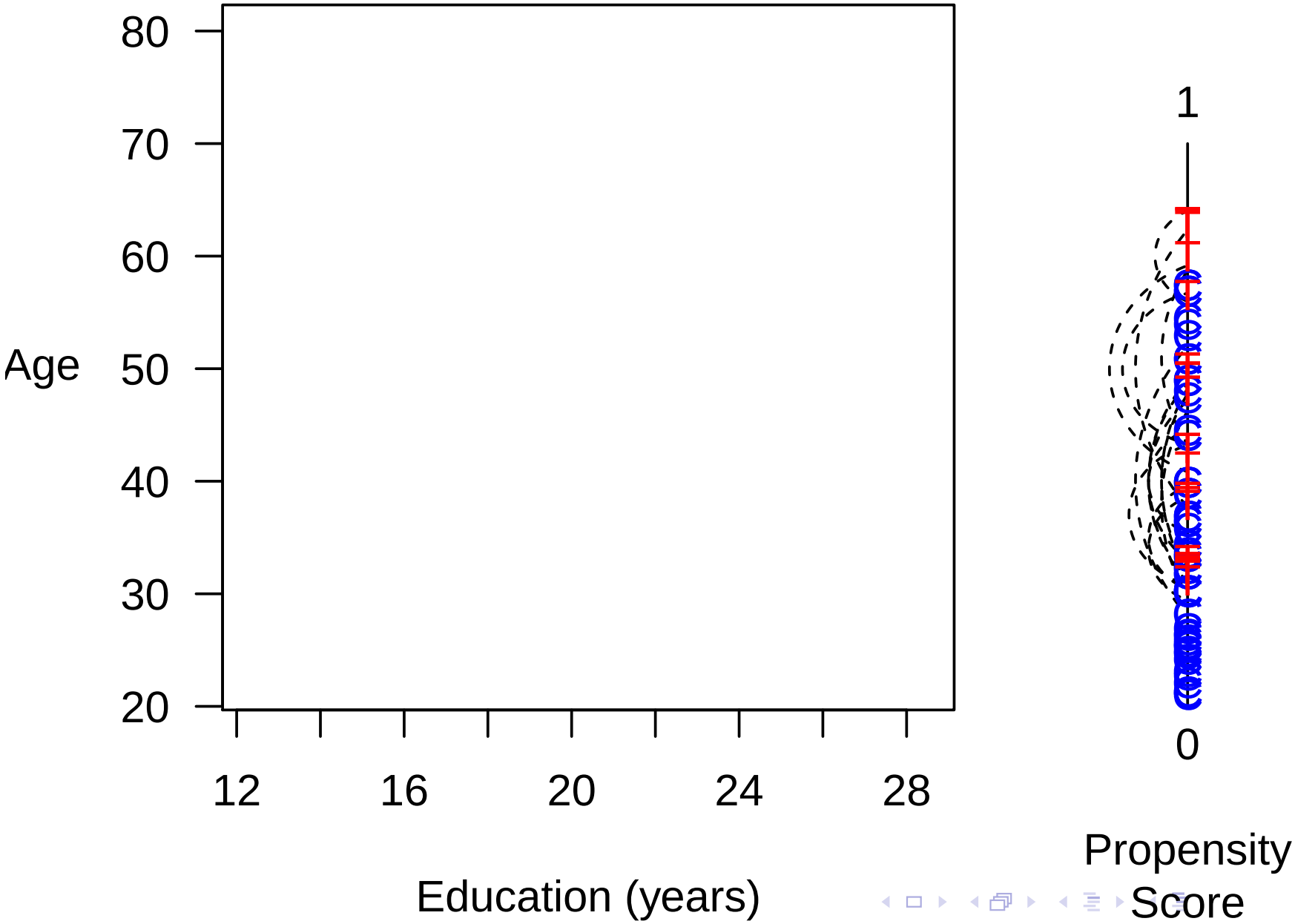

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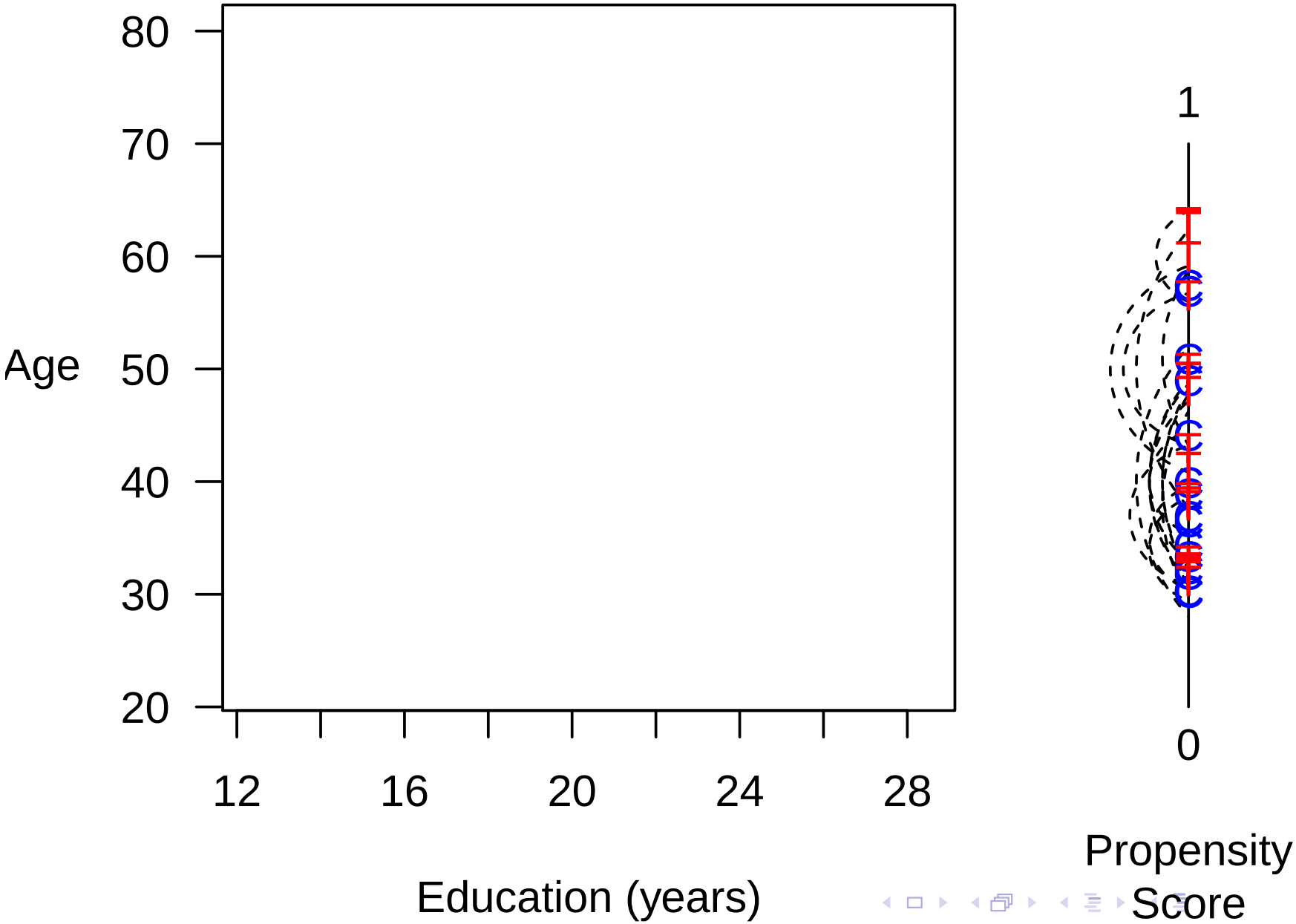

