Advanced Quantitative Research Methodology, Lecture Notes: Multiple Equation Models[1]

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- Partially identified models: the likelihood is informative but not about a single point
- Non-identified models: include those that make little sense, even if hard to tell.

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3. A likelihood with a plateau can be informative, but a unique MLE doesn't exist

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So $\{\beta_2 = 2, \beta_3 = 5\}$ gives the same likelihood as $\{\beta_2 = 5, \beta_3 = 2\}$.

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(BTW, you now know how to form the likelihood for multiple equation models!)

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Also assume parametric independence, and you can estimate the equations separately.

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$$= \prod_{i=1}^{n} (2\pi)^{-1}|\Sigma|^{-1/2} \exp\left[-\frac{1}{2}(y_i - \mu_i)'\Sigma^{-1}(y_i - \mu_i)\right]$$

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7. $\implies$ identification of extra parameters in multiple equation models with identical X's comes solely from model assumptions

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$$\begin{aligned} \text{Vote:} \quad & \mu_{1i} = x_{1i}\beta_1 + x_{2i}\beta_2 + \mu_{2i}\beta_3 \\ \text{PID:} \quad & \mu_{2i} = x_{1i}\gamma_1 + x_{3i}\gamma_2 + \mu_{1i}\gamma_3 \end{aligned}$$

Where,

x_1 demographics

x_2 candidate characteristics (affecting vote but not PID)

Reciprocal Causation

Stochastic component:

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Where,

x_1 demographics

x_2 candidate characteristics (affecting vote but not PID)

x_3 parents PID (affecting PID but not vote)

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Likelihood:

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$$f(y|\mu,\Sigma) = \prod_{i=1}^{n} N(y_i|\mu_i,\Sigma)$$
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Where,

$$y_i = (y_{1i}, y_{2i})',$$

Reciprocal Causation

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$$

Where,

$$
\begin{aligned}
y_i &= (y_{1i}, y_{2i})',\\
\mu_i &= (\mu_{1i}, \mu_{2i})'\\
\Sigma &= \begin{pmatrix} \sigma_1 & \sigma_{12} \\ \sigma_{12} & \sigma_2 \end{pmatrix}
\end{aligned}
$$

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2. Time series models are recursively reparameterized
3. This model requires *multiple equation reparameterization*.

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\mu_{1i} &= x_{1i}\beta_1 + x_{2i}\beta_2 + \mu_{2i}\beta_3 \\
&= x_{1i}\beta_1 + x_{2i}\beta_2 + (x_{1i}\gamma_1 + x_{3i}\gamma_2 + \mu_{1i}\gamma_3)\beta_3
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&= \left(\frac{1}{1-\gamma_3\beta_3}\right)\left[x_{1i}\beta_1 + (x_{1i}\gamma_1 + x_{3i}\gamma_2)\beta_3 + x_{2i}\beta_2\right]
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Suppose we drop X_2 and X_3

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$$\mu_{1i} = \left(\frac{1}{1-\gamma_3\beta_3}\right)[x_{1i} + x_{1i}\gamma_1\beta_3]$$

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\begin{aligned}
\mu_{1i} &= \left(\frac{1}{1-\gamma_3\beta_3}\right)[x_{1i} + x_{1i}\gamma_1\beta_3] \\
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- $\{\beta_1 = 1,\ \gamma_1 = 30,\ \beta_3 = 555,\ \gamma_3 = -30\} \implies \mu_{1i} = -x_i.$

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$$\begin{aligned}\mu_{1i} &= \left(\frac{1}{1-\gamma_3\beta_3}\right)[x_{1i} + x_{1i}\gamma_1\beta_3] \\ &= x_{1i}\left(\frac{\beta_1+\gamma_1\beta_3}{1-\gamma_3\beta_3}\right)\end{aligned}$$

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- As long as $\beta_1 = 1$ and $\gamma_1 = -\gamma_3$, for any value of β_3, $\mu_{1i} = -x_i$.

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- As long as $\beta_1 = 1$ and $\gamma_1 = -\gamma_3$, for any value of β_3, $\mu_{1i} = -x_i$.
- Estimates are highly sensitive to assumptions about X_2 and X_3

Multinomial Choice Models

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1. $k \geq 2$ nominal choices, from which one is chosen.

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$$\mu_{ij} = x_{ij} \beta_j$$

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$$U_i^* \sim N(u_i^* | \mu_i, \Sigma)$$
$$\mu_{ij} = x_{ij}\beta_j$$

with observation mechanism:

$$Y_{ij} = \begin{cases} 1 & \text{if } U_{ij}^* > U_{ij'}^* \ \forall \ j \neq j' \\ 0 & \text{otherwise} \end{cases}$$

The stochastic component:

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$$\Pr(Y_{ij} = 1) = \pi_{ij}, \quad \text{s.t.} \quad \sum_{j=1}^{J} \pi_{ij} = 1 \quad \text{for} \quad i = 1, \ldots, n$$

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$$Y_{ij} = \begin{cases} 1 & \text{if } Y^*_{ij} > 0 \\ 0 & \text{otherwise} \end{cases}$$

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$$\pi_{ij} = \Pr(y_{ij} = 1)$$

The stochastic component:

$$\Pr(Y_{ij}=1)=\pi_{ij}, \quad \text{s.t.} \quad \sum_{j=1}^{J}\pi_{ij}=1 \quad \text{for} \quad i=1,\dots,n$$

