Advanced Quantitative Research Methodology, Lecture Notes: Matching Methods for Causal Inference[1]

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- Formal Assumptions: We save for last

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  - If treated and control groups are better balanced than when you started, due to pruning, model dependence is reduced

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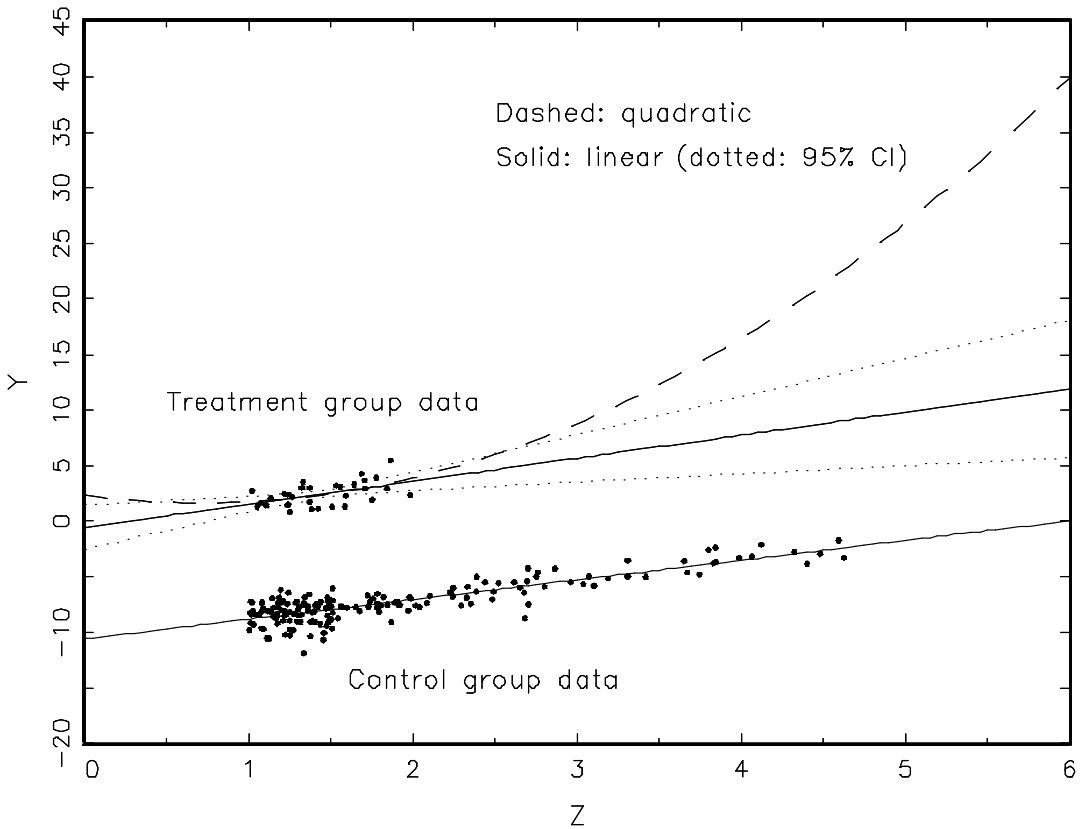

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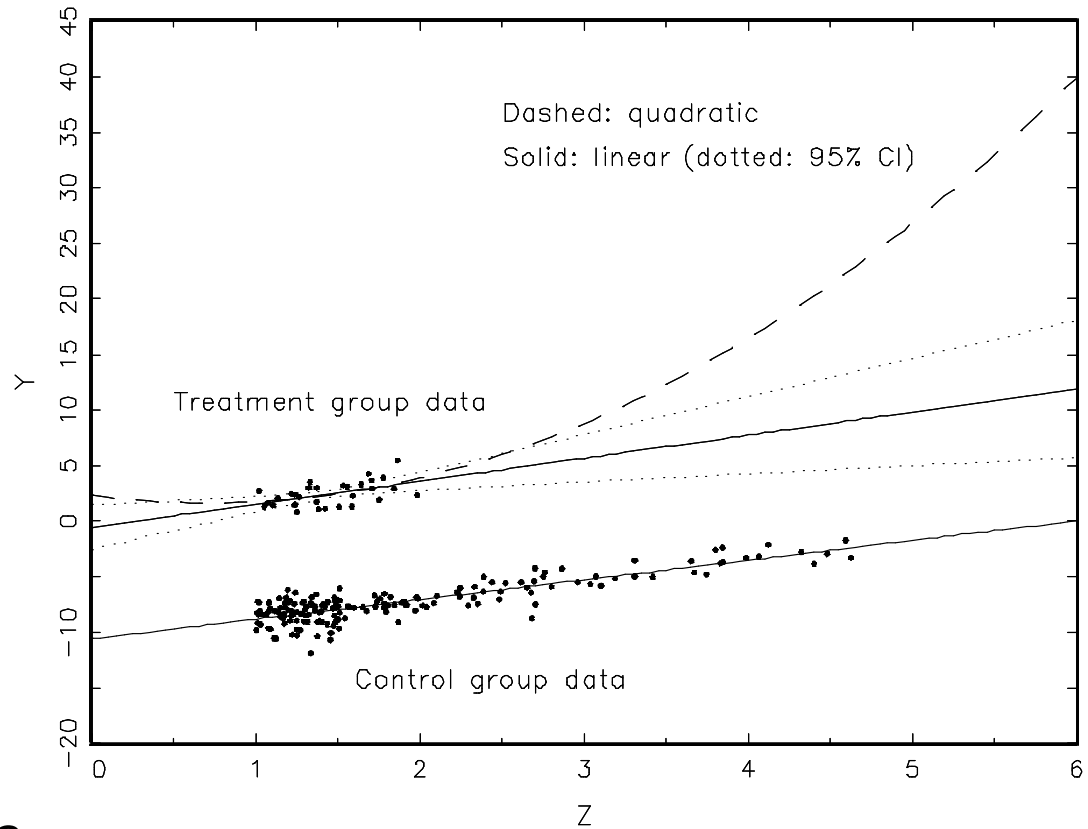

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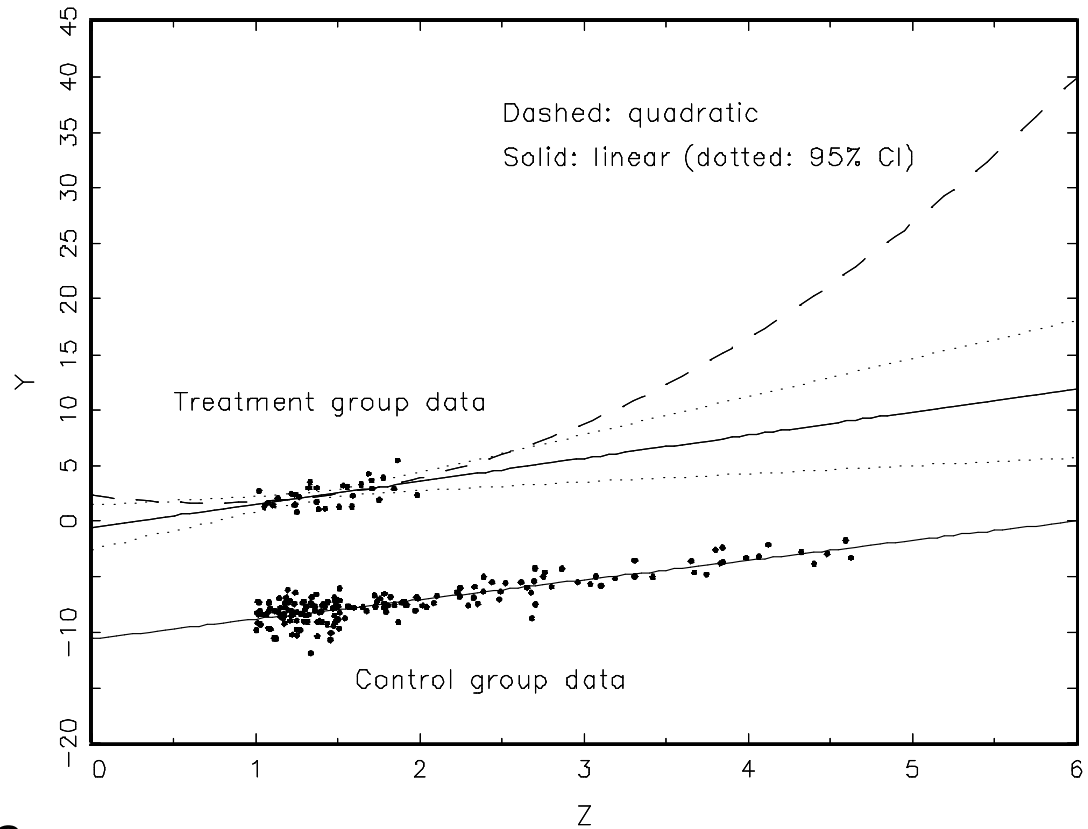

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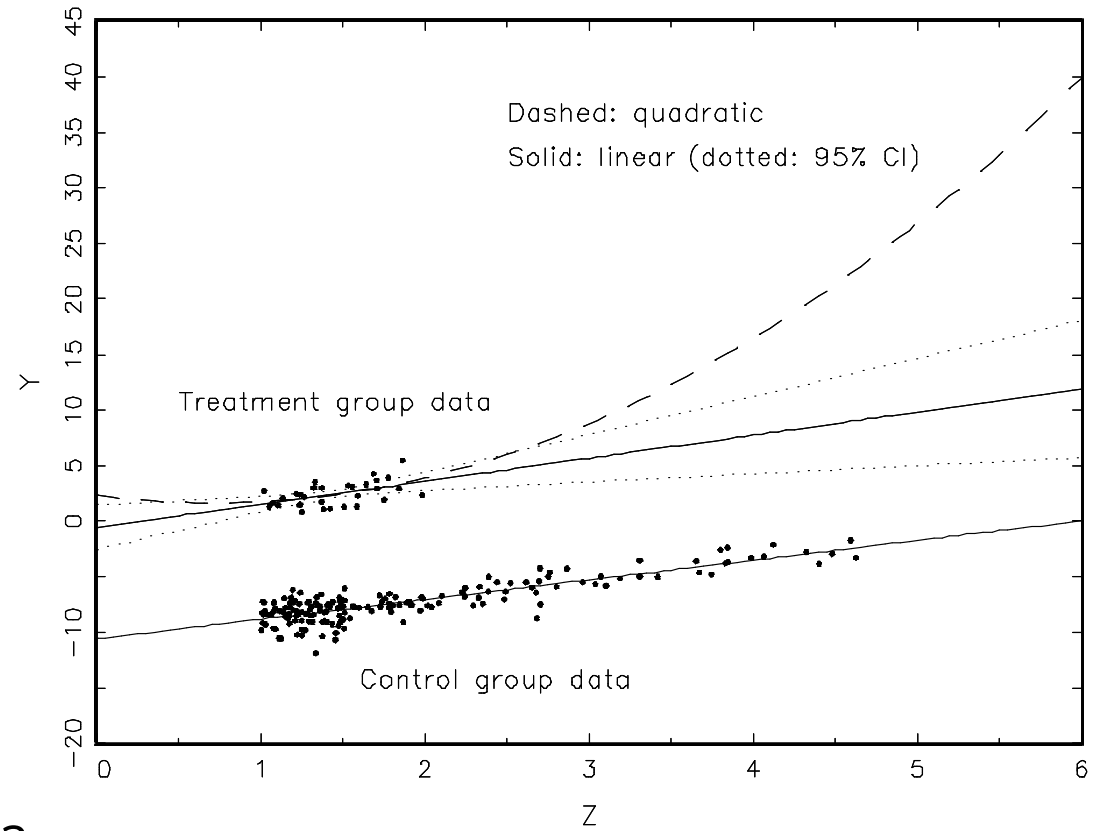

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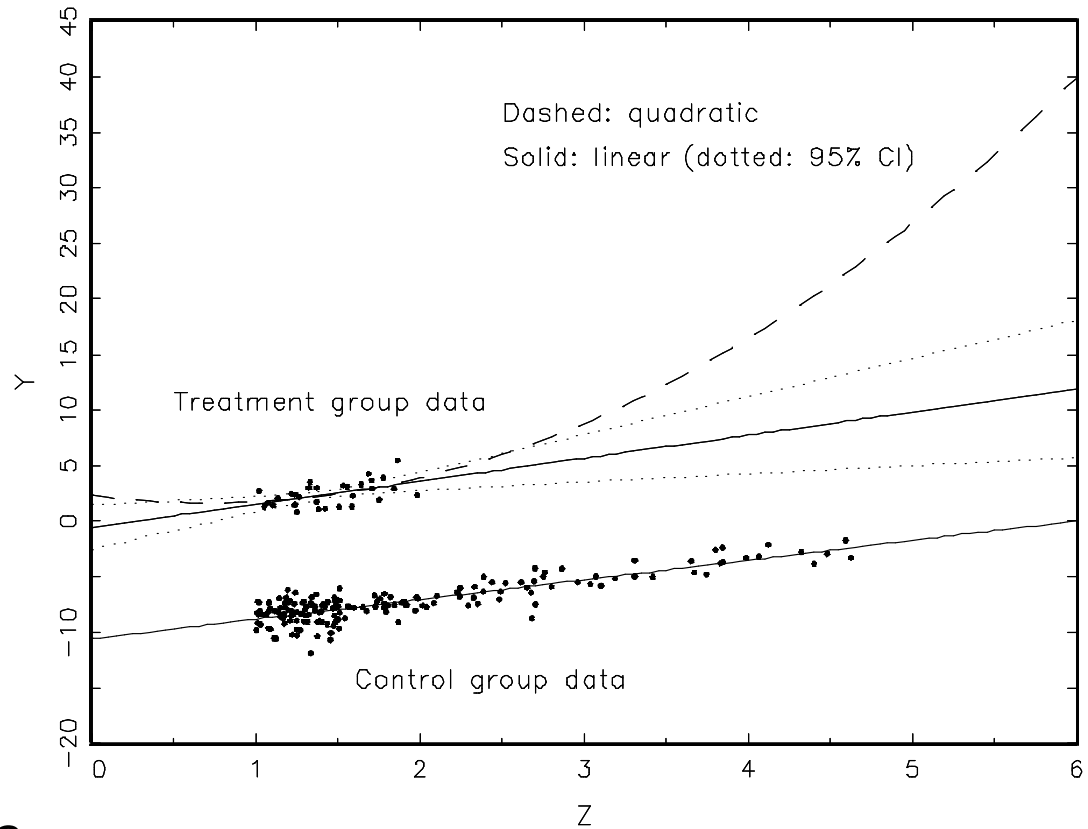

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- Model remaining imbalance (as you would w/o matching)

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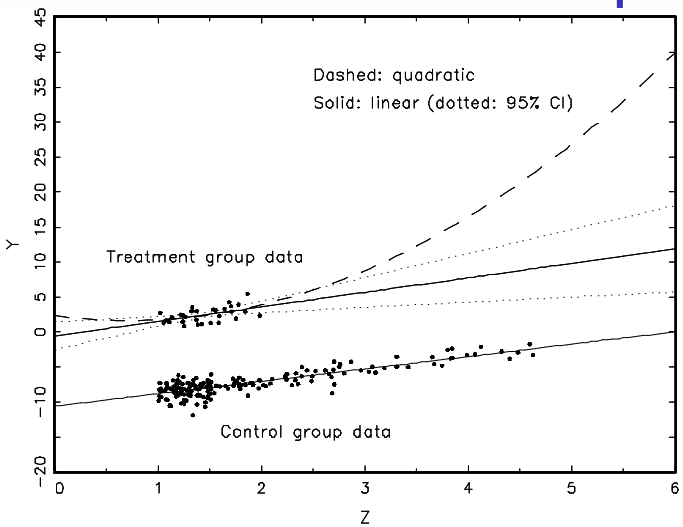

