

Advanced Quantitative Research Methodology, Lecture Notes: Research Designs for Causal Inference¹

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April 1, 2012

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- Kosuke Imai, Gary King, and Elizabeth Stuart. **Misunderstandings among Experimentalists and Observationalists: Balance Test Fallacies in Causal Inference** *Journal of the Royal Statistical Society, Series A* Vol. 171, Part 2 (2008): Pp. 1-22
<http://gking.harvard.edu/files/abs/matchse-abs.shtml>

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- **Fundamental problem of causal inference.** Only one potential outcome is ever observed:
If $T_i = 0$, $Y_i(0) = Y_i$ $Y_i(1) = ?$
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If $T_i = 0$, $Y_i(0) = Y_i$ $Y_i(1) = ?$
If $T_i = 1$, $Y_i(0) = ?$ $Y_i(1) = Y_i$
- (I_i, T_i, Y_i) are random; $Y_i(1)$ and $Y_i(0)$ are fixed.

Quantities of Interest

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- Sample Average Treatment Effect:

$$SATE \equiv \frac{1}{n} \sum_{i \in \{I_i=1\}} TE_i$$

Decomposition of Causal Effect Estimation Error

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- Difference in means estimator:

$$D \equiv \left(\frac{1}{n/2} \sum_{i \in \{I_i=1, T_i=1\}} Y_i \right) - \left(\frac{1}{n/2} \sum_{i \in \{I_i=1, T_i=0\}} Y_i \right).$$

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- Estimation Error:

$$\Delta \equiv \text{PATE} - D$$

- Pretreatment confounders: X are observed and U are unobserved
- Decomposition:

$$\begin{aligned} \Delta &= \Delta_S + \Delta_T \\ &= (\Delta_{S_X} + \Delta_{S_U}) + (\Delta_{T_X} + \Delta_{T_U}) \end{aligned}$$

Error due to Δ_S (sample selection), Δ_T (treatment imbalance), and each due to observed (X_i) and unobserved (U_i) covariates

Selection Error

- Definition:

$$\begin{aligned}\Delta_S &\equiv \text{PATE} - \text{SATE} \\ &= \frac{N - n}{N}(\text{NATE} - \text{SATE}),\end{aligned}$$

where NATE is the nonsample average treatment effect.

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- Δ_S vanishes if:
 - 1 The sample is a census ($I_i = 1$ for all observations and $n = N$);
 - 2 $\text{SATE} = \text{NATE}$; or
 - 3 Switch quantity of interest from PATE to SATE.

Decomposing Selection Error

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- conditions are unverifiable: X is observed only in sample and U is not observed at all.
- Δ_{S_X} vanishes if weighting on X
- Δ_{S_U} cannot be corrected after the fact

Balance in Observed and Unobserved Covariates

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Balance in Observed and Unobserved Covariates

- Decomposition:

$$\Delta_T = \Delta_{T_X} + \Delta_{T_U}$$

- $\Delta_{T_X} = 0$ when X is balanced between treateds and controls:

$$\tilde{F}(X \mid T = 1, I = 1) = \tilde{F}(X \mid T = 0, I = 1).$$

Verifiable from data; can be generated ex ante by blocking or enforced ex post via matching or parametric adjustment

Balance in Observed and Unobserved Covariates

- Decomposition:

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Verifiable from data; can be generated ex ante by blocking or enforced ex post via matching or parametric adjustment

- $\Delta_{T_U} = 0$ when U is balanced between treateds and controls:

$$\tilde{F}(U \mid T = 1, I = 1) = \tilde{F}(U \mid T = 0, I = 1).$$

Unverifiable. Achieved only by assumption or, on average, by random treatment assignment

Alternative Quantities of Interest

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$$\text{SATT} \equiv \frac{1}{n/2} \sum_{i \in \{I_i=1, T_i=1\}} \text{TE}_i$$

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- Population average treatment effect on the treated:

$$\text{PATT} \equiv \frac{1}{N^*} \sum_{i \in \{T_i=1\}} \text{TE}_i$$

where $N^* = \sum_{i=1}^N T_i$ is the number of treated units in the population.