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$$\begin{aligned}\pi_{ij}&=\Pr(y_{ij}=1)\\ &=\Pr(Y^*_{i1}\le 0,\dots,{\color{red}Y^*_{ij}>0},\dots,Y^*_{iJ}\le 0)\end{aligned}$$

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$$\begin{aligned}\pi_{ij} &= \Pr(y_{ij}=1)\\ &= \Pr(Y^*_{i1}\le 0,\ldots,Y^*_{ij}>0,\ldots,Y^*_{iJ}\le 0)\\ &= \int_{-\infty}^{0}\cdots\int_{0}^{\infty}\cdots\int_{-\infty}^{0} N(y|\mu_i,\Sigma)dy_{i1}\cdots dy_{ij}\cdots dy_{iJ}\end{aligned}$$

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- All elements of Σ are not identified.
- The entire model is only weakly identified
- Model is a straightforward generalization of SURM, or of univariate probit

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  - Identical to binary logit when $J = 2$, but it generalizes
- The Likelihood: $L(\beta|y) = \prod_{i=1}^{n} \left[\prod_{j=1}^{J} \pi_{ij}^{y_{ij}}\right]$

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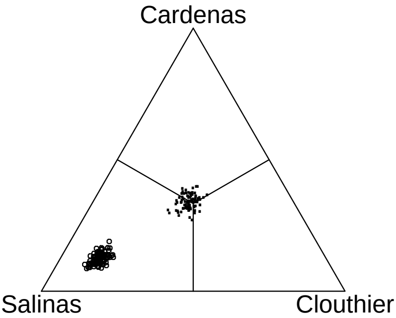
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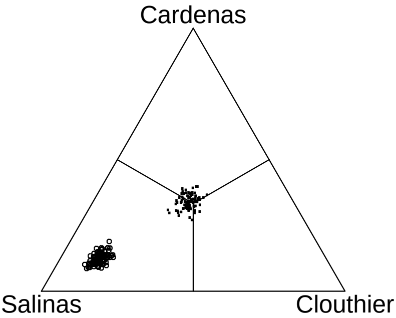
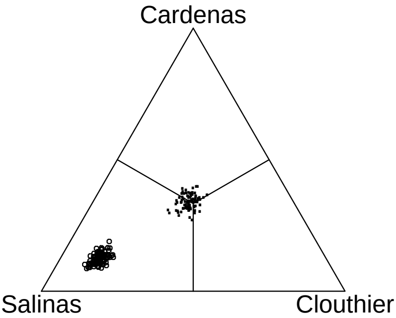


- Each point in the figure is an election outcome drawn randomly from a world in which all voters believe Salinas' PRI party is strengthing (for the "o"'s in the bottom left) or weakening (for the "."'s in the middle), with other variables held constant at their means. (100 of each).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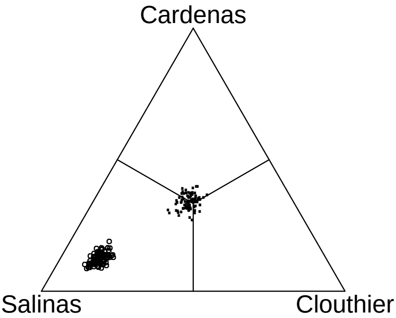

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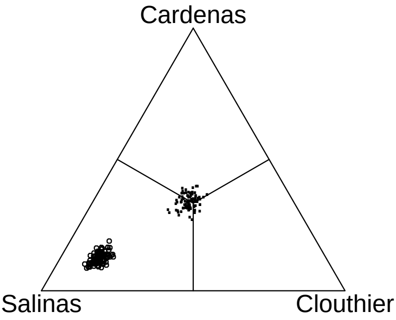

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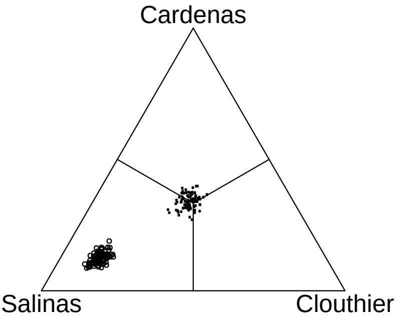

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- The PRI in fact lost the next election (finally, after 72 years in power)

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  where $X_{ij}\beta = \alpha + \beta x_{ij}$.
- Applying binary logit: telling the computer you have $2n$ observations (husbands and wives) when you really only have n (families) — known as the double barreled data extender!

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  - Reading: King, Gary. "Proper Nouns and Methodological Propriety: Pooling Dyads in International Relations Data," *International Organization*, Vol. 55, No. 2 (Fall, 2001): Pp. 497–507.

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- Ethan Katz (*Political Analysis*, 2000) showed that if $T \geq 16$, you're ok.

- Could run binary logit, but need to control for hard-to-measure country-specific variables (e.g., war-proneness, historical animosity, etc.)
- One possible solution: include dummy variables for countries to control for *all* country-specific variables
- But, for MLE's to be consistent, the number of parameters must stay fixed as n increases (or not increase faster than n).
- This makes sense: if the estimates do not encompass more information as n increases, the sampling distribution will not collapse to a spike.
- The inconsistency is serious: $\hat{\beta}$ can be off by a factor of 2.
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- The alternative is used, but its not a very happy solution!