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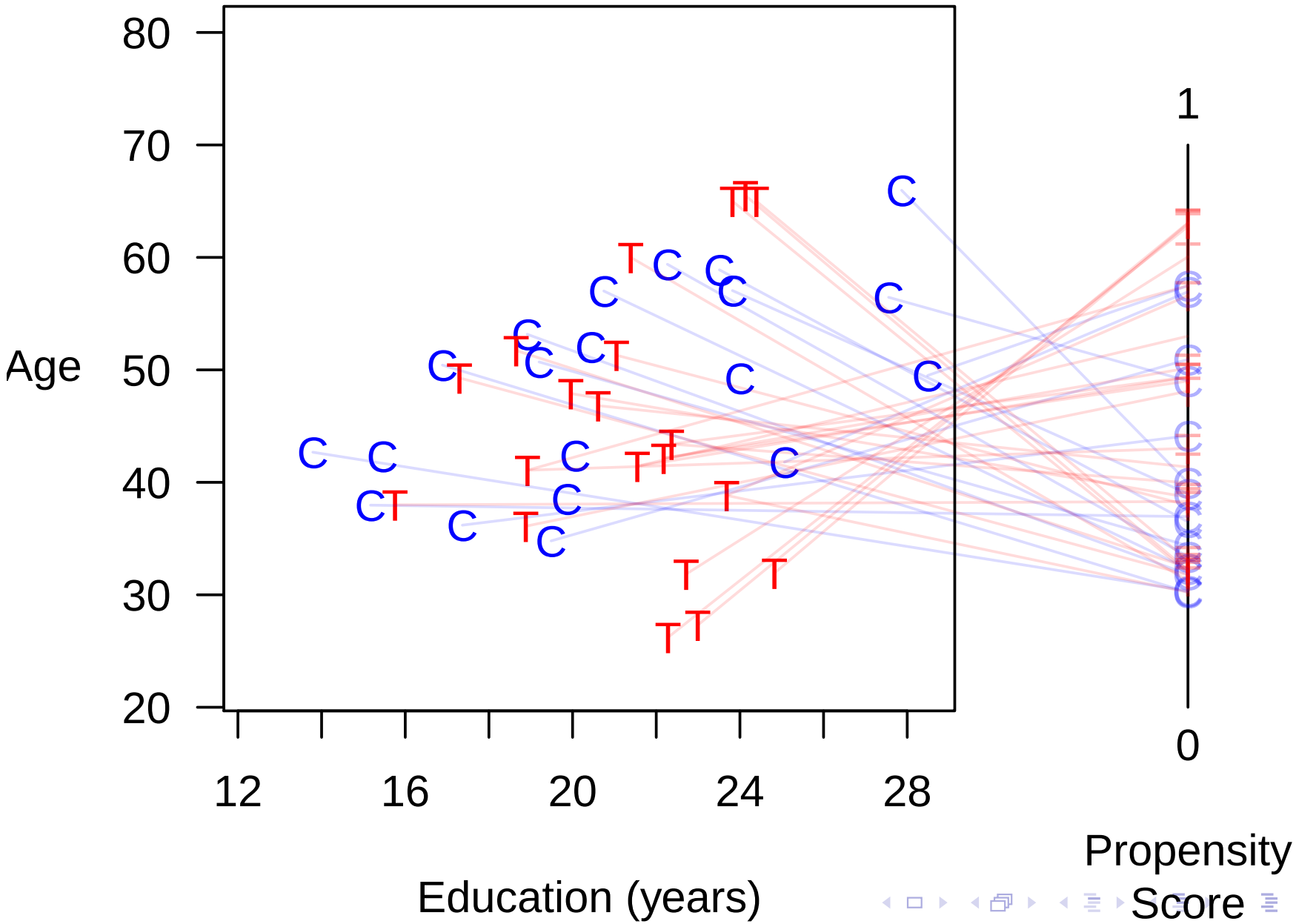

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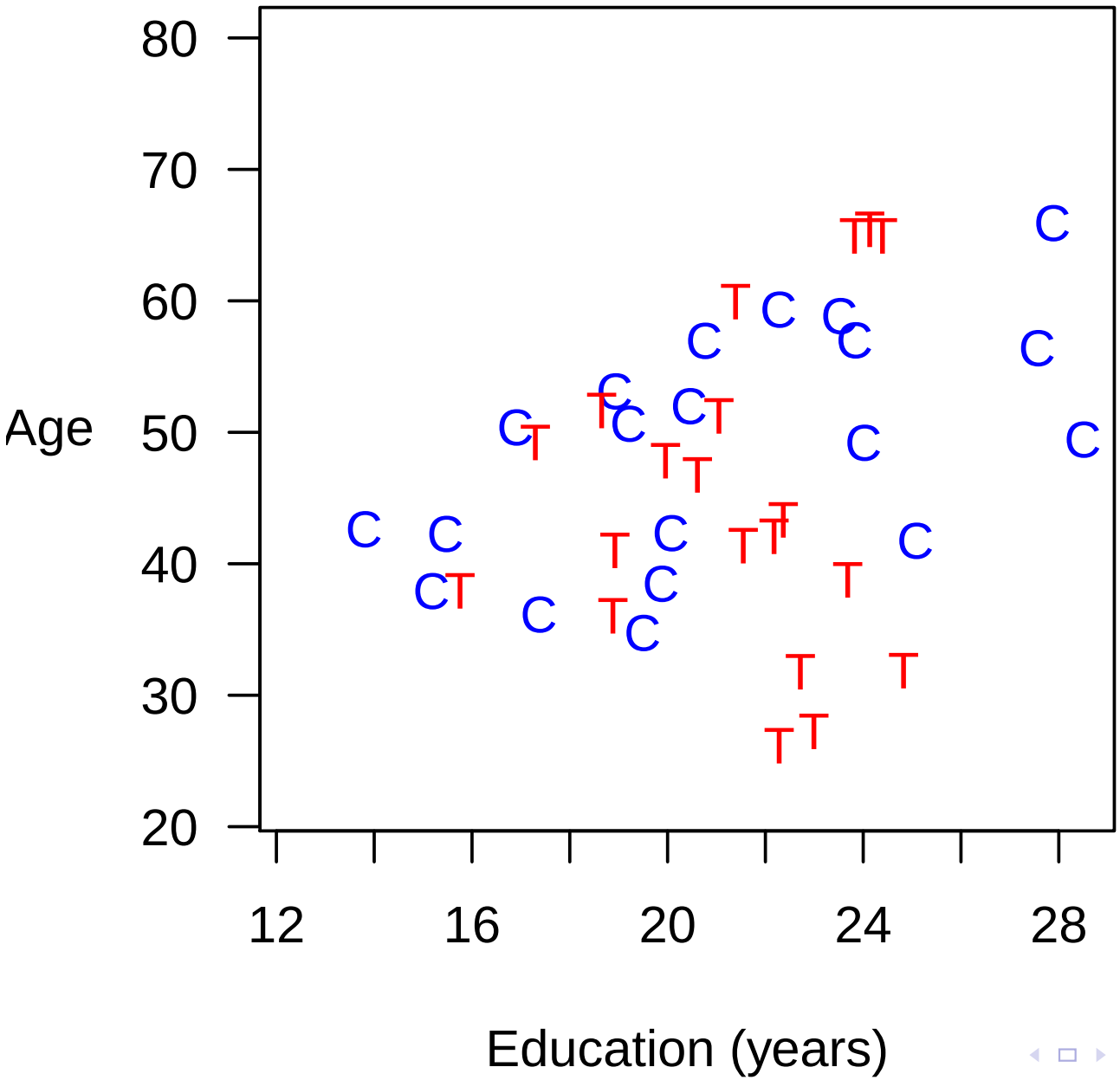