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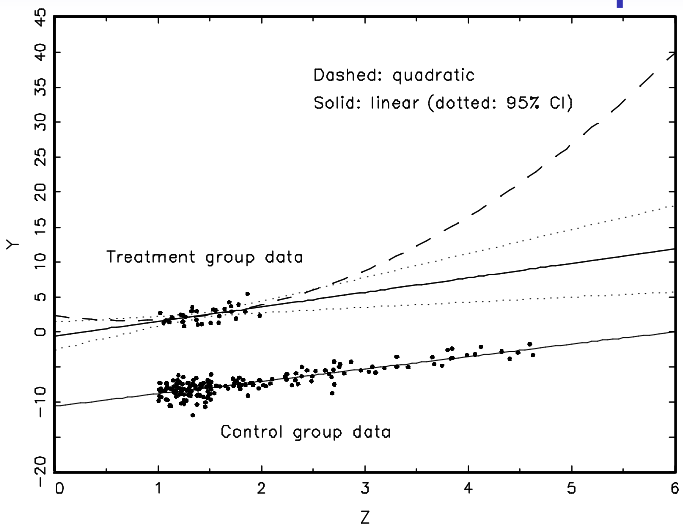

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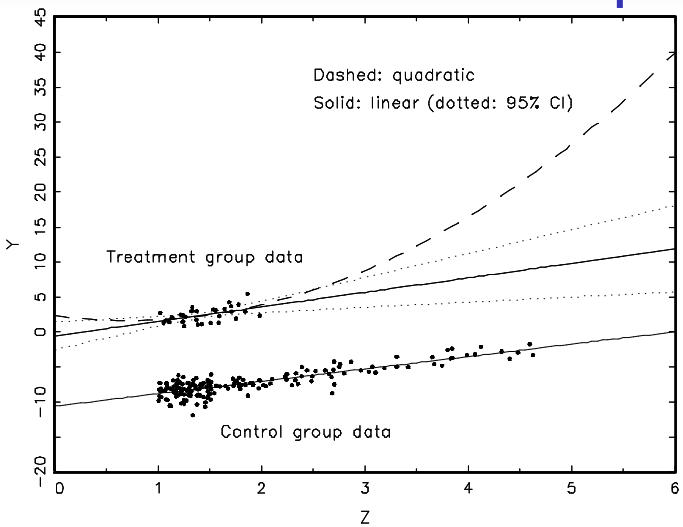

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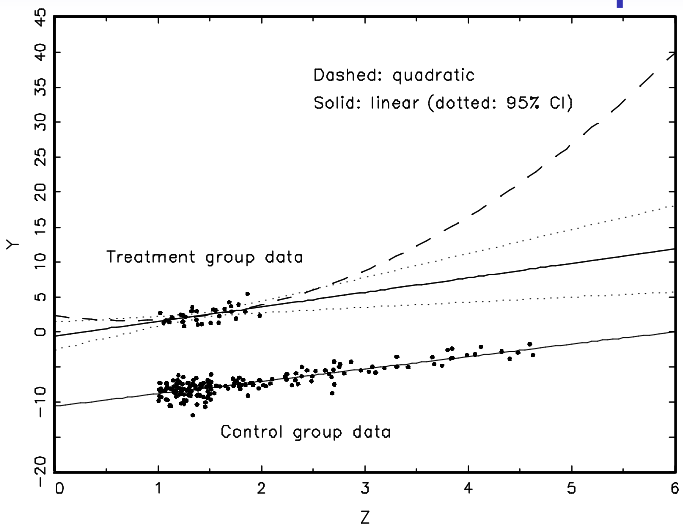

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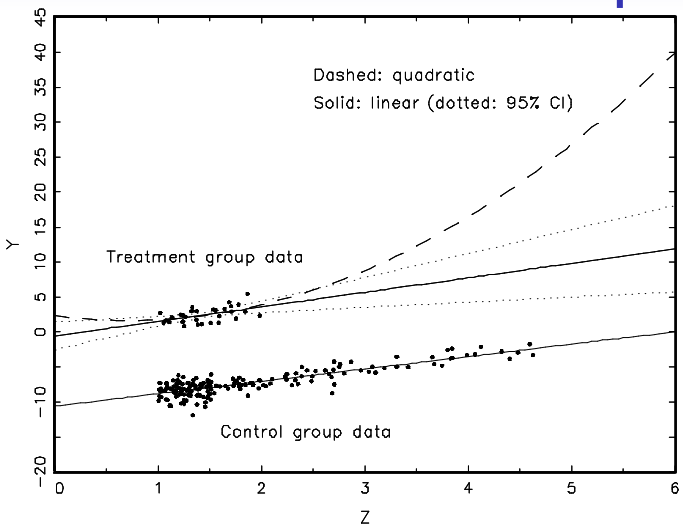

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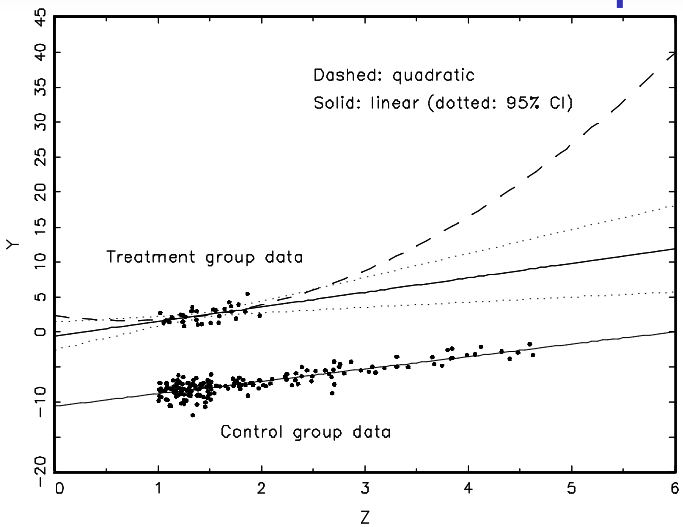

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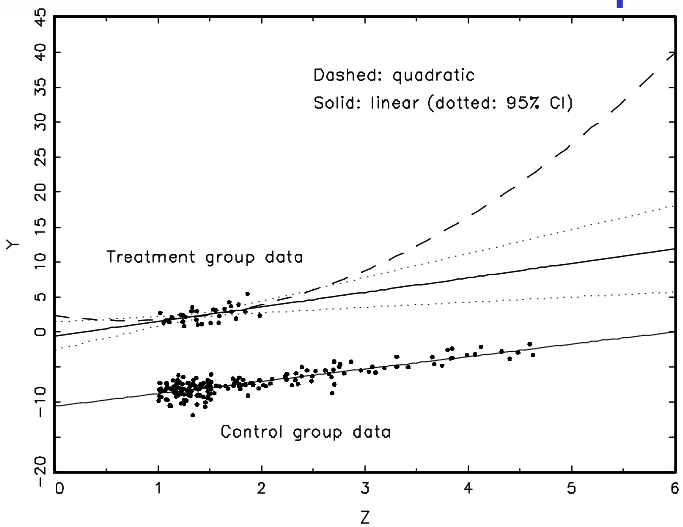

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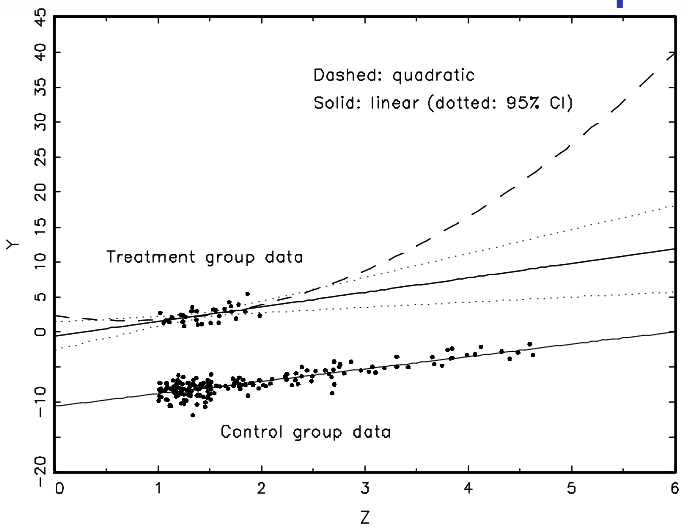

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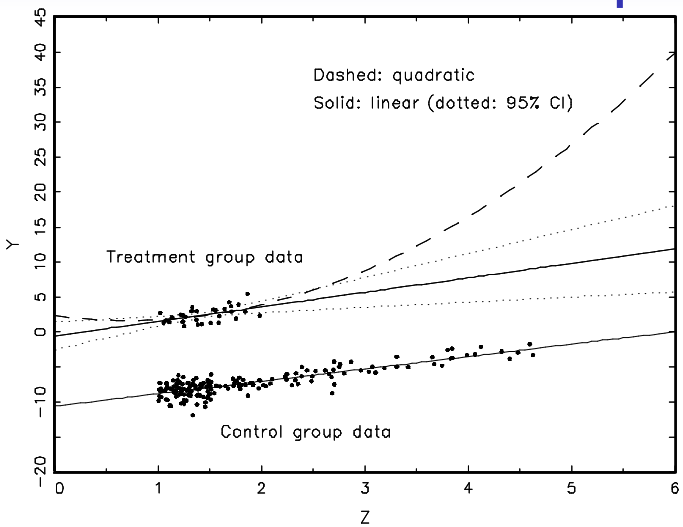

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  4. Easiest: Coarsened Exact Matching, no separate step needed

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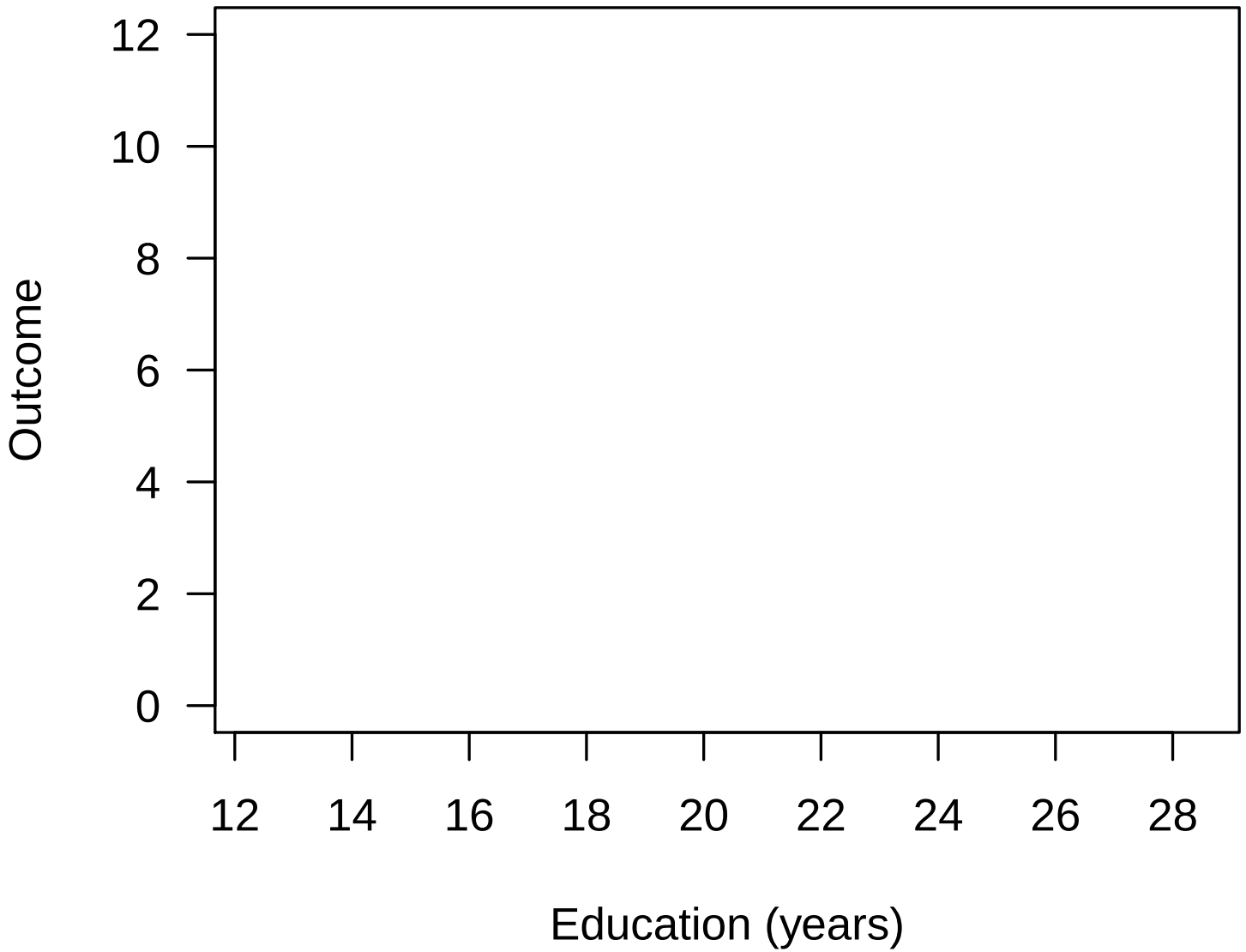

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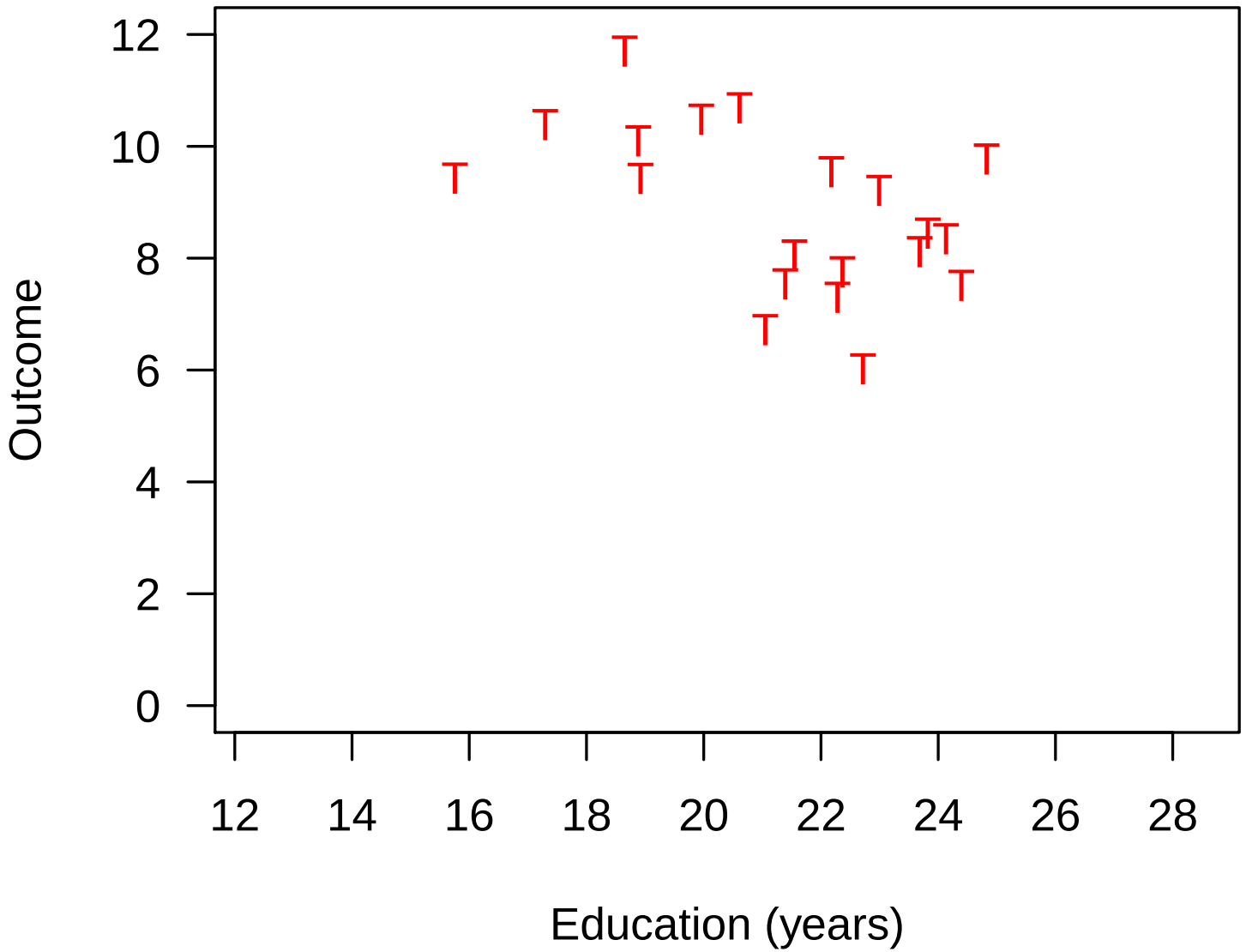