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where $N^* = \sum_{i=1}^N T_i$ is the number of treated units in the population.

- When are these of more interest than PATE and SATE? Why never for randomized experiments? Why usually for matching?

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where $N^* = \sum_{i=1}^N T_i$ is the number of treated units in the population.

- When are these of more interest than PATE and SATE? Why never for randomized experiments? Why usually for matching?
- Analogous estimation error decomposition: $\Delta' = \text{PATT} - D$, holds:

$$\Delta' = (\Delta'_{S_X} + \Delta'_{S_U}) + (\Delta'_{T_X} + \Delta'_{T_U})$$

Effects of Design Components on Estimation Error

Design Choice

$$\Delta_{S_X}$$

$$\Delta_{S_U}$$

$$\Delta_{T_X}$$

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Design Choice

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Effects of Design Components on Estimation Error

Design Choice

Random sampling

$$\begin{array}{cccc} \Delta_{S_X} & \Delta_{S_U} & \Delta_{T_X} & \Delta_{T_U} \\ \stackrel{\text{avg}}{=} 0 & \stackrel{\text{avg}}{=} 0 & & \end{array}$$

Effects of Design Components on Estimation Error

Design Choice

	Δ_{S_X}	Δ_{S_U}	Δ_{T_X}	Δ_{T_U}
Random sampling	$\underset{\text{avg}}{=} 0$	$\underset{\text{avg}}{=} 0$		
Complete stratified random sampling	$= 0$	$\underset{\text{avg}}{=} 0$		

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Focus on SATE rather than PATE	$= 0$	$= 0$		

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Focus on SATE rather than PATE	$= 0$	$= 0$		
Weighting for nonrandom sampling	$= 0$	$= ?$		

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Weighting for nonrandom sampling	$= 0$	$= ?$		
Large sample size	$\rightarrow ?$	$\rightarrow ?$	$\rightarrow ?$	$\rightarrow ?$

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Weighting for nonrandom sampling	$= 0$	$= ?$		
Large sample size	$\rightarrow ?$	$\rightarrow ?$	$\rightarrow ?$	$\rightarrow ?$
Random treatment assignment			$\stackrel{\text{avg}}{=} 0$	$\stackrel{\text{avg}}{=} 0$

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Weighting for nonrandom sampling	$= 0$	$= ?$		
Large sample size	$\rightarrow ?$	$\rightarrow ?$	$\rightarrow ?$	$\rightarrow ?$
Random treatment assignment			$\stackrel{\text{avg}}{=} 0$	$\stackrel{\text{avg}}{=} 0$
Complete blocking			$= 0$	$= ?$

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Large sample size	$\rightarrow ?$	$\rightarrow ?$	$\rightarrow ?$	$\rightarrow ?$
Random treatment assignment			$\stackrel{\text{avg}}{=} 0$	$\stackrel{\text{avg}}{=} 0$
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Exact matching			$= 0$	$= ?$

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Assumption

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No selection bias	$\stackrel{\text{avg}}{=} 0$	$\stackrel{\text{avg}}{=} 0$
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No selection bias	$\stackrel{\text{avg}}{=} 0$	$\stackrel{\text{avg}}{=} 0$		
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Ignorability				$\underset{\text{avg}}{=} 0$
No omitted variables				$= 0$

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 - 4 In the worst case scenerio, matching (just like parametric adjustment) can increase bias (this cannot occur with blocking plus random assignment)
- Adding matching to a parametric model almost always reduces model dependence and bias, and sometimes variance too