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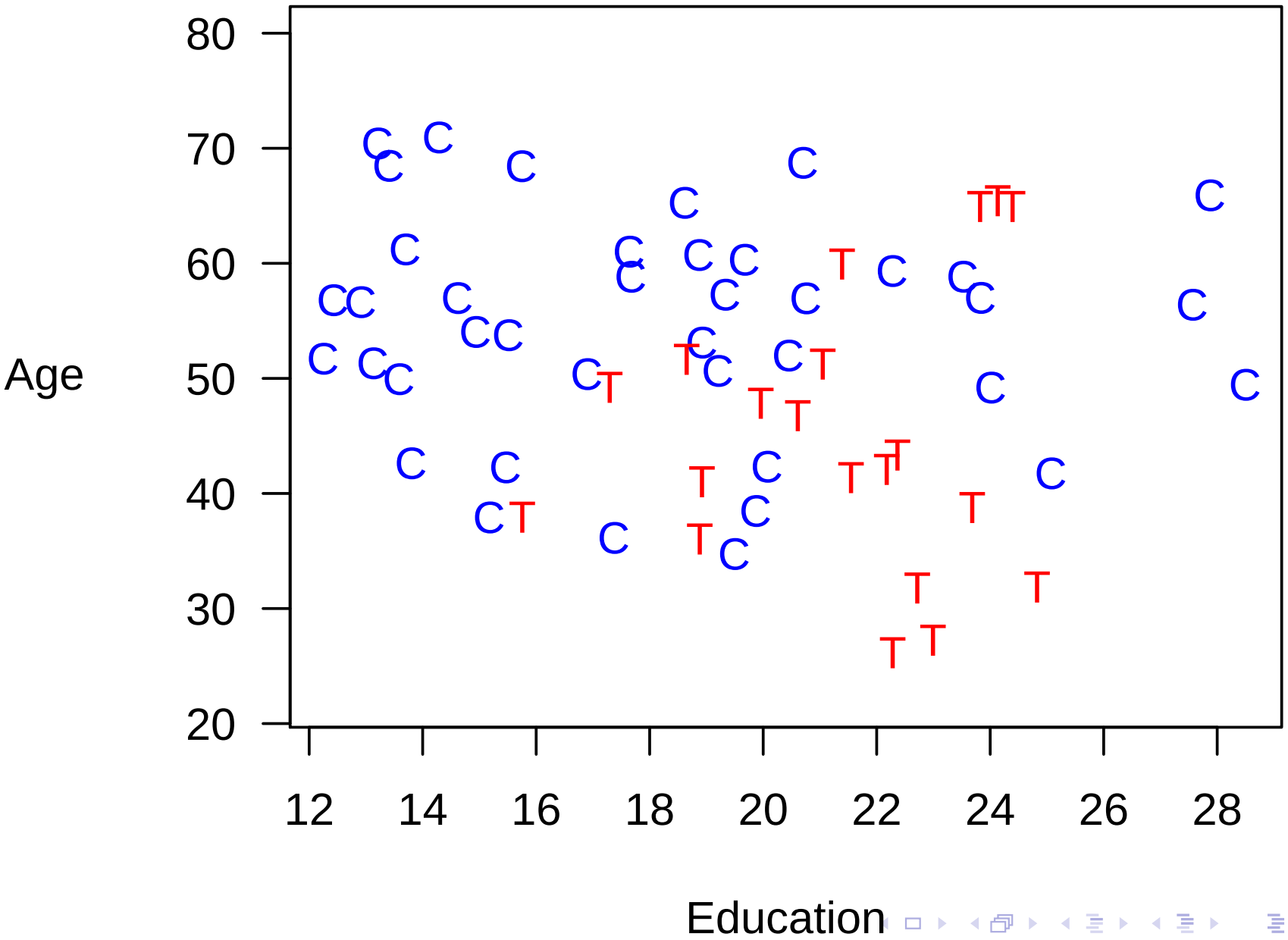

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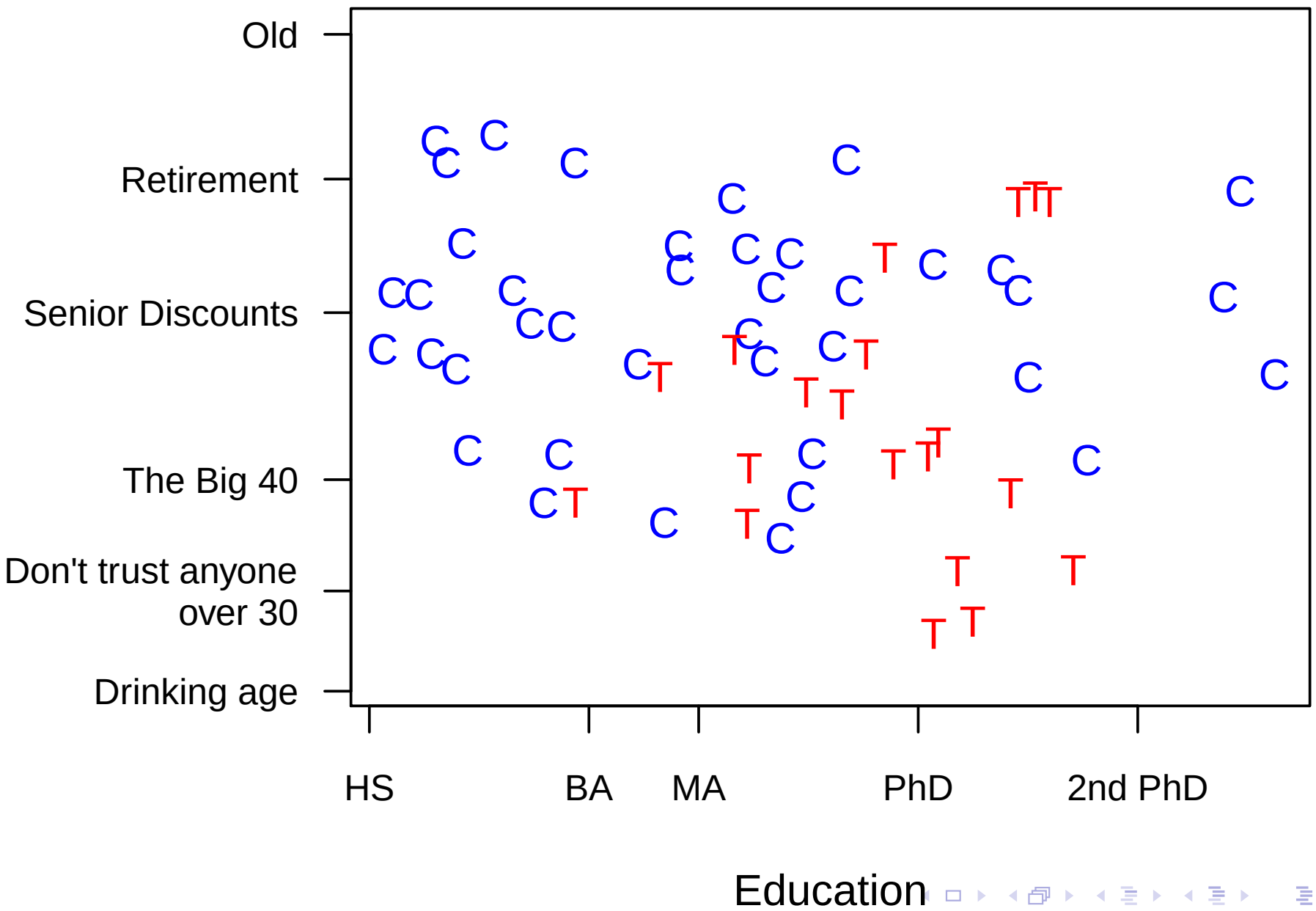

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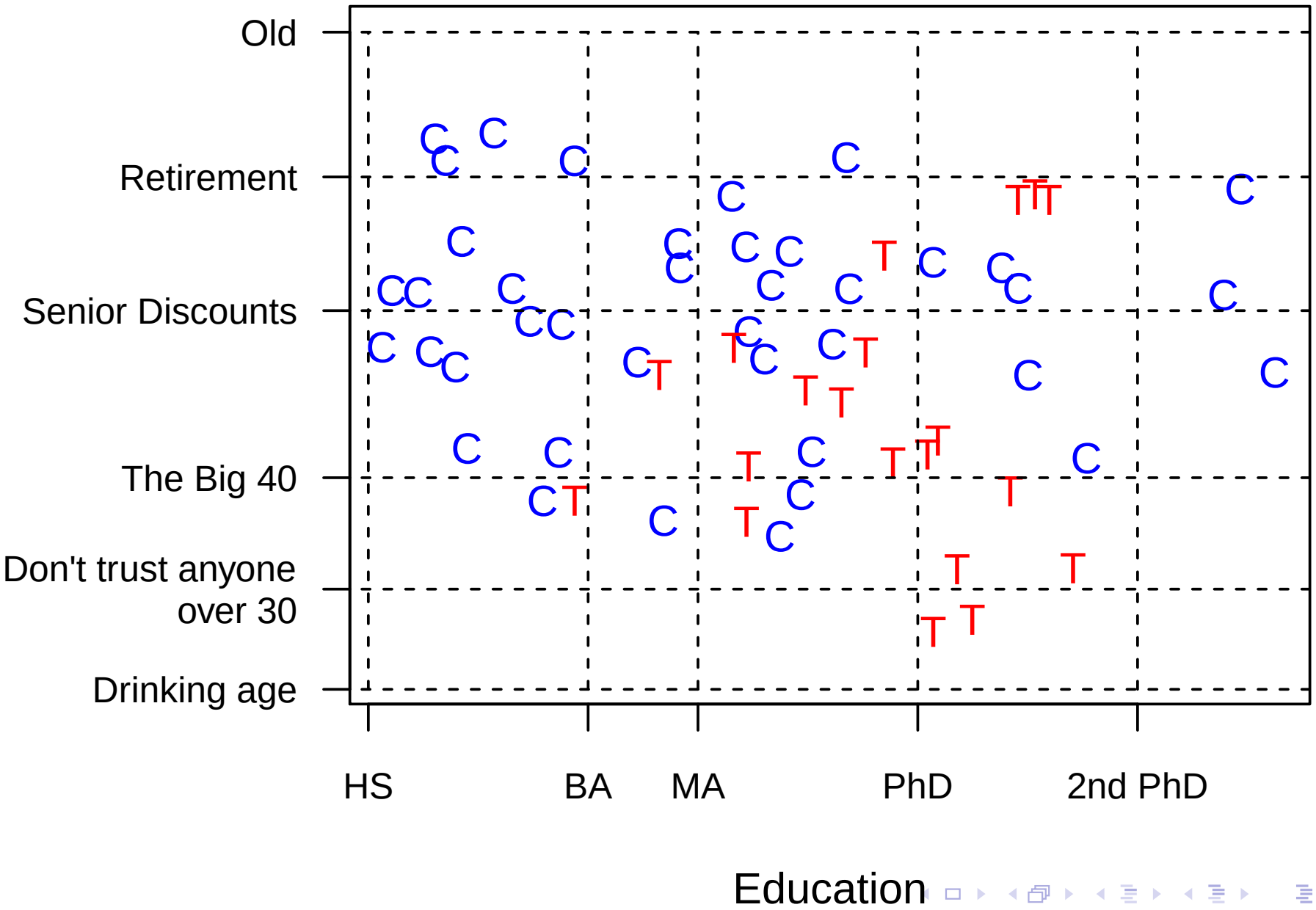