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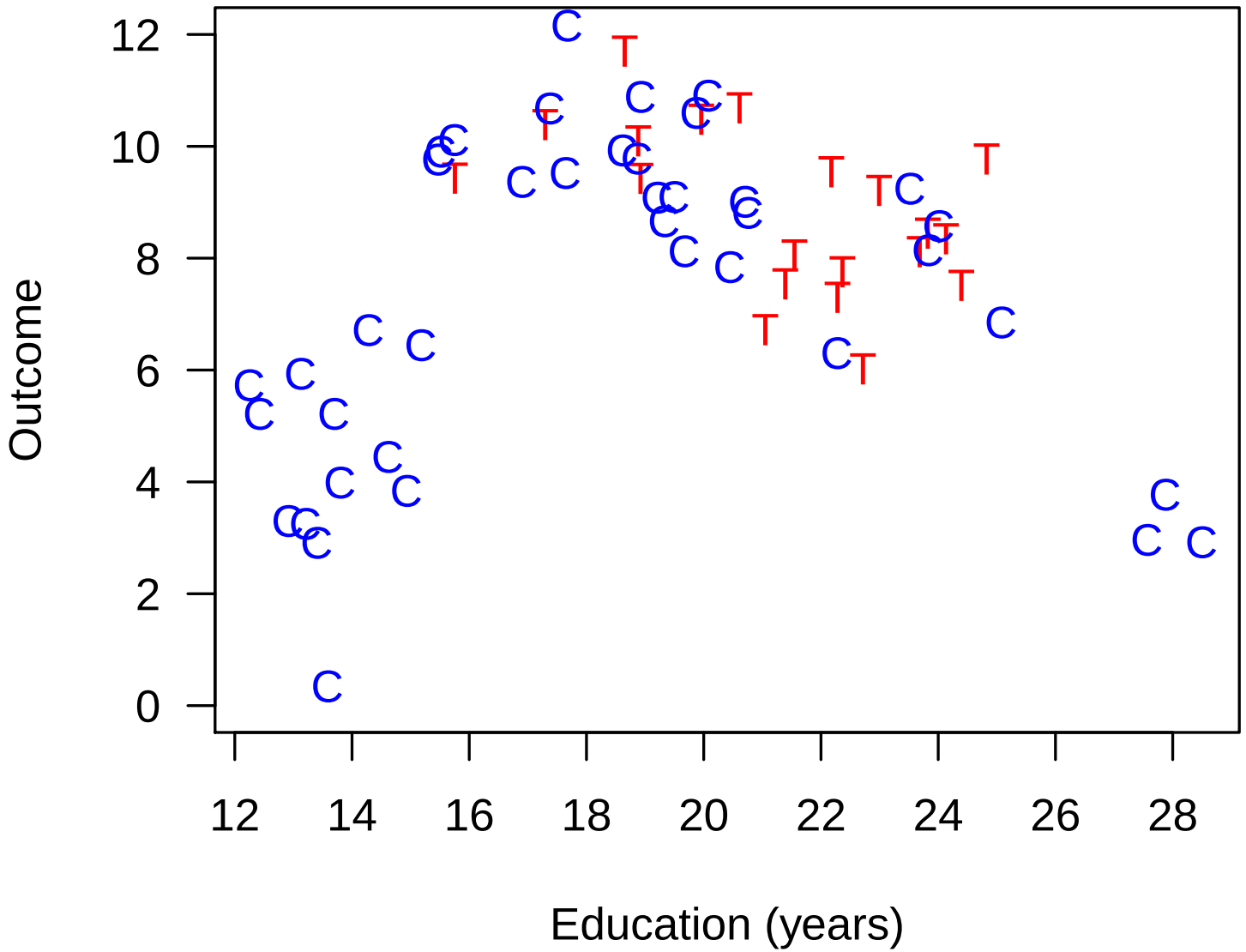

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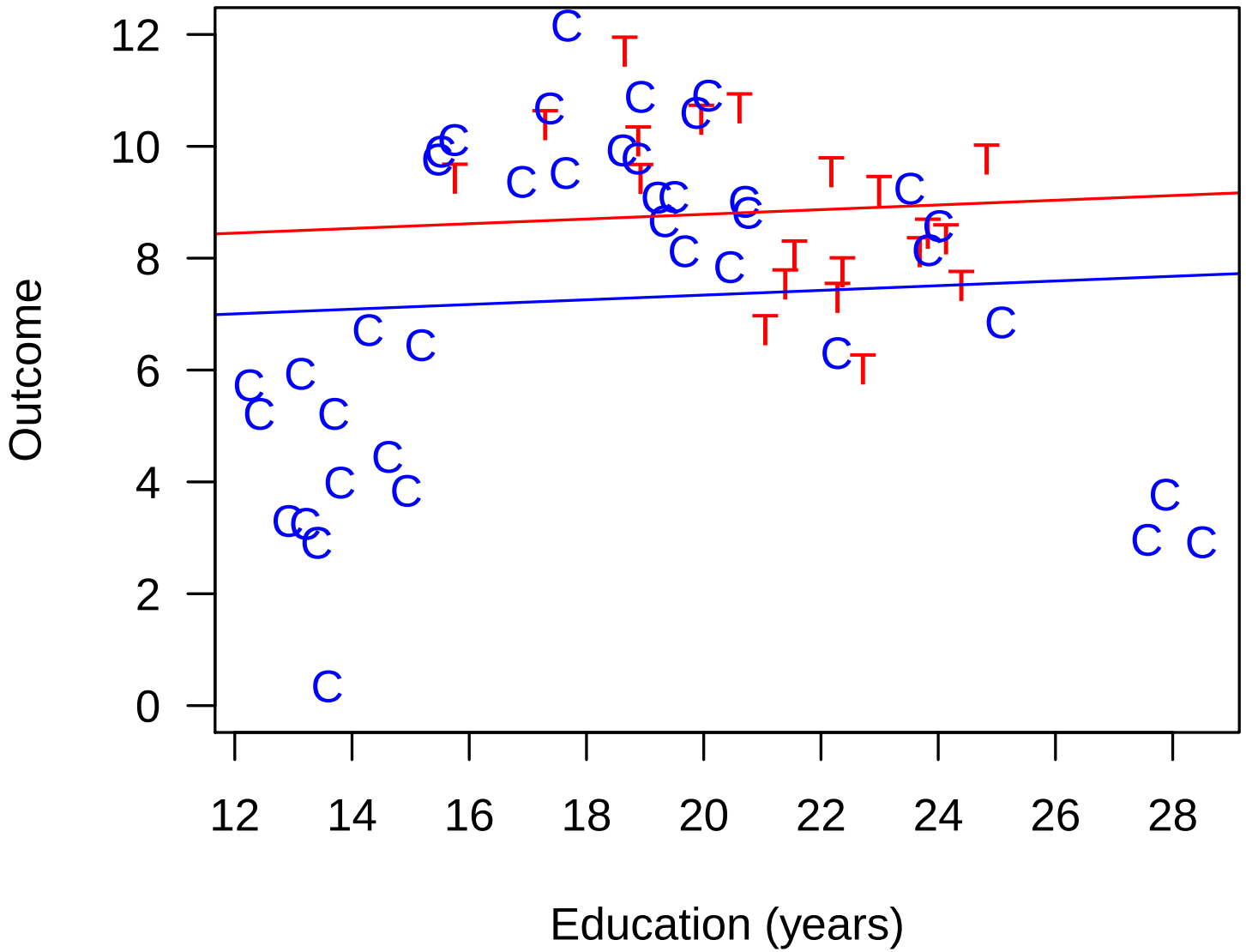

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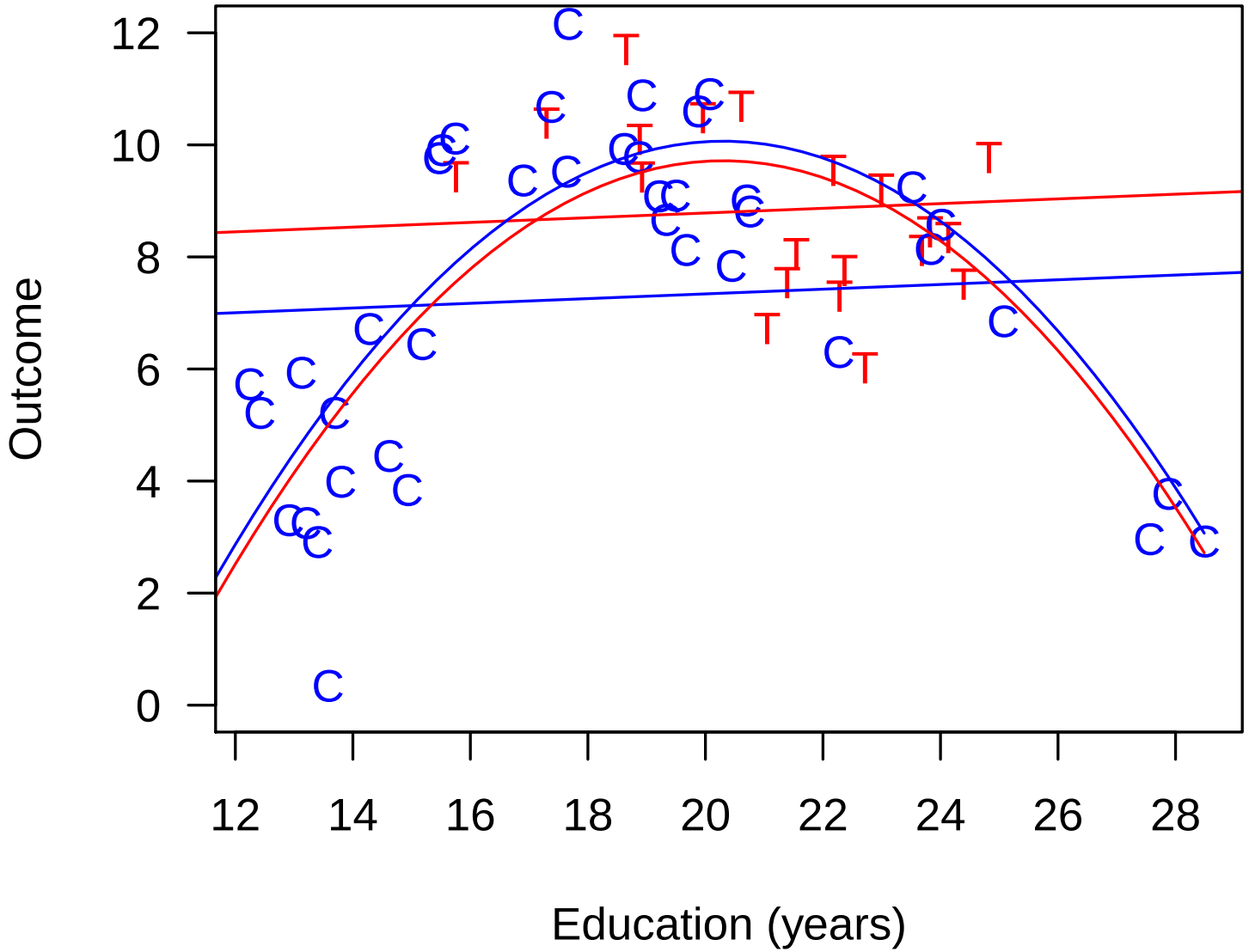

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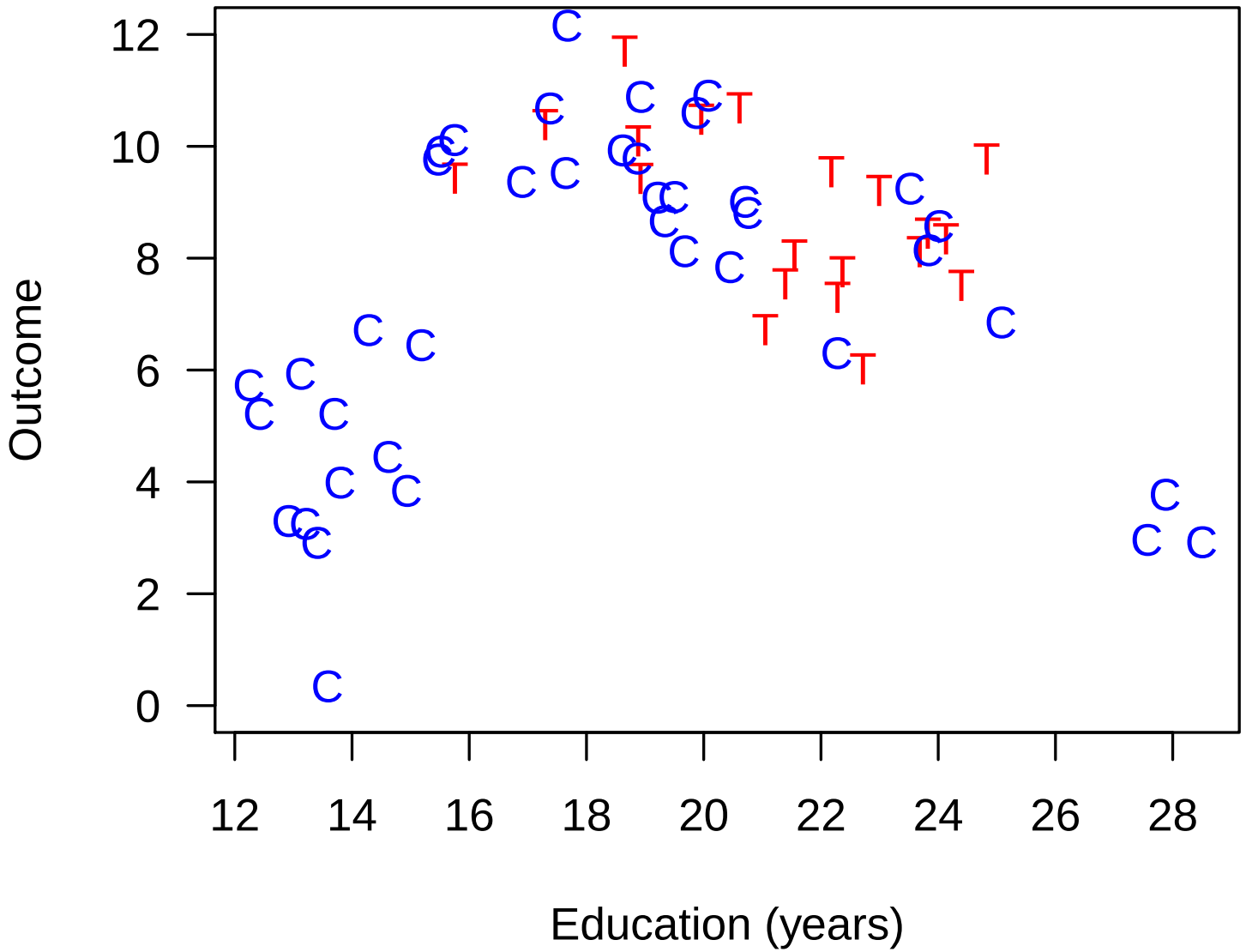