The Benefits of Major Research Designs: Overview

	Δ_{S_X}	Δ_{S_U}	Δ_{T_X}	Δ_{T_U}
Ideal experiment	$\rightarrow 0$	$\rightarrow 0$	$= 0$	$\rightarrow 0$
Randomized clinical trials (Limited or no blocking)	$\neq 0$	$\neq 0$	$\overset{\text{avg}}{=} 0$	$\overset{\text{avg}}{=} 0$
Randomized clinical trials (Full blocking)	$\neq 0$	$\neq 0$	$= 0$	$\overset{\text{avg}}{=} 0$
Social Science Field Experiment (Limited or no blocking)	$\neq 0$	$\neq 0$	$\rightarrow 0$	$\rightarrow 0$
Survey Experiment (Limited or no blocking)	$\rightarrow 0$	$\rightarrow 0$	$\rightarrow 0$	$\rightarrow 0$
Observational Study (Representative data set, Well-matched)	≈ 0	≈ 0	≈ 0	$\neq 0$
Observational Study (Unrepresentative but partially, correctable data, well-matched)	≈ 0	$\neq 0$	≈ 0	$\neq 0$
Observational Study (Unrepresentative data set, Well-matched)	$\neq 0$	$\neq 0$	≈ 0	$\neq 0$

For column Q , “ $\rightarrow 0$ ” denotes $E(Q) = 0$ and $\lim_{n \rightarrow \infty} \text{Var}(Q) = 0$, whereas “ $\overset{\text{avg}}{=} 0$ ” indicates zero on average, or $E(Q) = 0$, for a design with a small n . Δ_S can be set to zero if we switch from PATE to SATE.

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The Ideal Experiment (according to the paper)

- Random selection from well-defined population
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- blocking on all known confounders
- random treatment assignment within blocks
- $E(\Delta_{S_X}) = 0, \lim_{n \rightarrow \infty} V(\Delta_{S_X}) = 0$
- $E(\Delta_{S_U}) = 0, \lim_{n \rightarrow \infty} V(\Delta_{S_U}) = 0$

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- random treatment assignment within blocks
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- $E(\Delta_{S_U}) = 0, \lim_{n \rightarrow \infty} V(\Delta_{S_U}) = 0$
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The Ideal Experiment (according to the paper)

- Random selection from well-defined population
- large n
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Randomized Clinical Trials (Little or no Blocking)

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Social Science Field Experiment

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Survey Experiment

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Observational Study, well-matched

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- $\Delta_{T_U} \neq 0$ except by assumption

What is the Best Design?

- The ideal design is rarely feasible
- the Achilles heal of experimental studies: Δ_S and small n
- the Achilles heal of observational studies: Δ_U
- Each design accomodates best to the applications for which it was designed
- Effort in experimental studies: random assignment
- Effort in observational studies: adjusting for X (via matching or parametric models)

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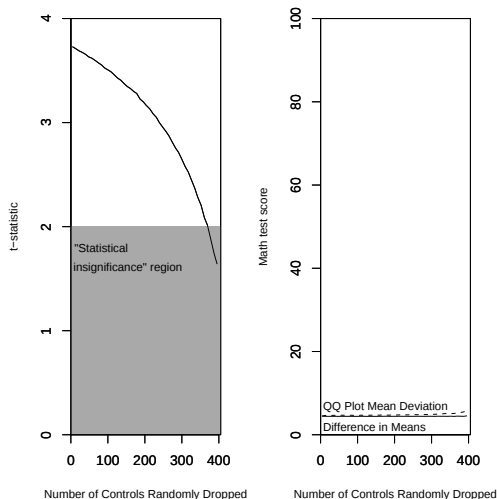
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 - if variables became available after treatment assignment, a t-test only checks for whether randomization was done appropriately
 - randomization balances on average; any one random assignment is not balanced exactly (which is why its better to block)

The Balance Test Fallacy in Matching Research



Randomly dropping observations “improves” balance???

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 - If you cannot control G (and we don't usually try to minimize γ so we don't cook the books), then G must be reduced without limit.
- No threshold level is safe.