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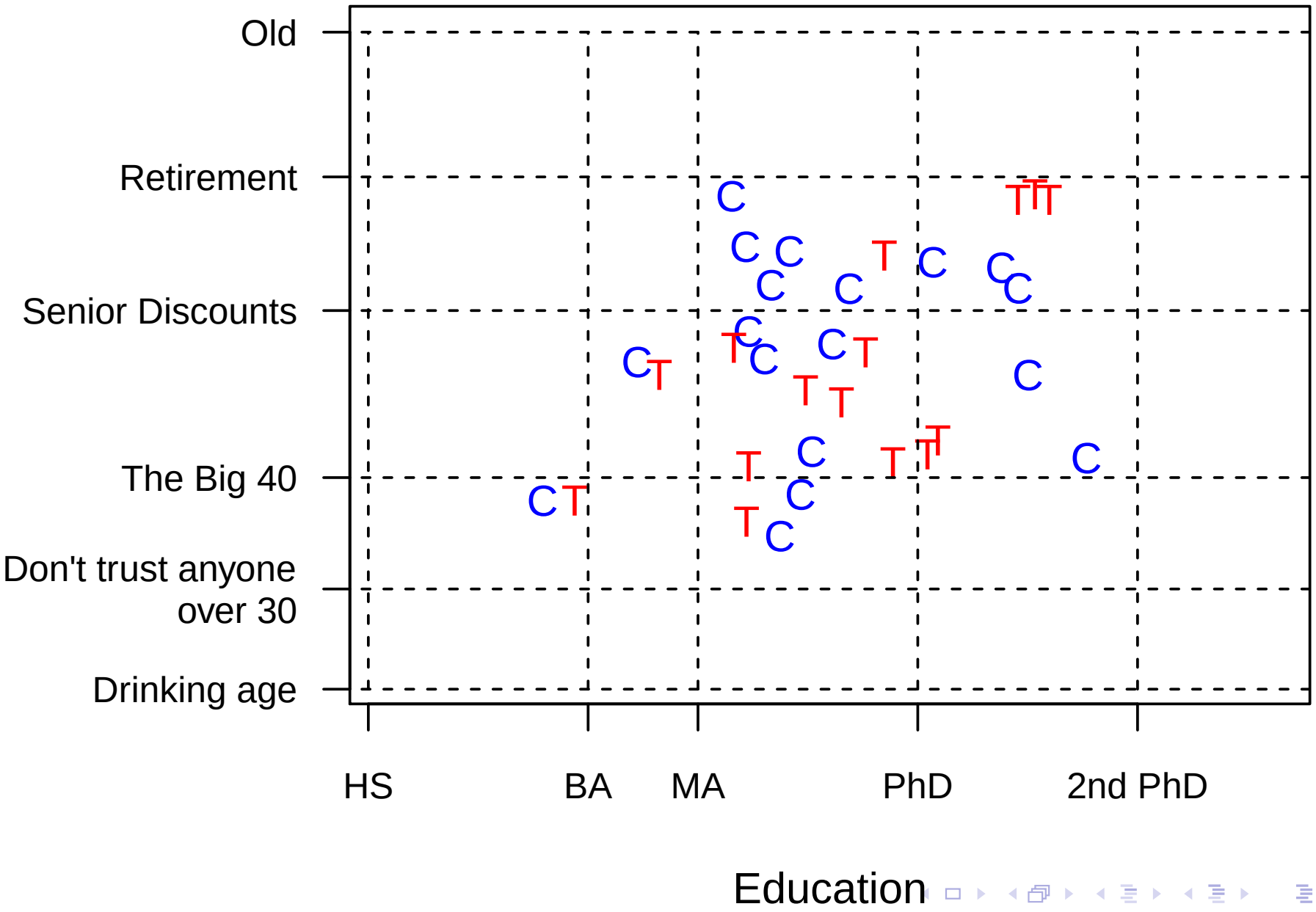

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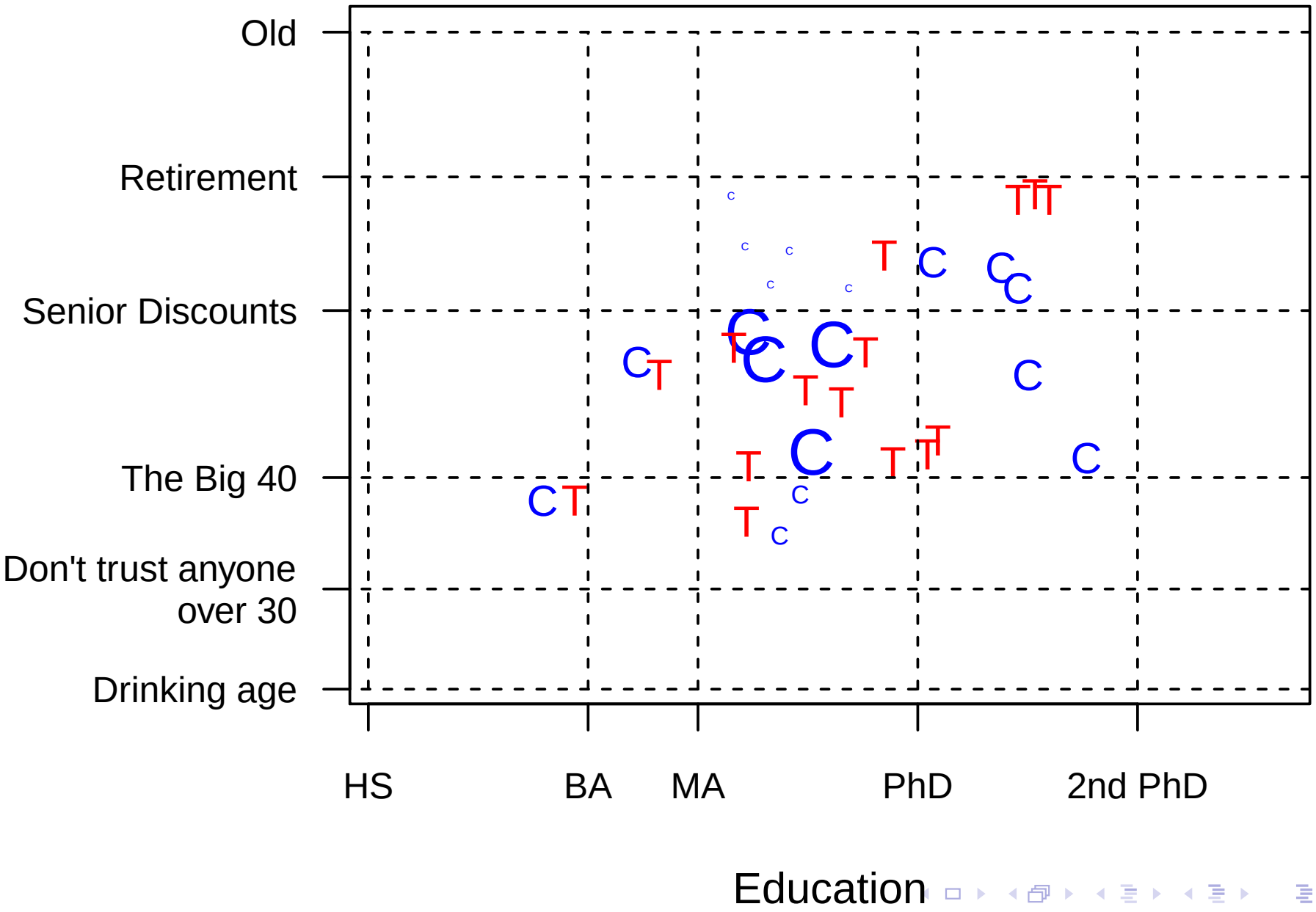