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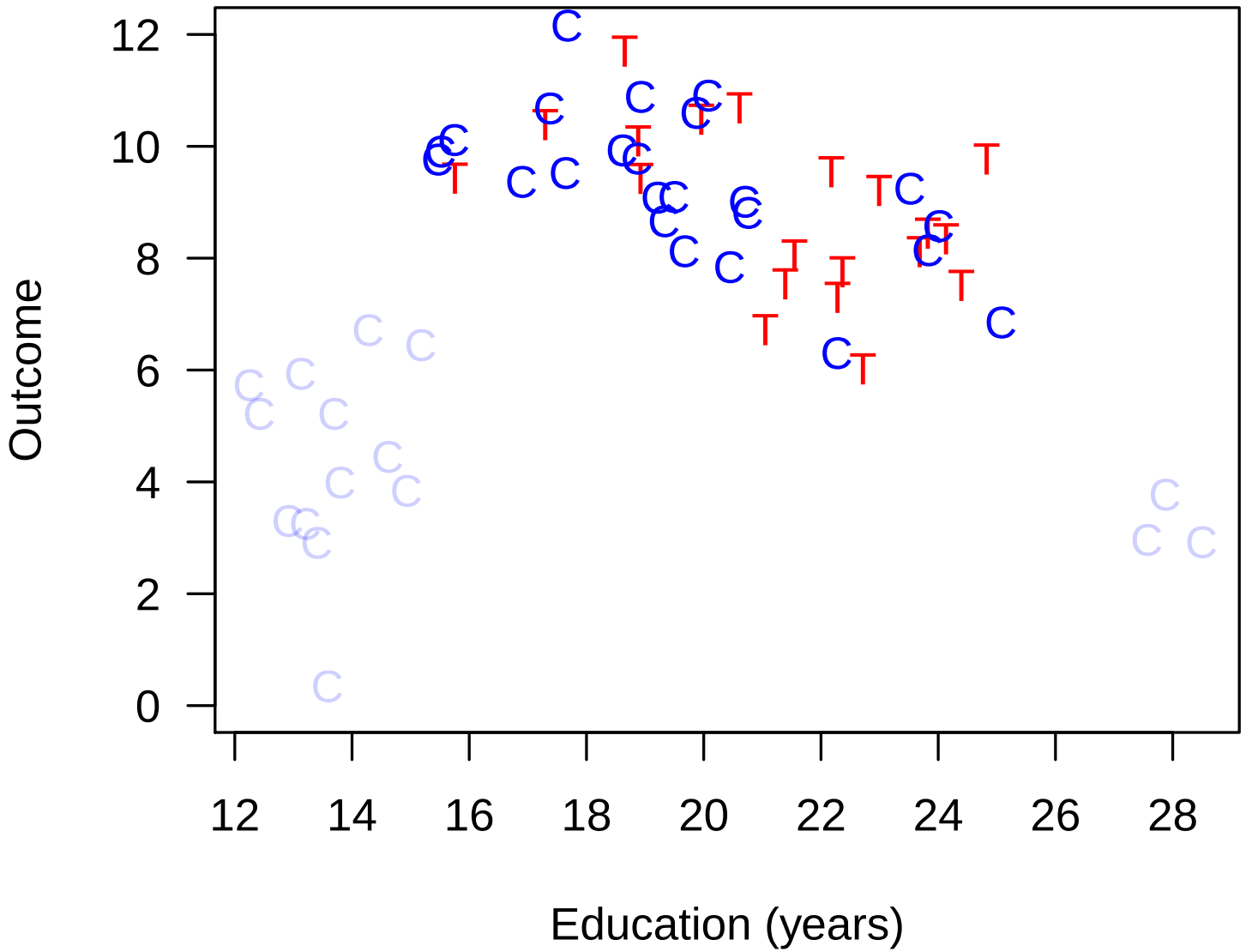

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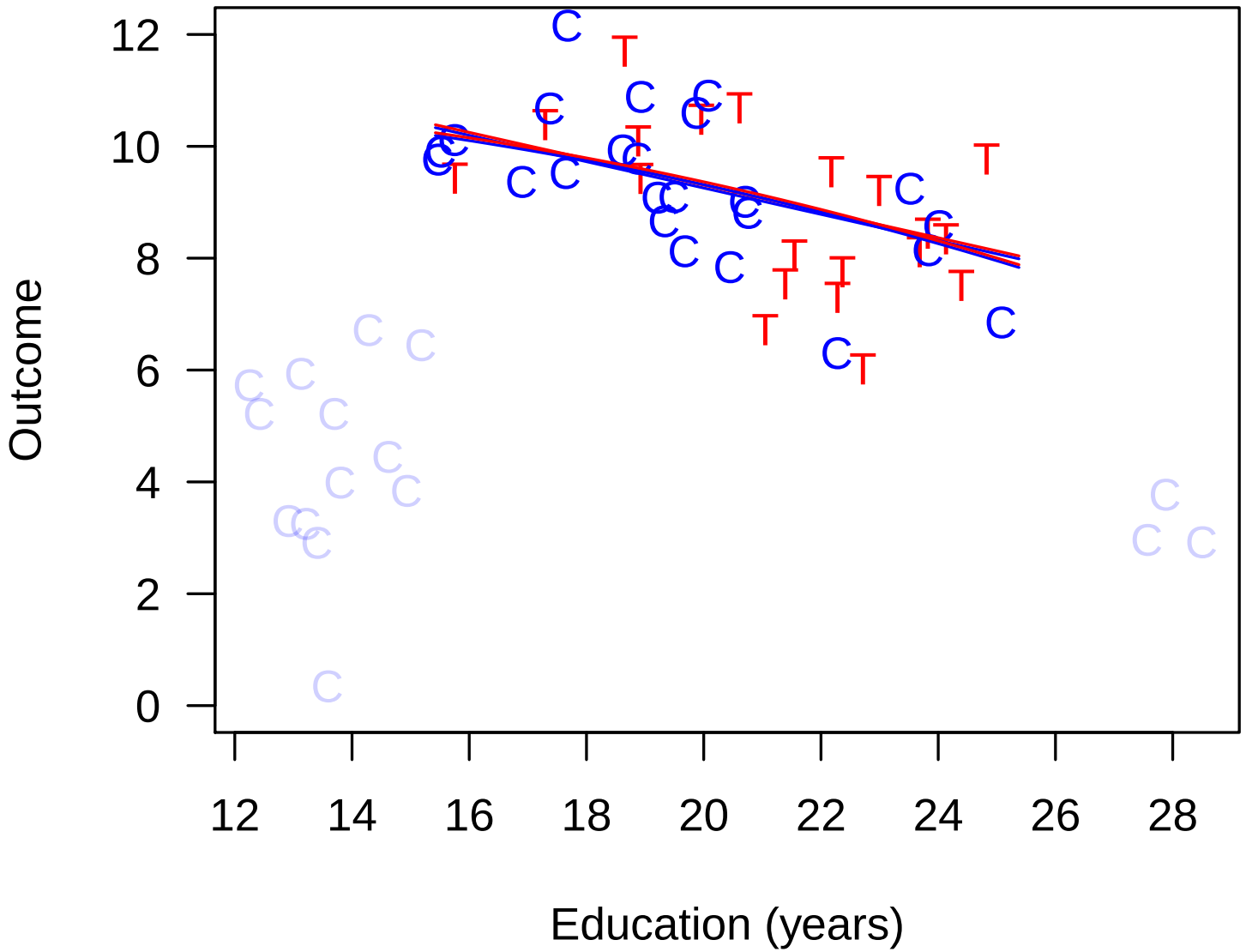

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Matching reduces model dependence, bias, and variance

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- 18 control variables (clinical factors, firm characteristics, media variables, etc.)

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- Look at *variability* in ATE estimate across specifications.
- (Normal applications would only use one or a few specifications.)

Reducing Model Dependence

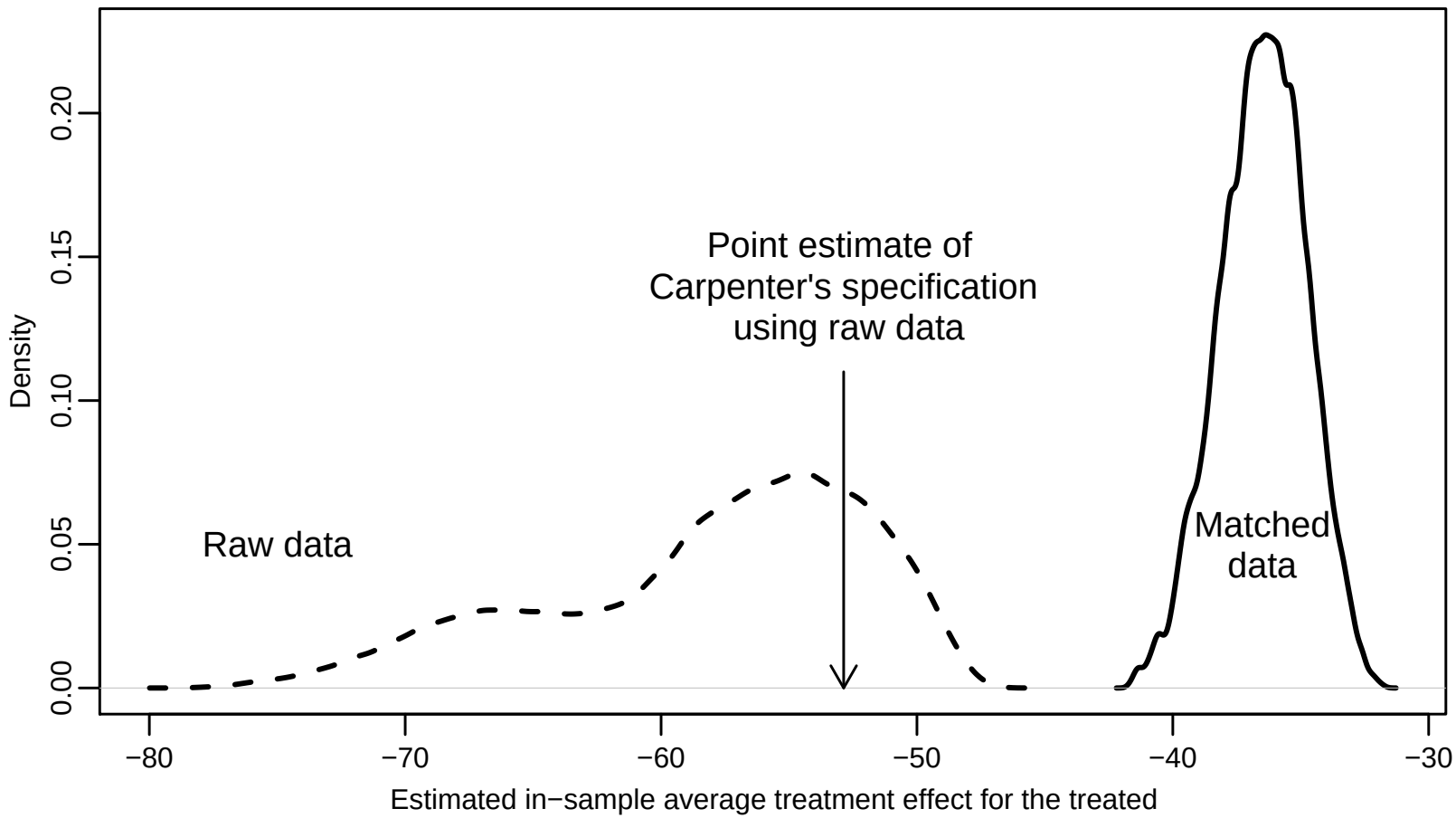

Figure: SATT Histogram: Effect of Democratic Senate majority on FDA drug approval time, across 262,143 specifications.

Another Example: Jeffrey Koch, AJPS, 2002

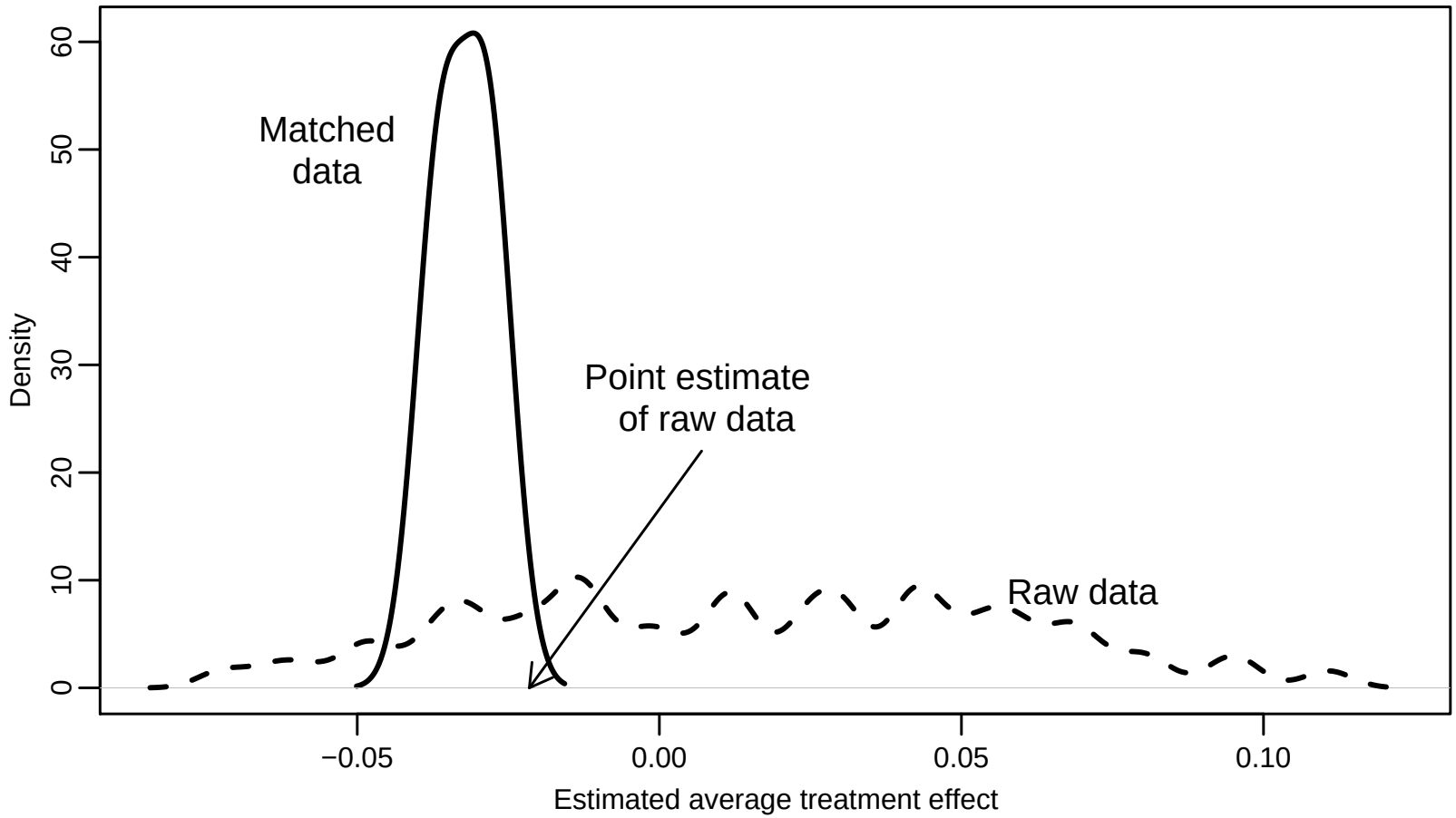

Figure: SATT Histogram: Effect of being a highly visible female Republican candidate across 63 possible specifications with the Koch