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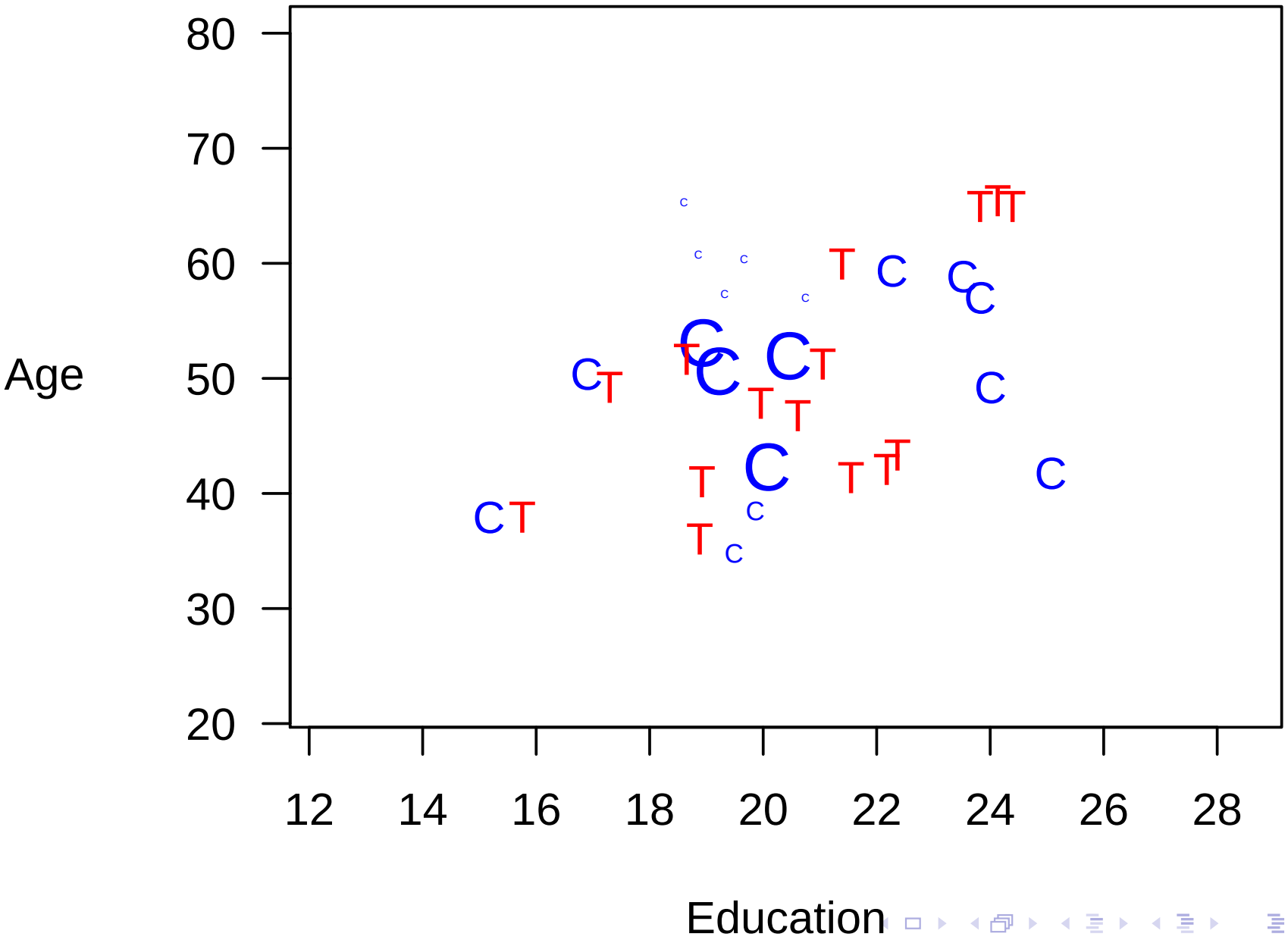

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  - Better measure: Difference of multivariate histograms,

$$\mathcal{L}_1(f, g; H) = \frac{1}{2} \sum_{\ell_1 \cdots \ell_k \in H(\mathbf{X})} |f_{\ell_1 \cdots \ell_k} - g_{\ell_1 \cdots \ell_k}|$$

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- An alternative approach

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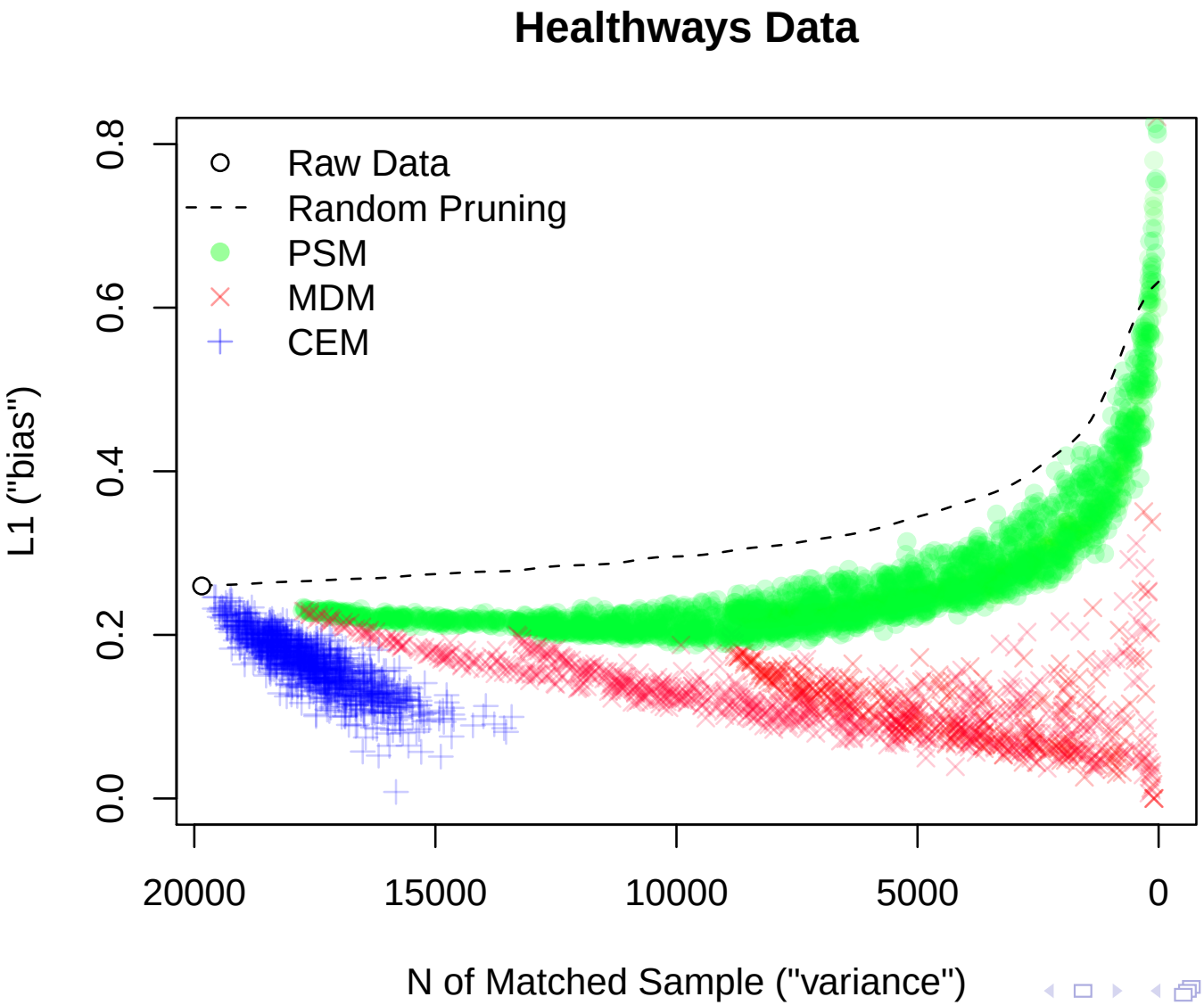
# A Space Graph: Real Data

King, Nielsen, Coberley, Pope, and Wells (2011)



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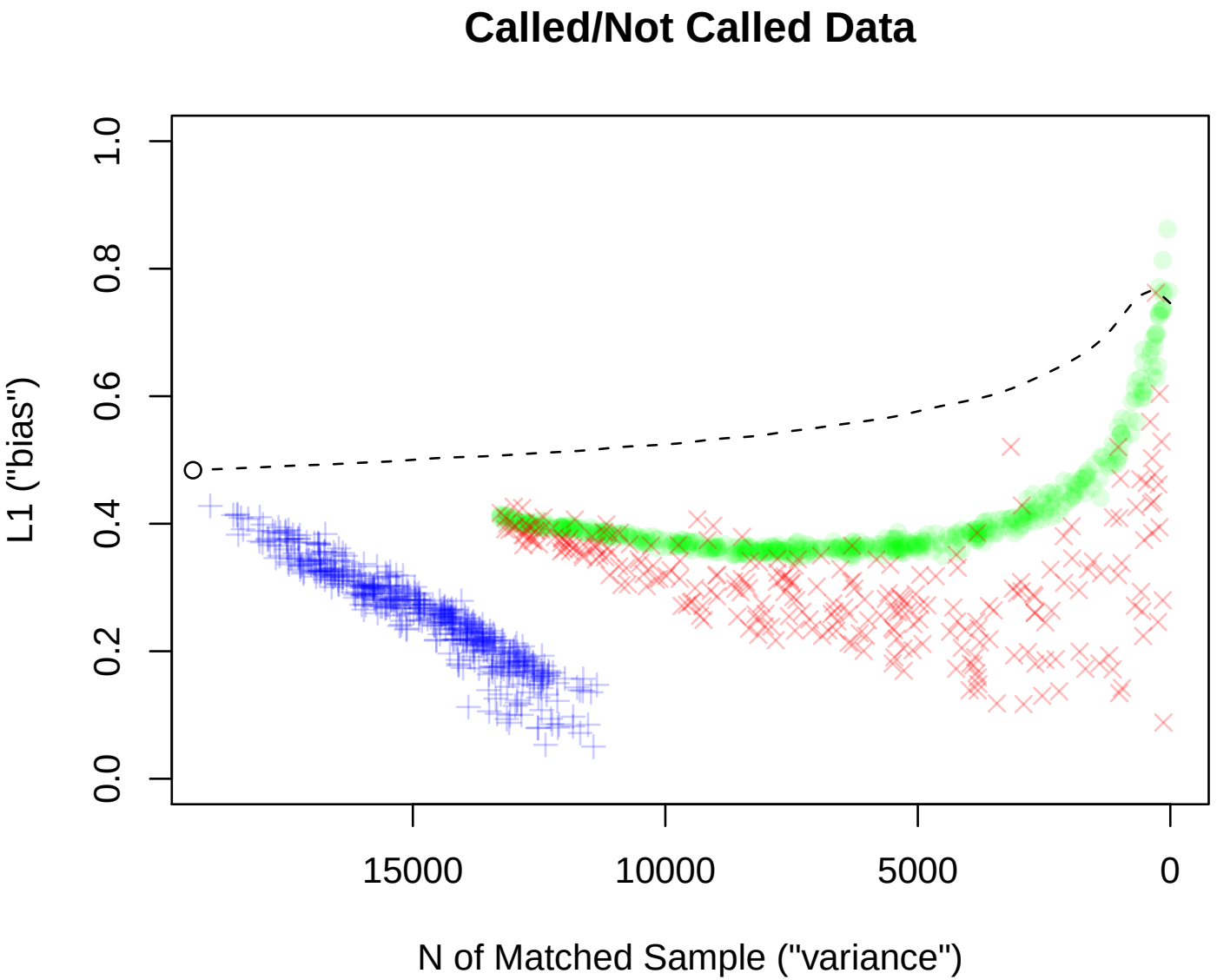

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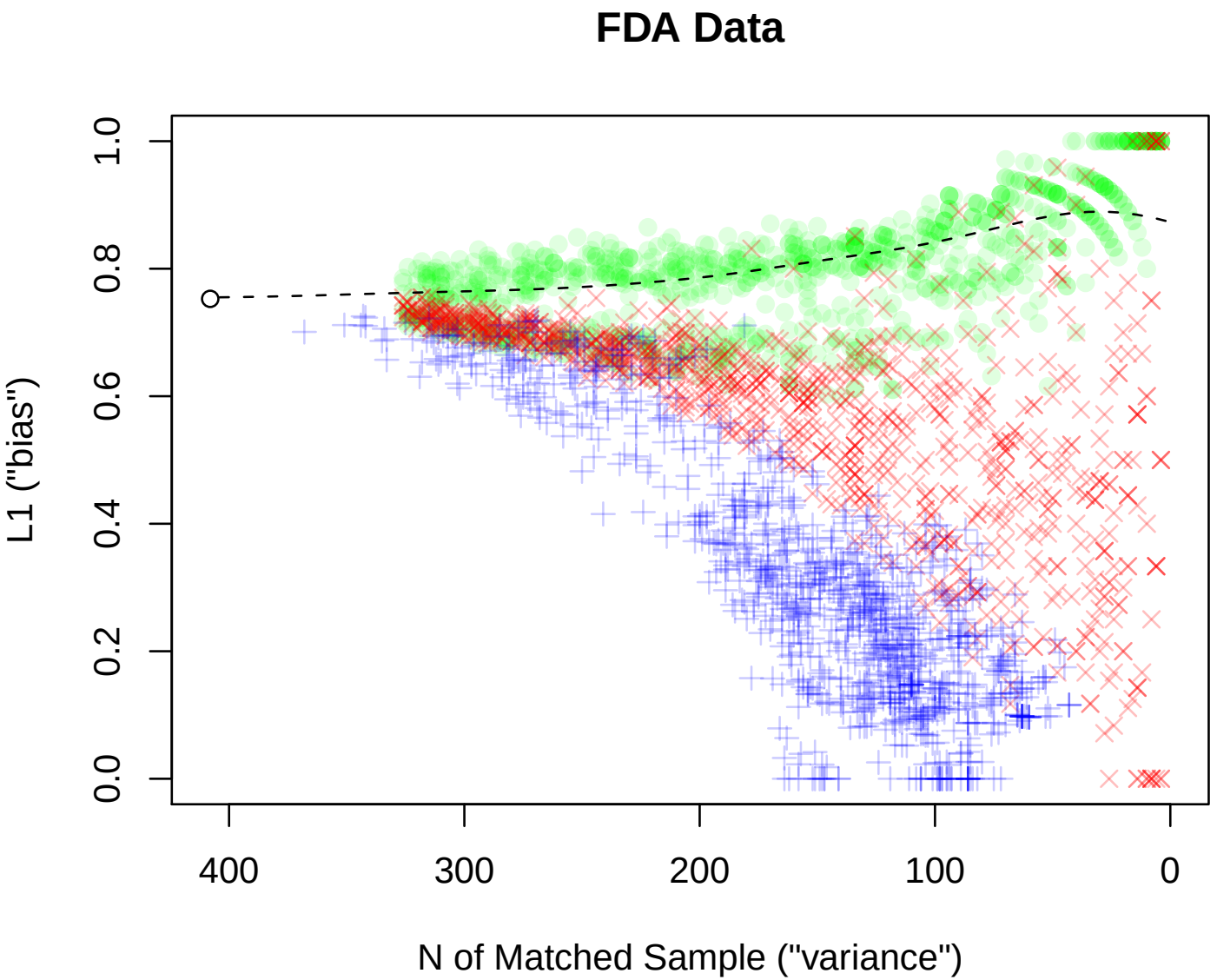