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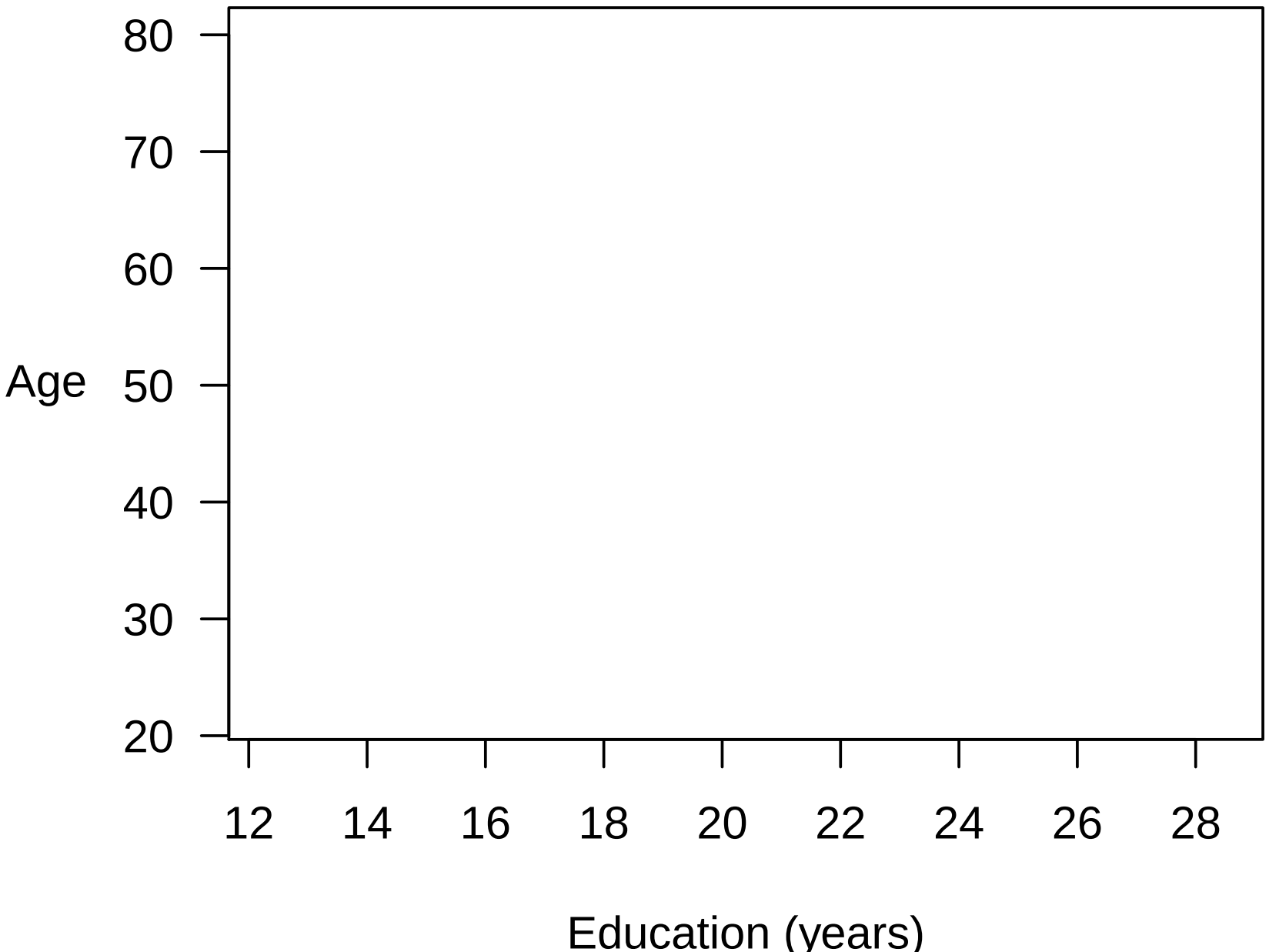

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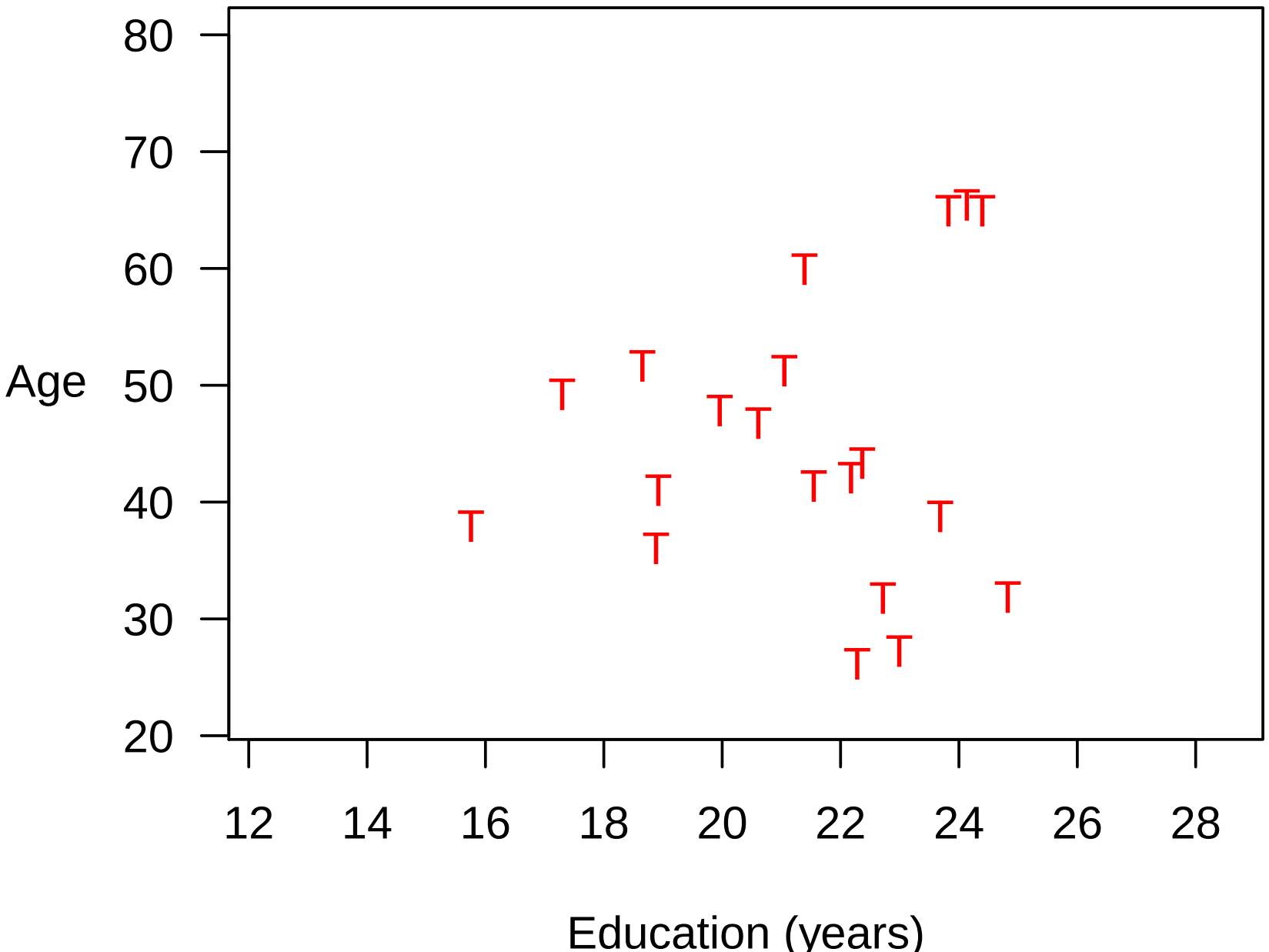

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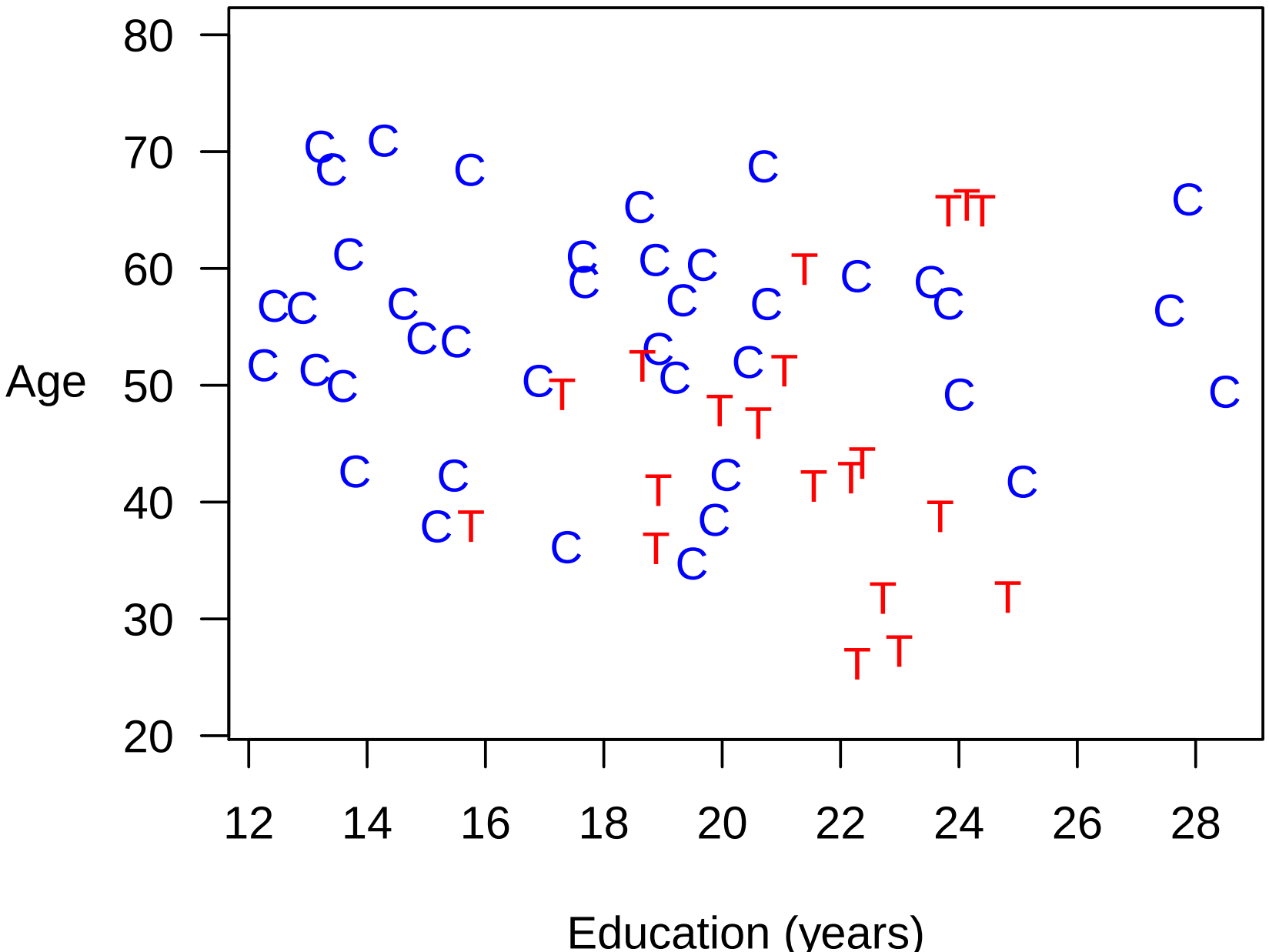

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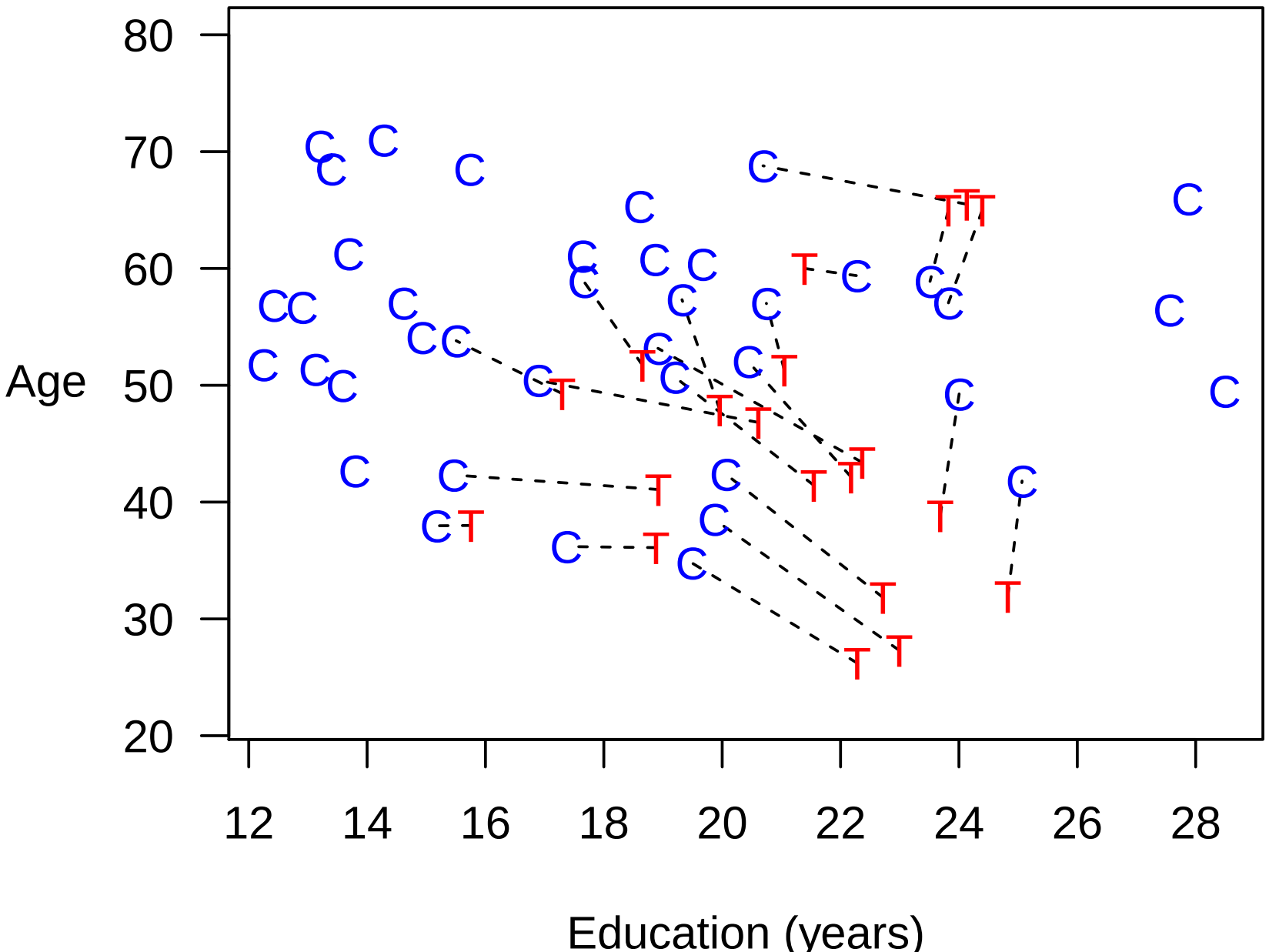