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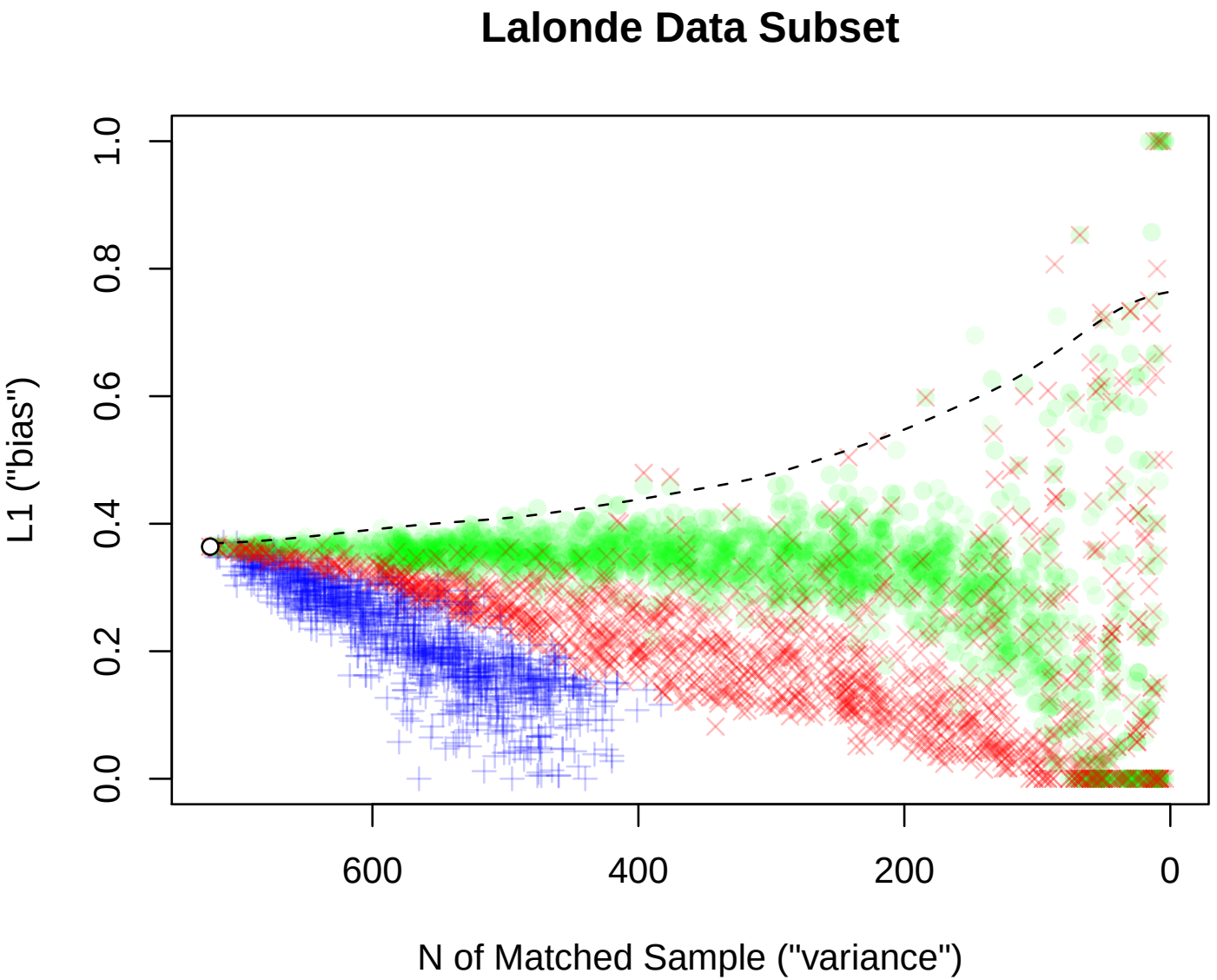
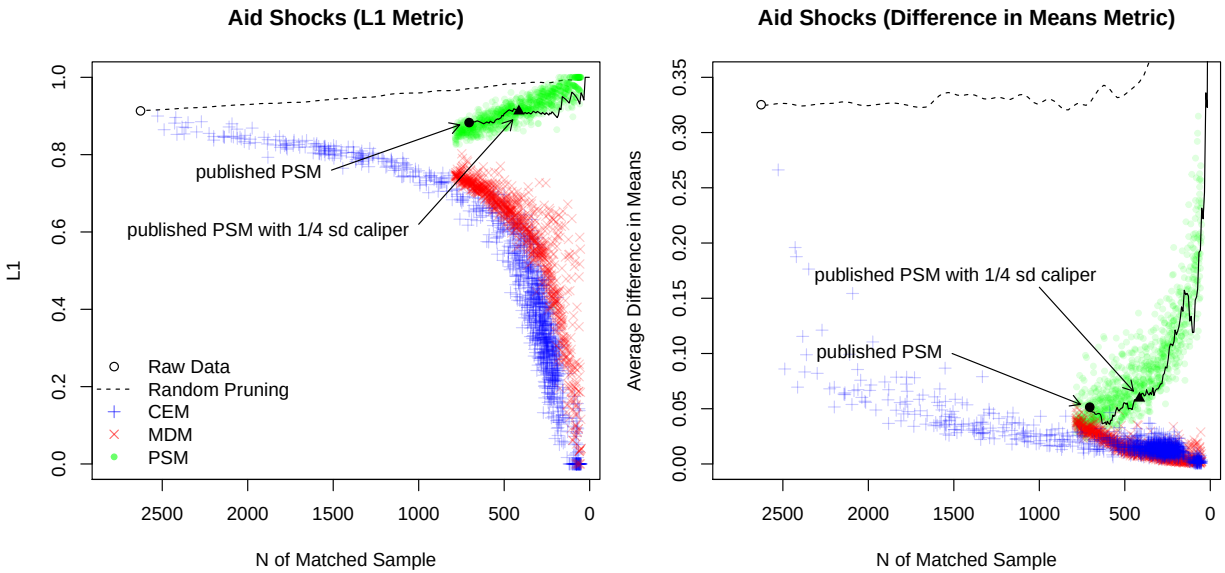
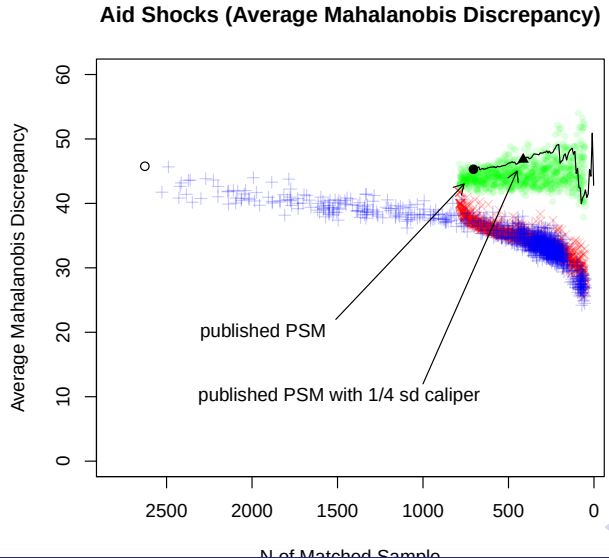
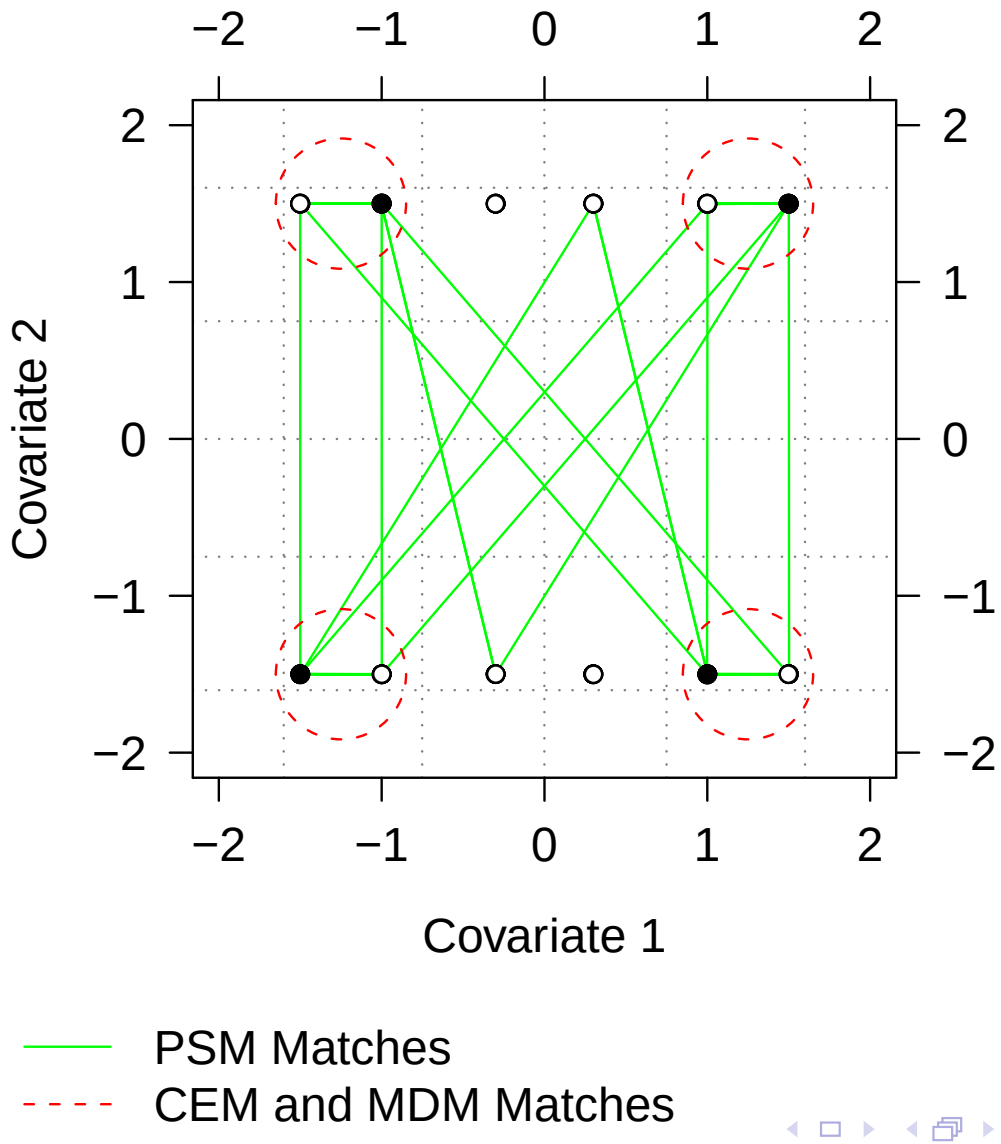