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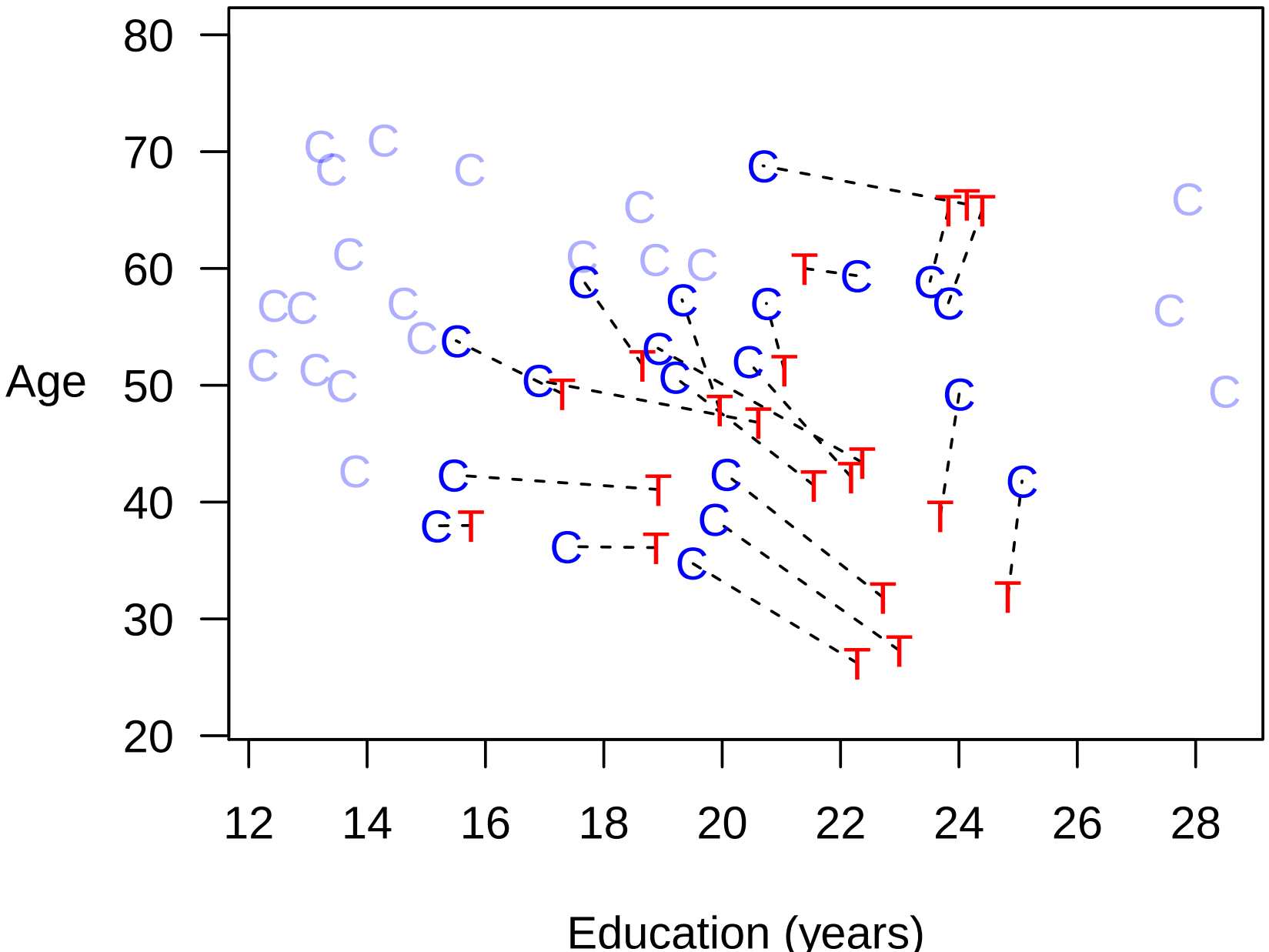

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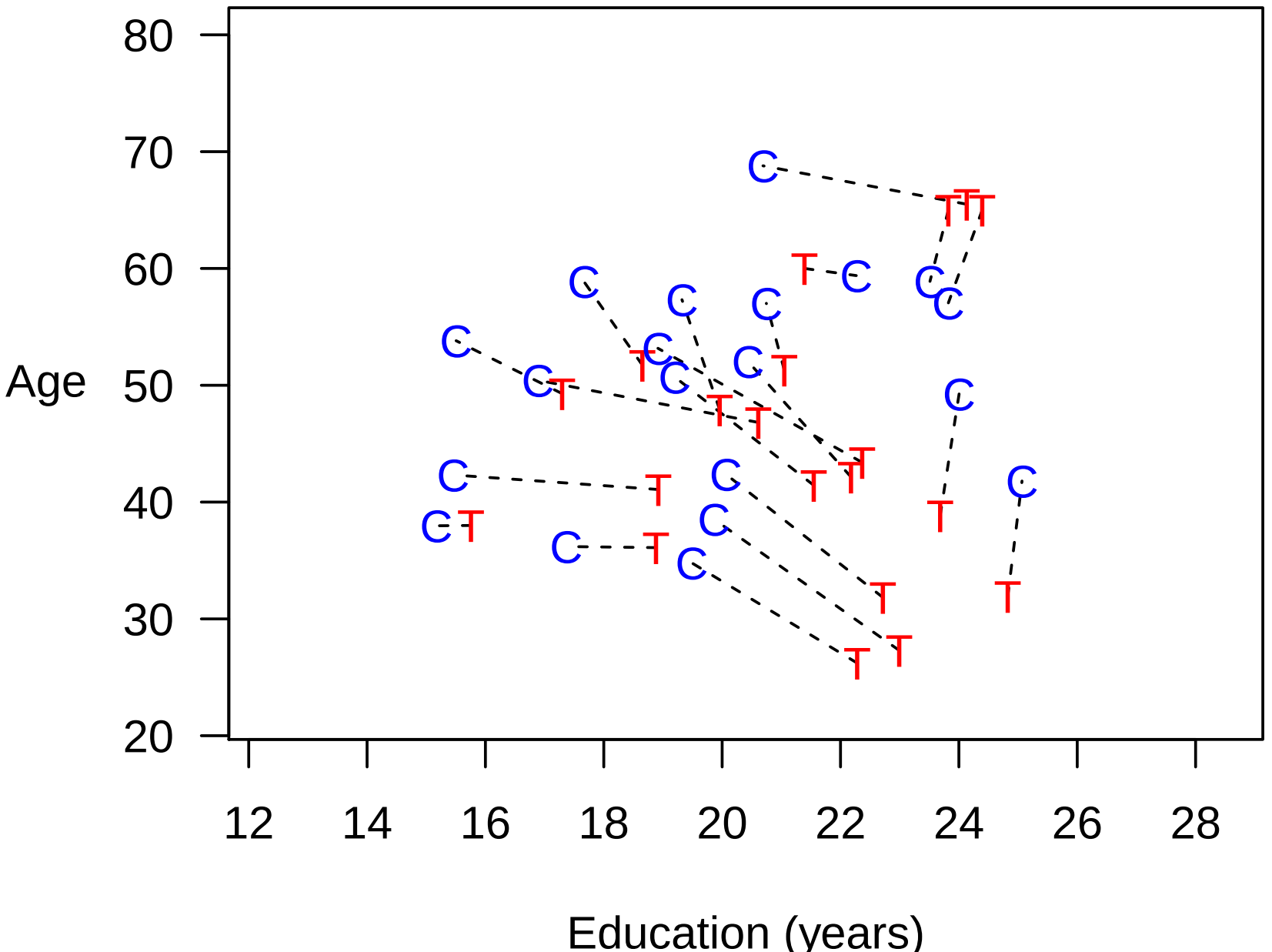

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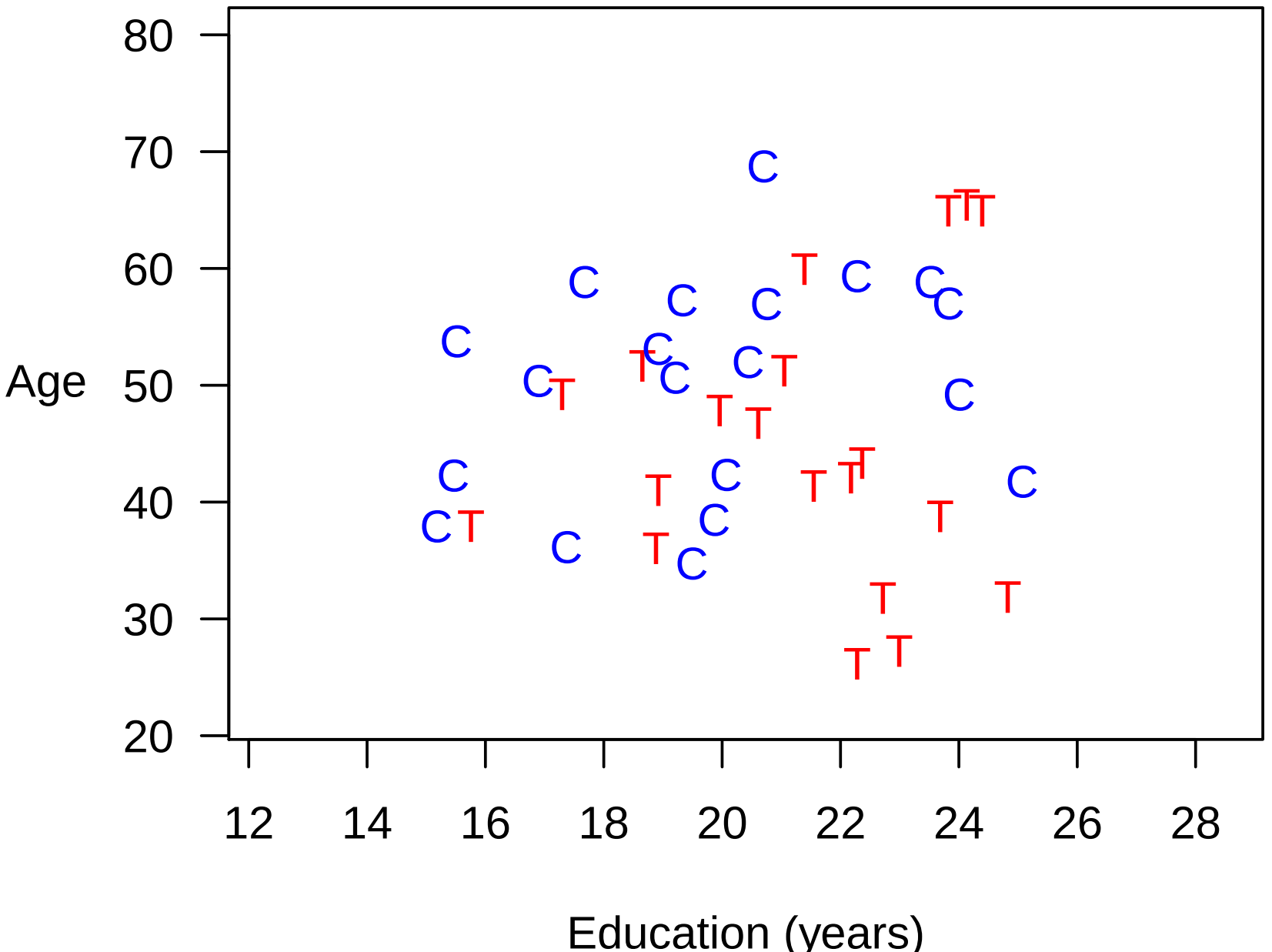

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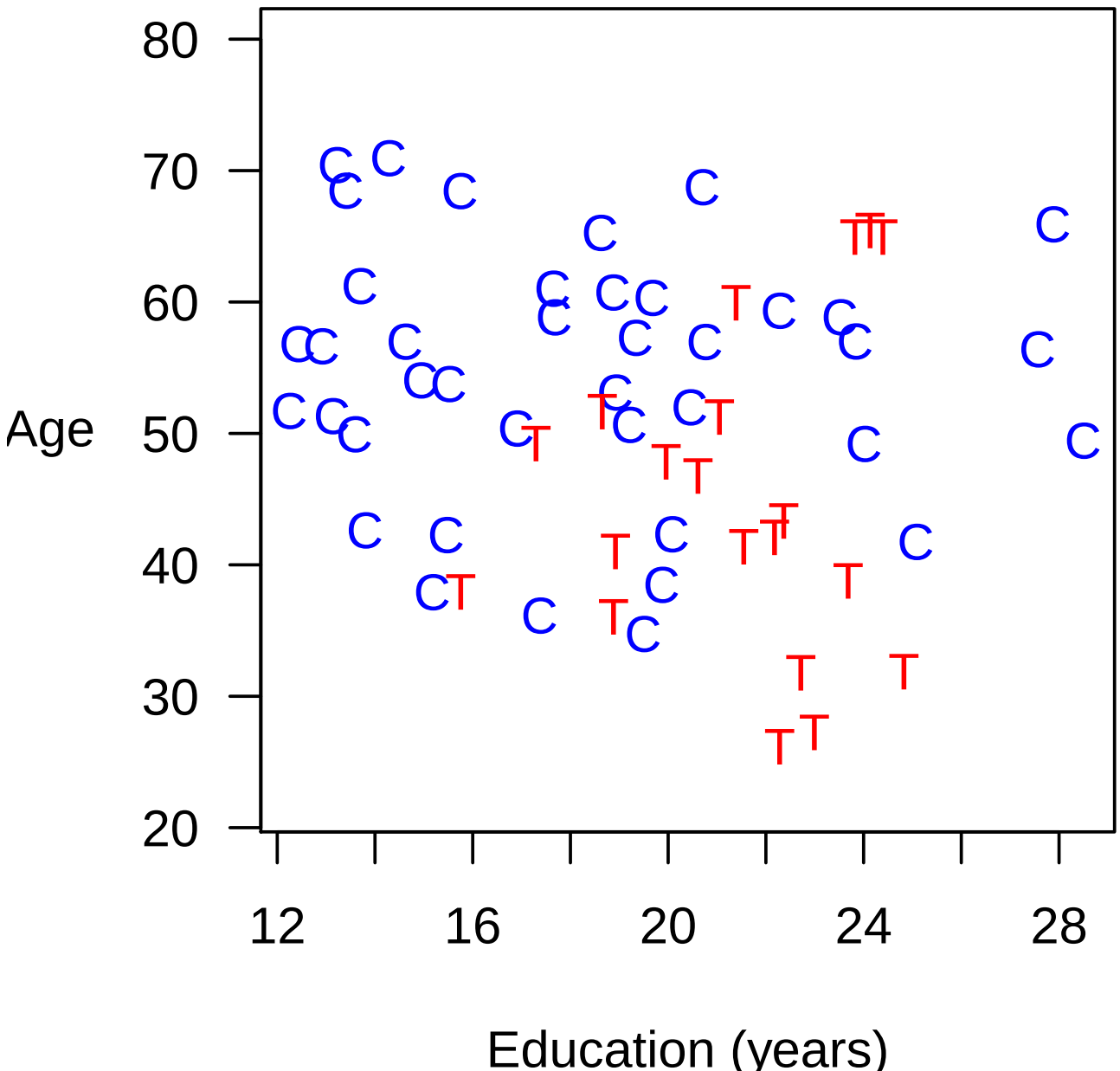

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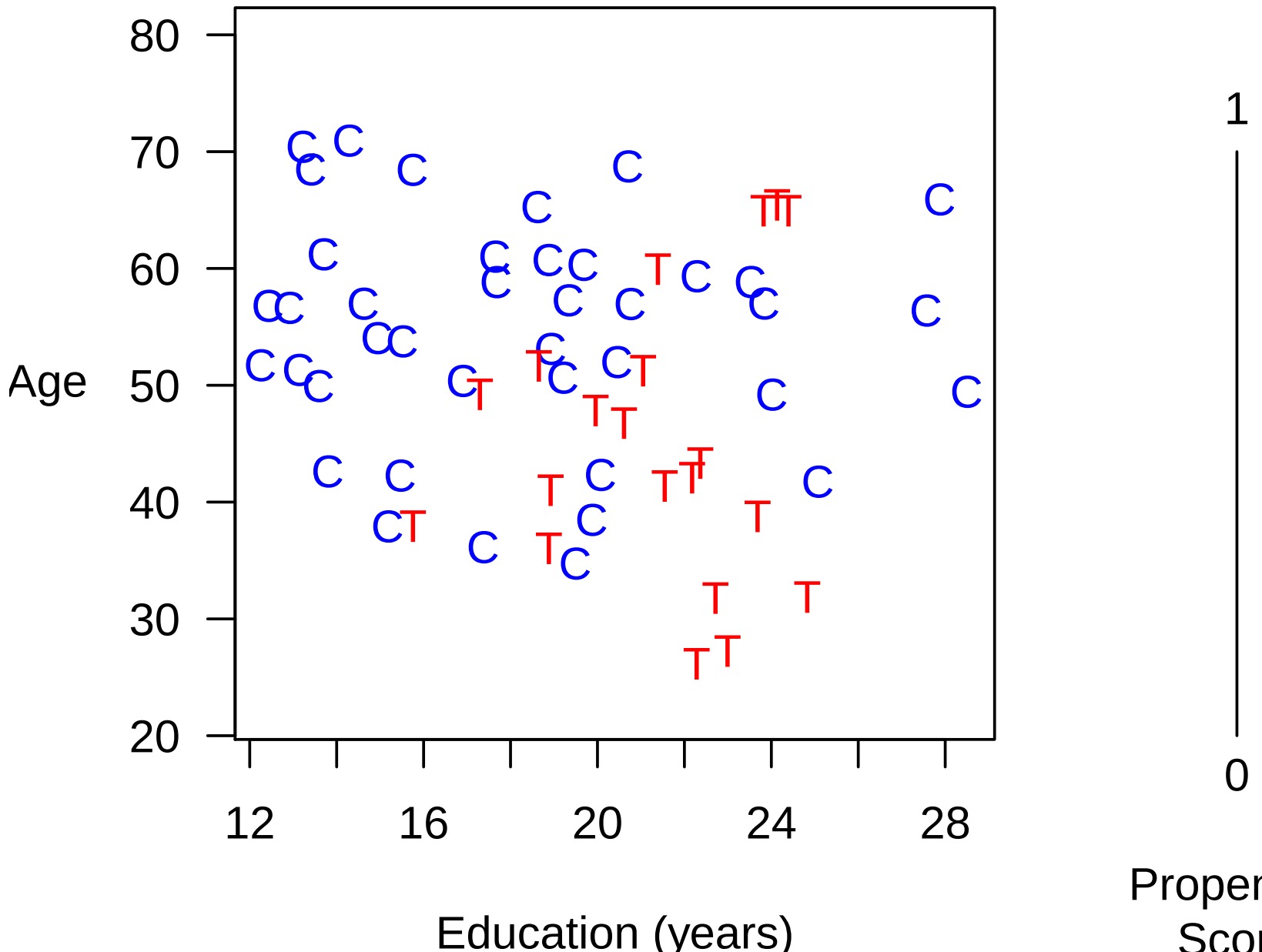

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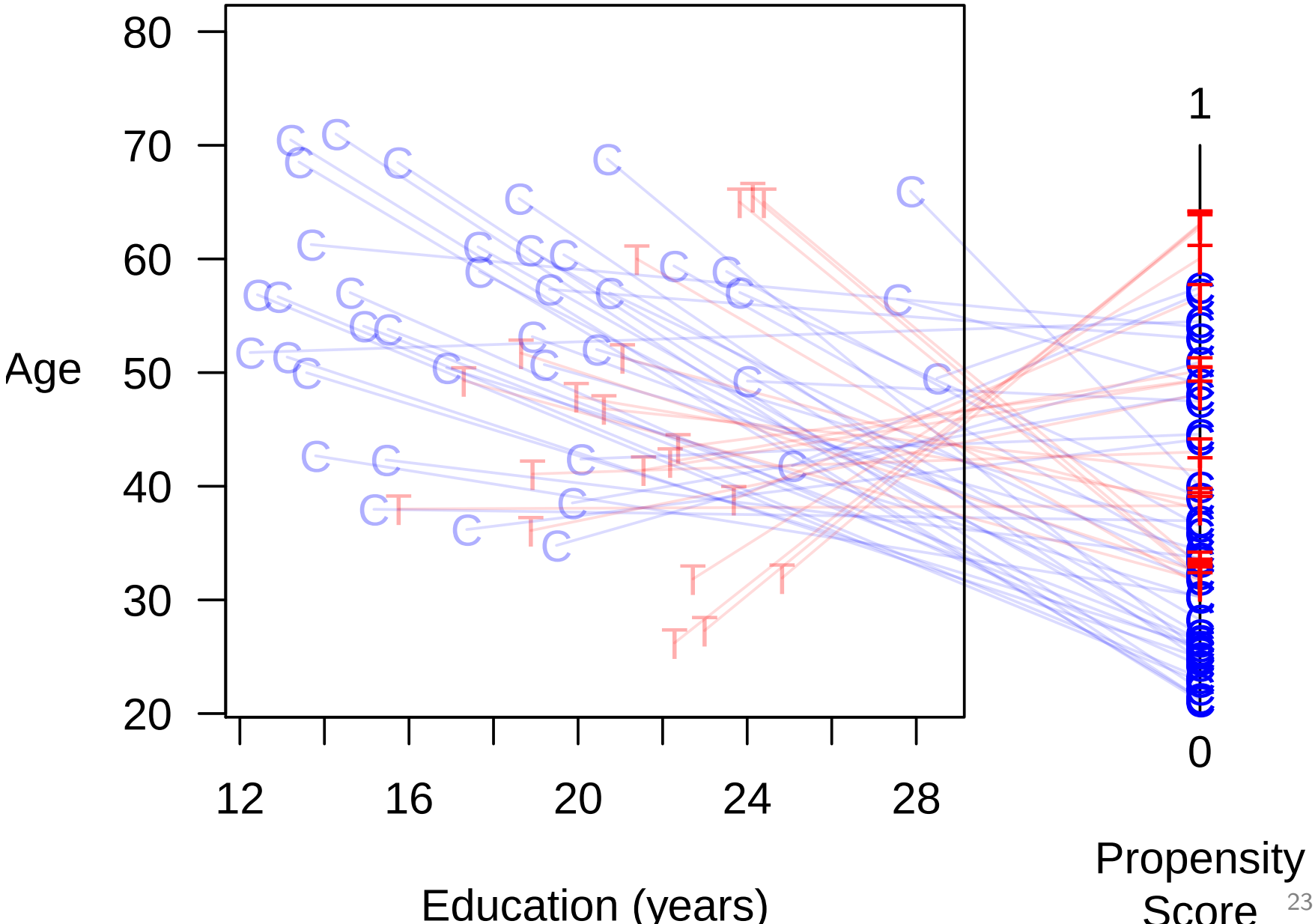

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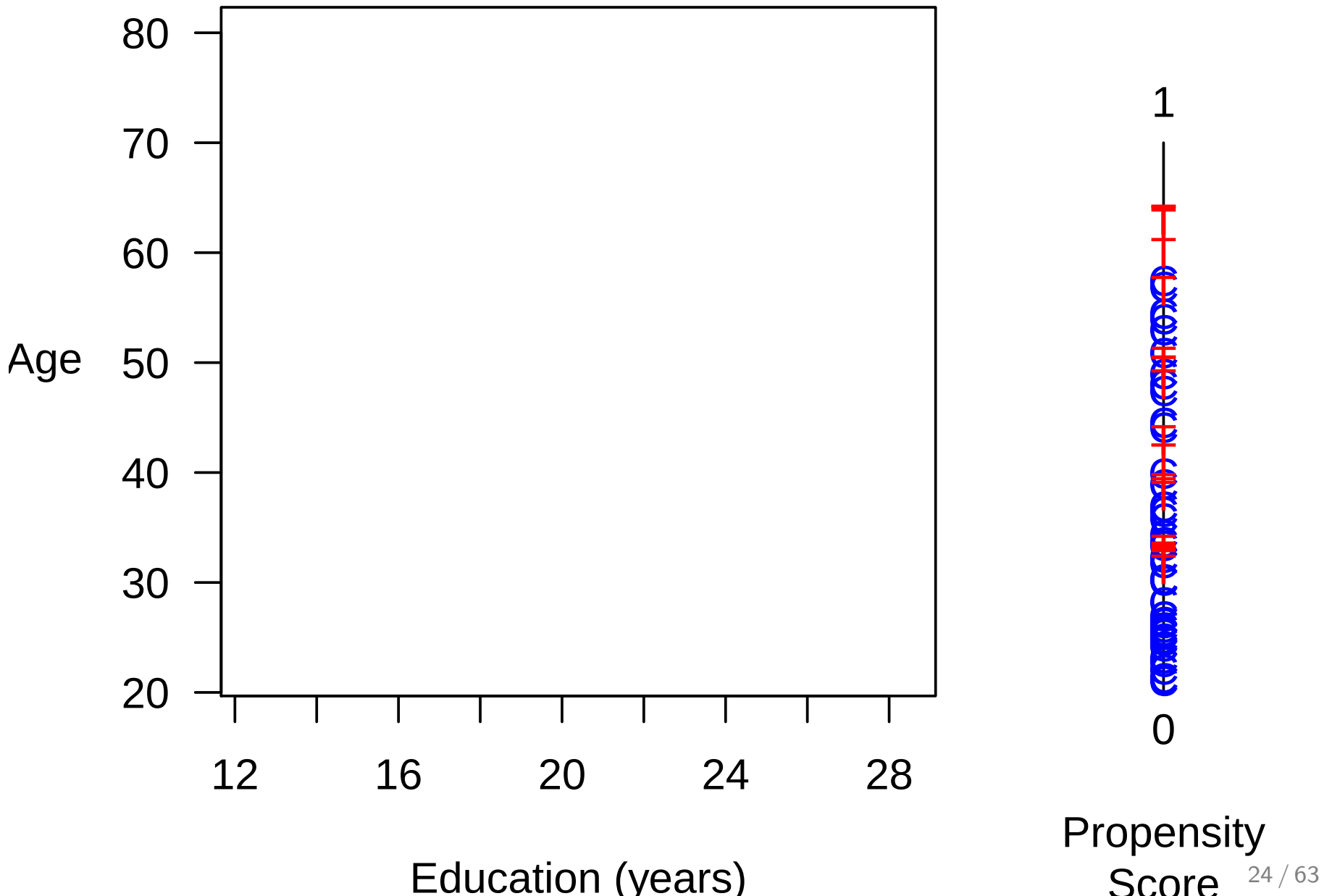

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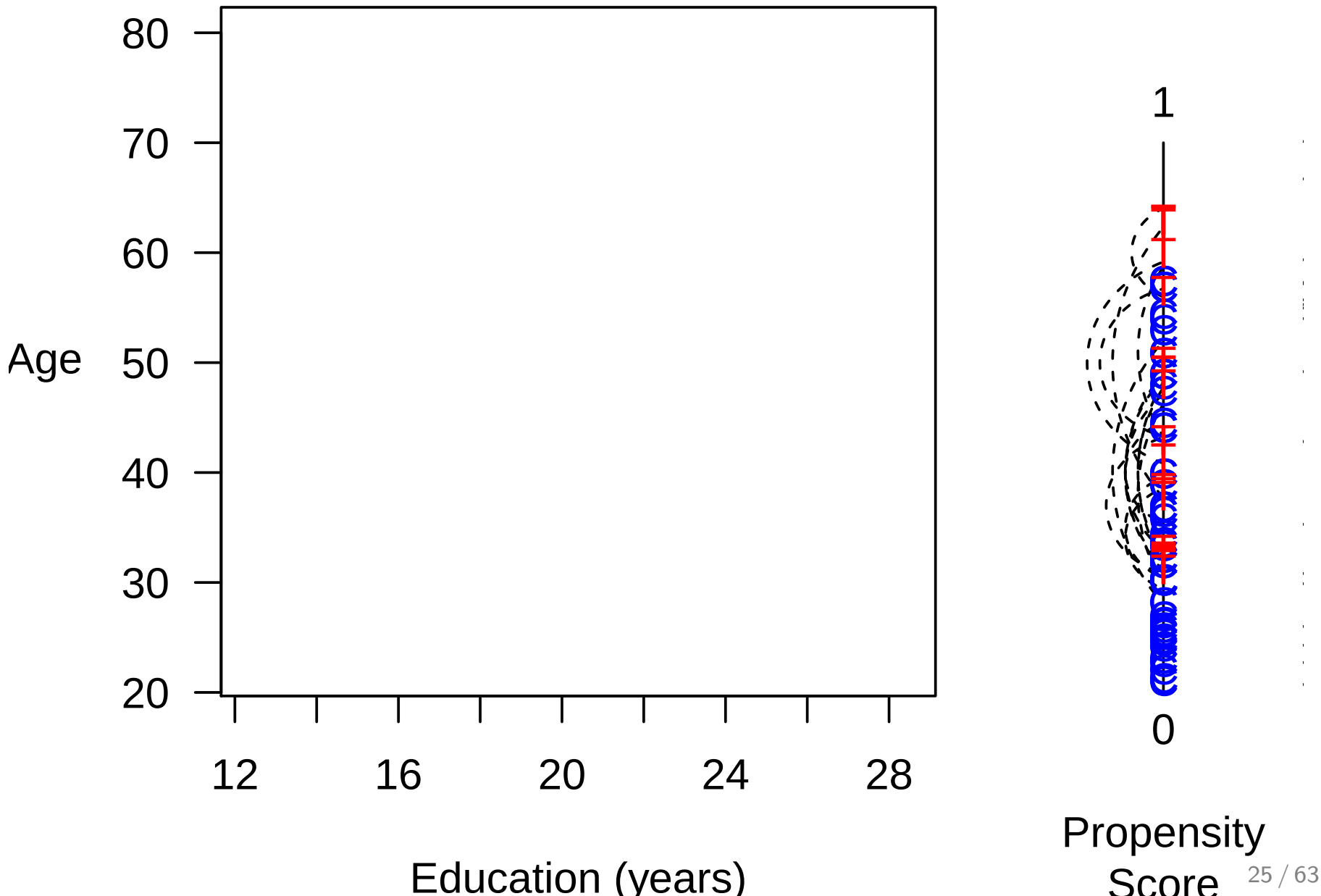

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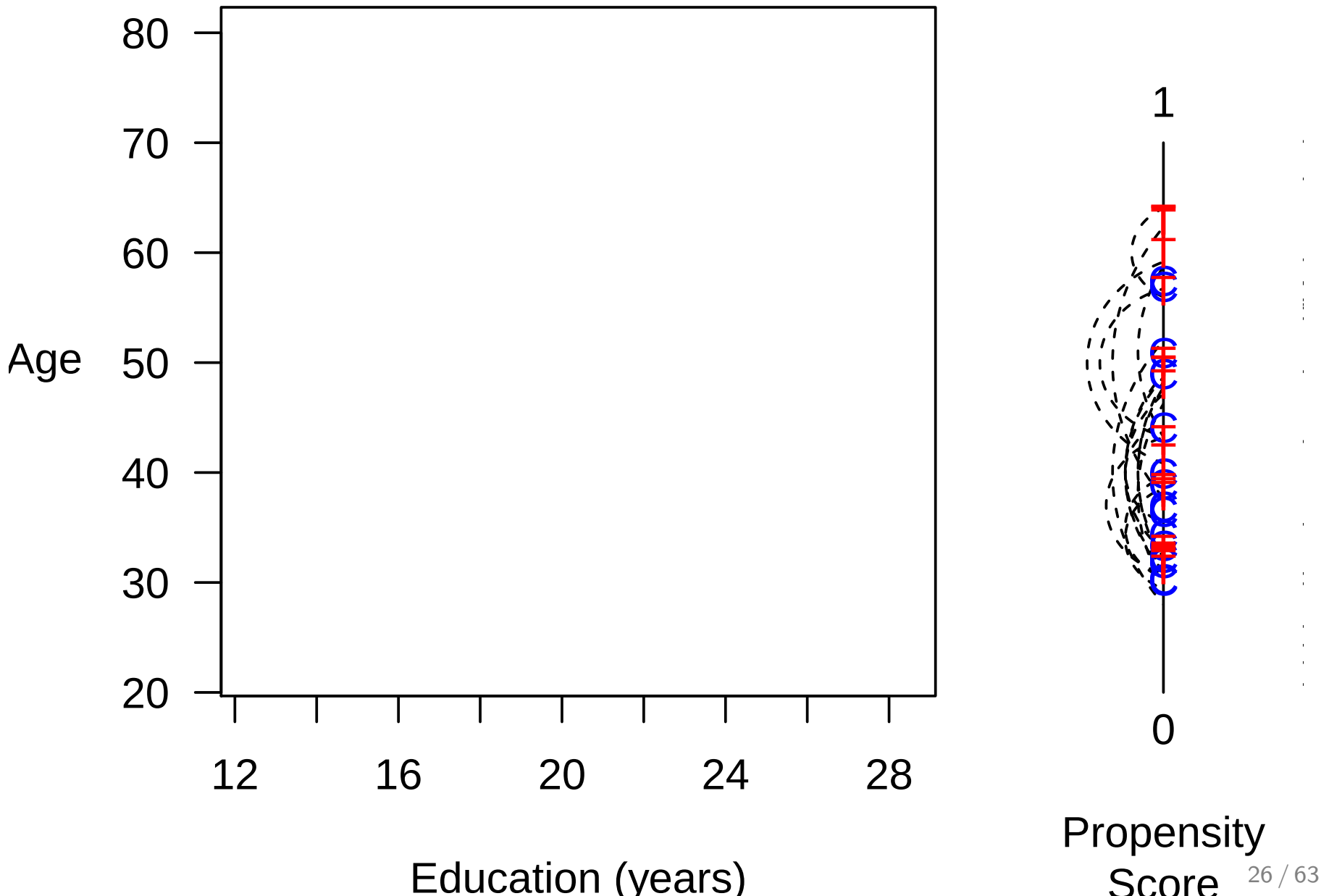