# Space Graphs: Different Imbalance Metrics

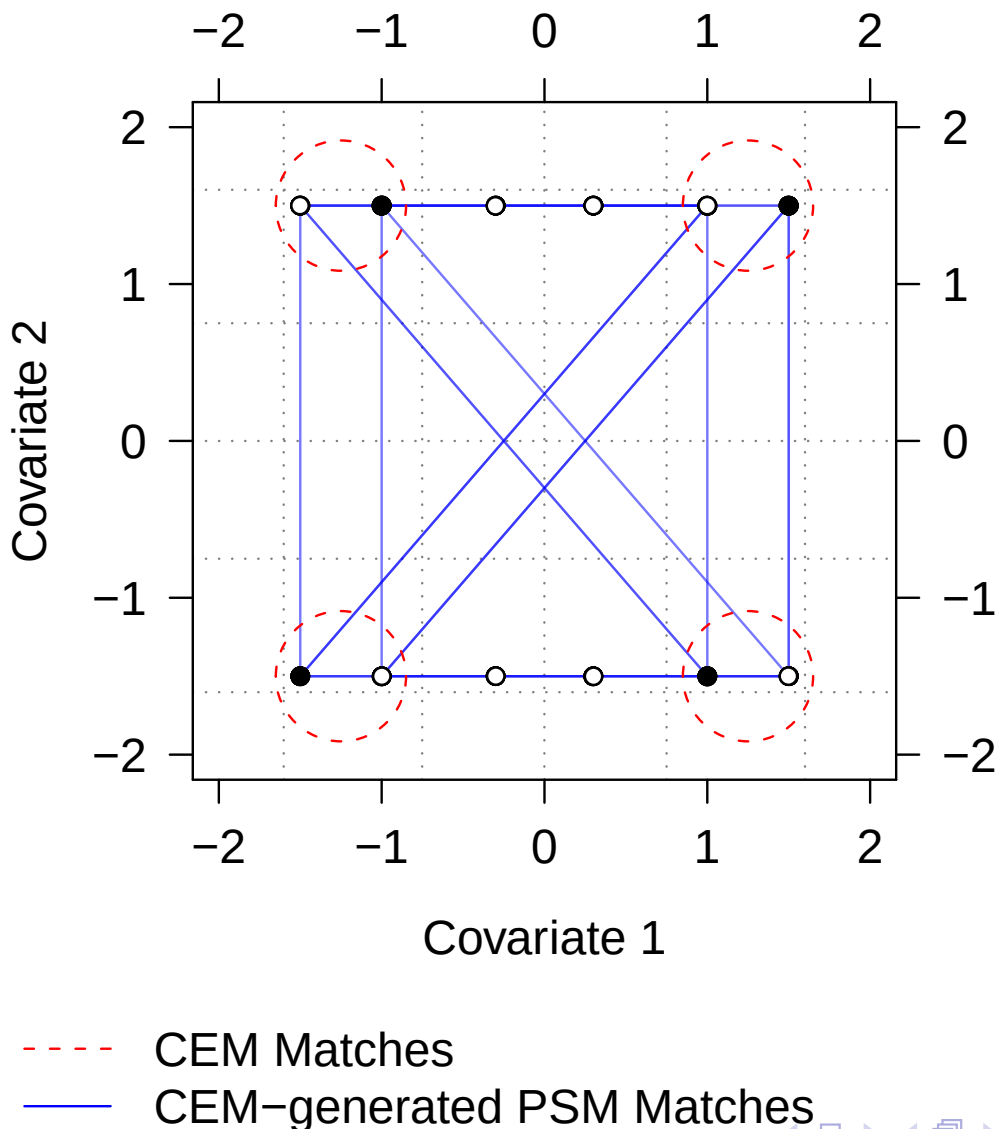


# PSM Approximates Random Matching in Balanced Data



# Destroying CEM with PSM's Two Step Approach



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End of planned slides for today; others follow

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- Simple to teach: coarsen, then exact match

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## Local Imbalance by Variable (given strata fixed by matching method)

$$I_2^{(j)} = \frac{1}{S} \sum_{s=1}^{S} \left| \bar{X}_{m_T^s}^{(j)} - \bar{X}_{m_C^s}^{(j)} \right|, \quad j = 1, \ldots, k$$

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| initial | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | |
| MAH | .20 | .20 | .20 | .20 | .20 | .28 |
| PSC | .11 | .06 | .03 | .06 | .03 | .16 |
| CEM | .04 | .02 | .06 | .06 | .04 | .08 |

Local ($I_2$) and multivariate $\mathcal{L}_1$ imbalance:

| | $X_1$ | $X_2$ | $X_3$ | $X_4$ | $X_5$ | $\mathcal{L}_1$ |
|---|---|---|---|---|---|---|
| initial | | | | | | 1.24 |
| PSC | 2.38 | 1.25 | .74 | 1.25 | .74 | 1.18 |

# CEM in Practice: EPBR-Compliant Data

Monte Carlo: $\mathbf{X}_T \sim N_5(\mathbf{0}, \Sigma)$ and $\mathbf{X}_C \sim N_5(\mathbf{1}, \Sigma)$. $n = 2,000$, reps=5,000
Allow MAH & PSC to match with replacement; use automated CEM

Difference in means ($I_1$):

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$\rightsquigarrow$ CEM dominates EPBR-methods in EPBR Data

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$\rightsquigarrow$ CEM works well in non-EPBR data too

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- Automating user choices Histogram bin size calculations, *Estimated* SATT error bound, Progressive Coarsening
- Detecting Extreme Counterfactuals

# For papers, software (for R and Stata), tutorials, etc.

http://GKing.Harvard.edu/cem