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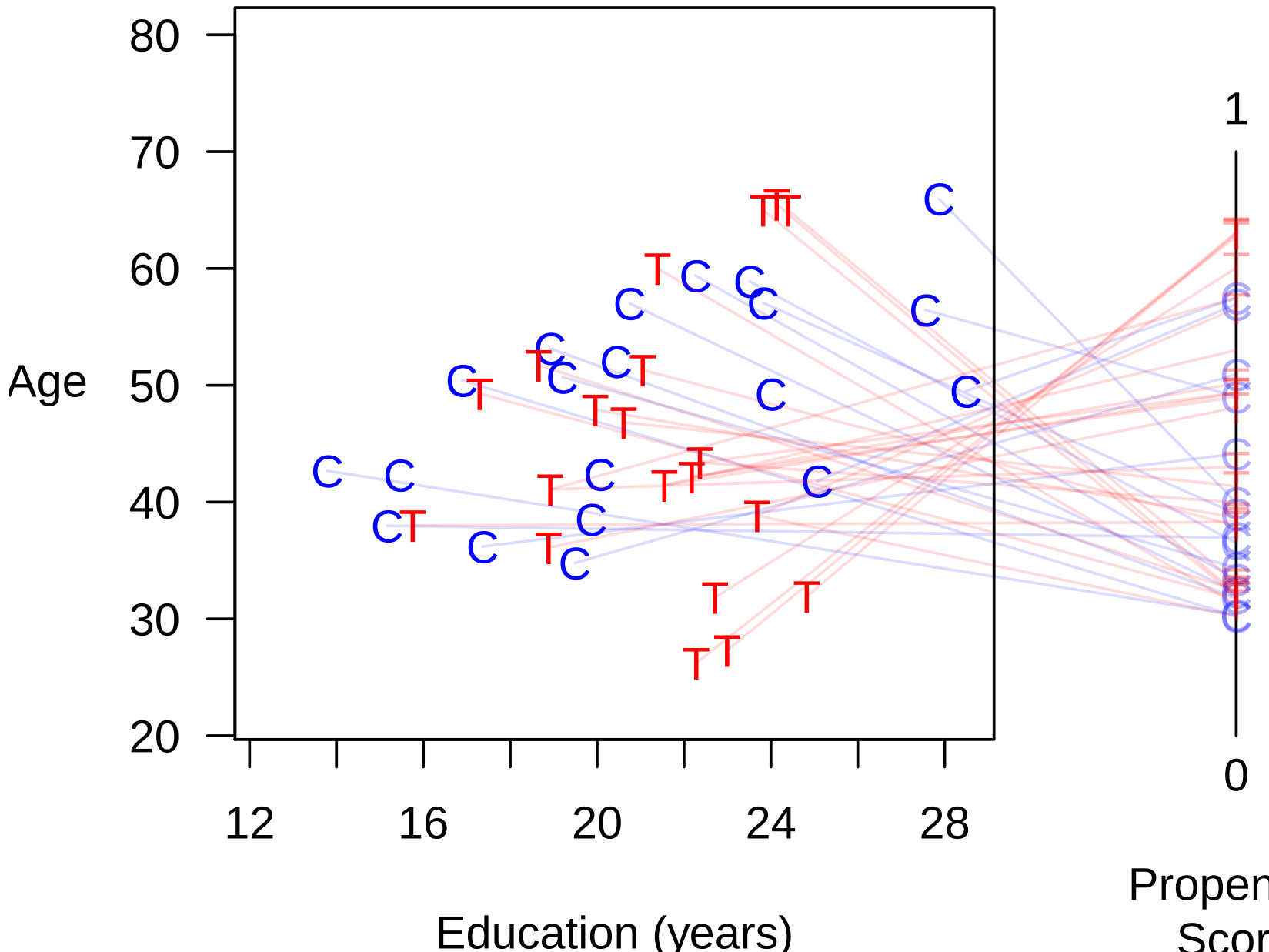

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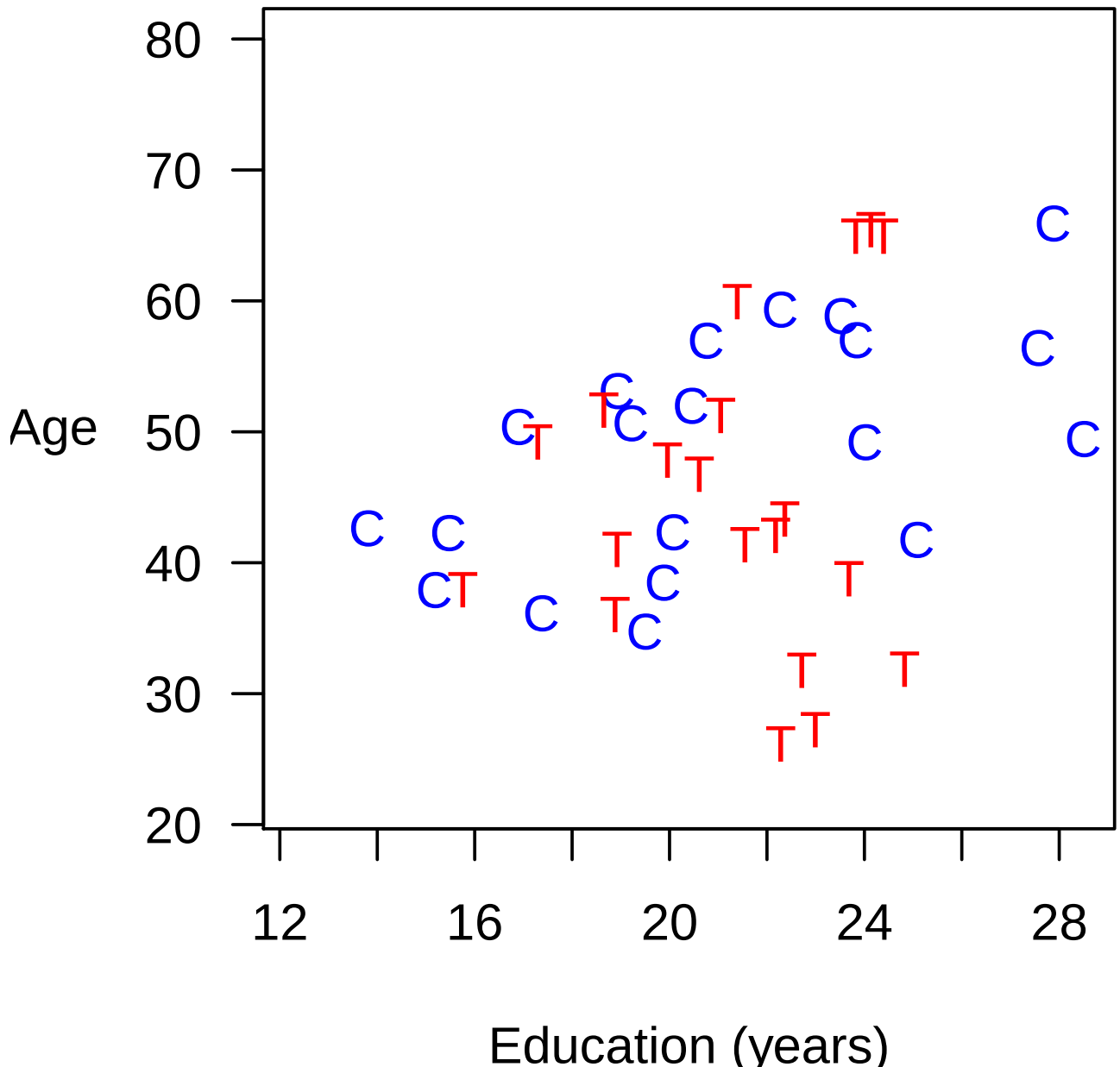

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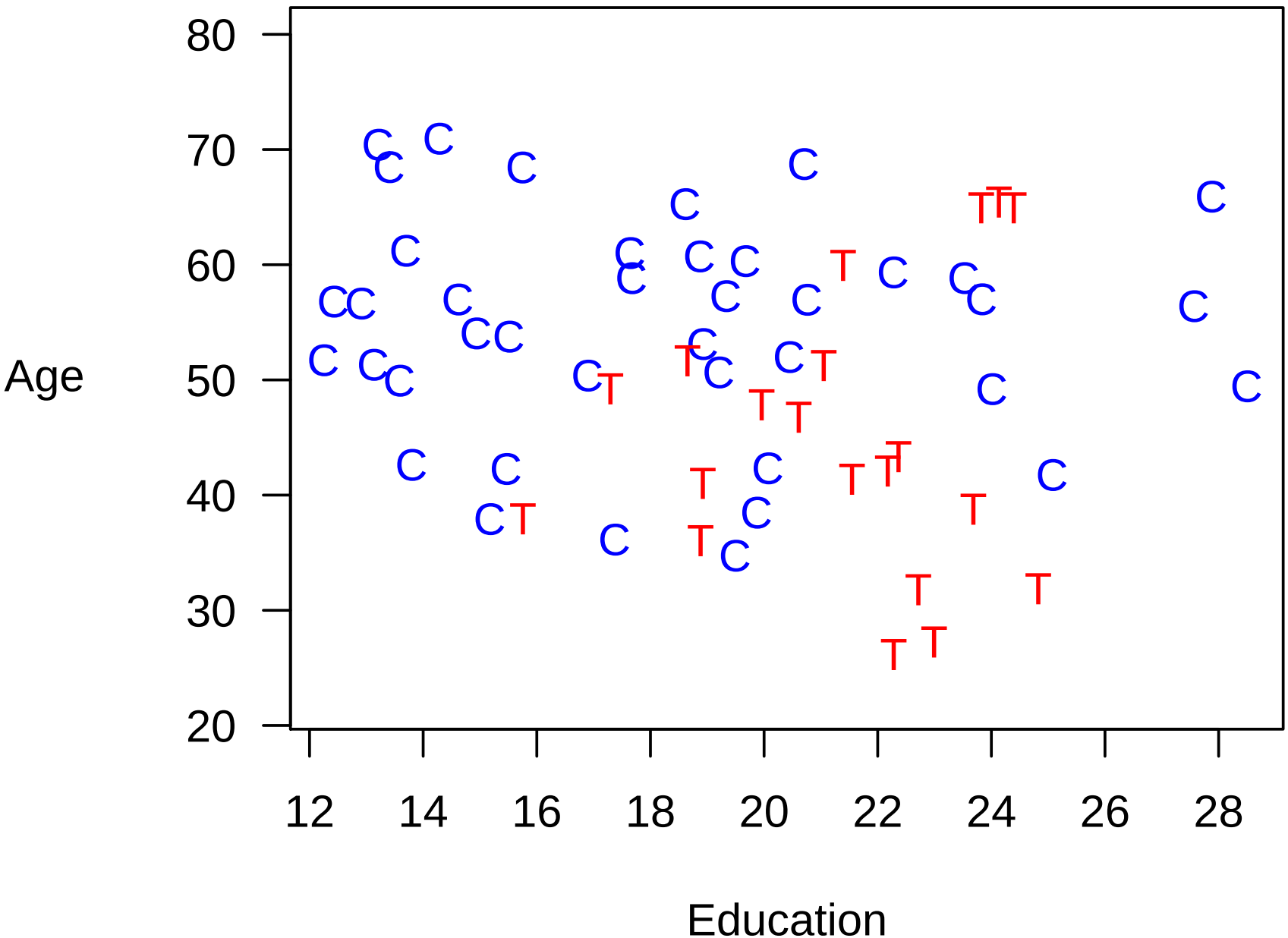

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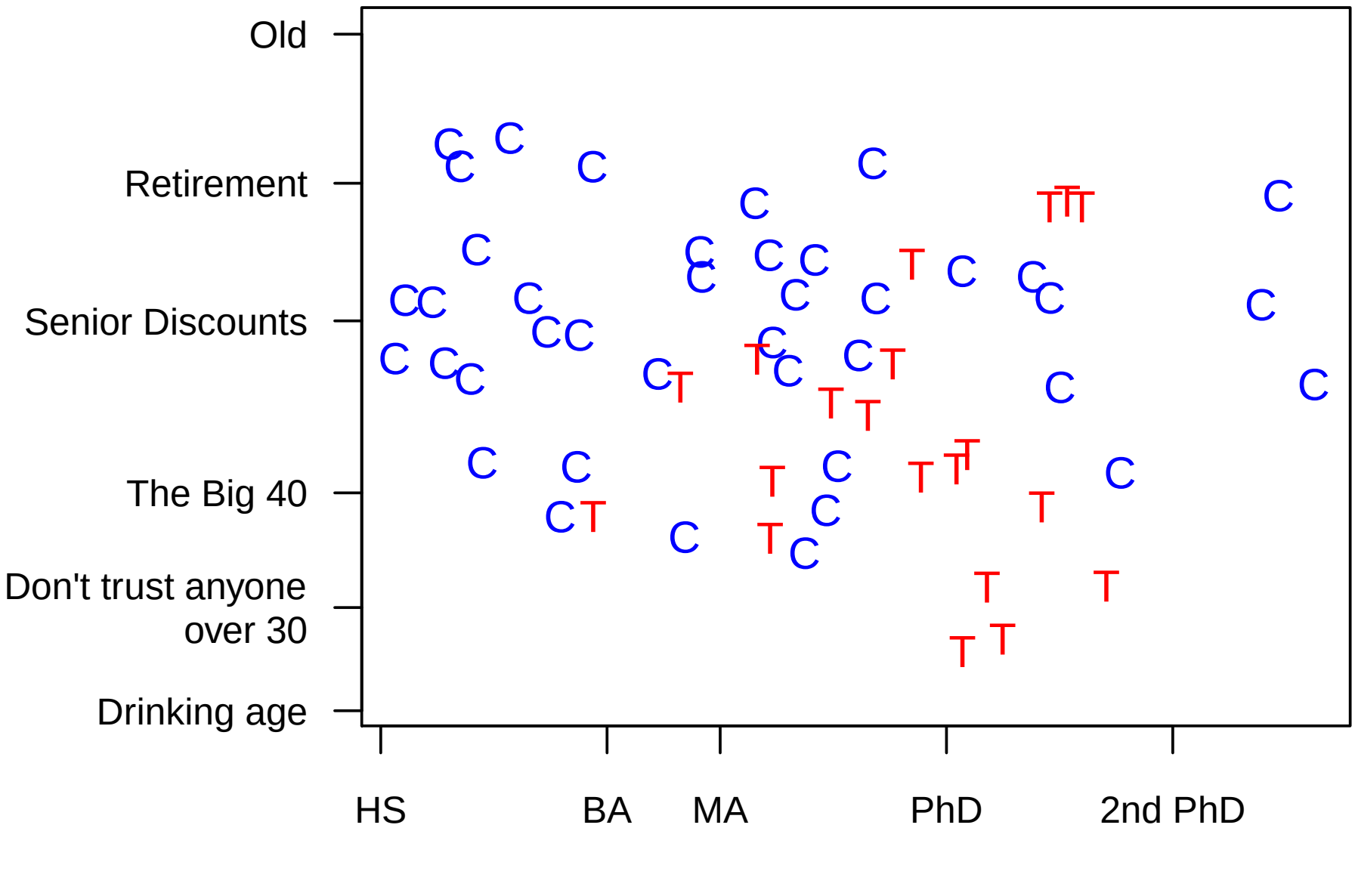

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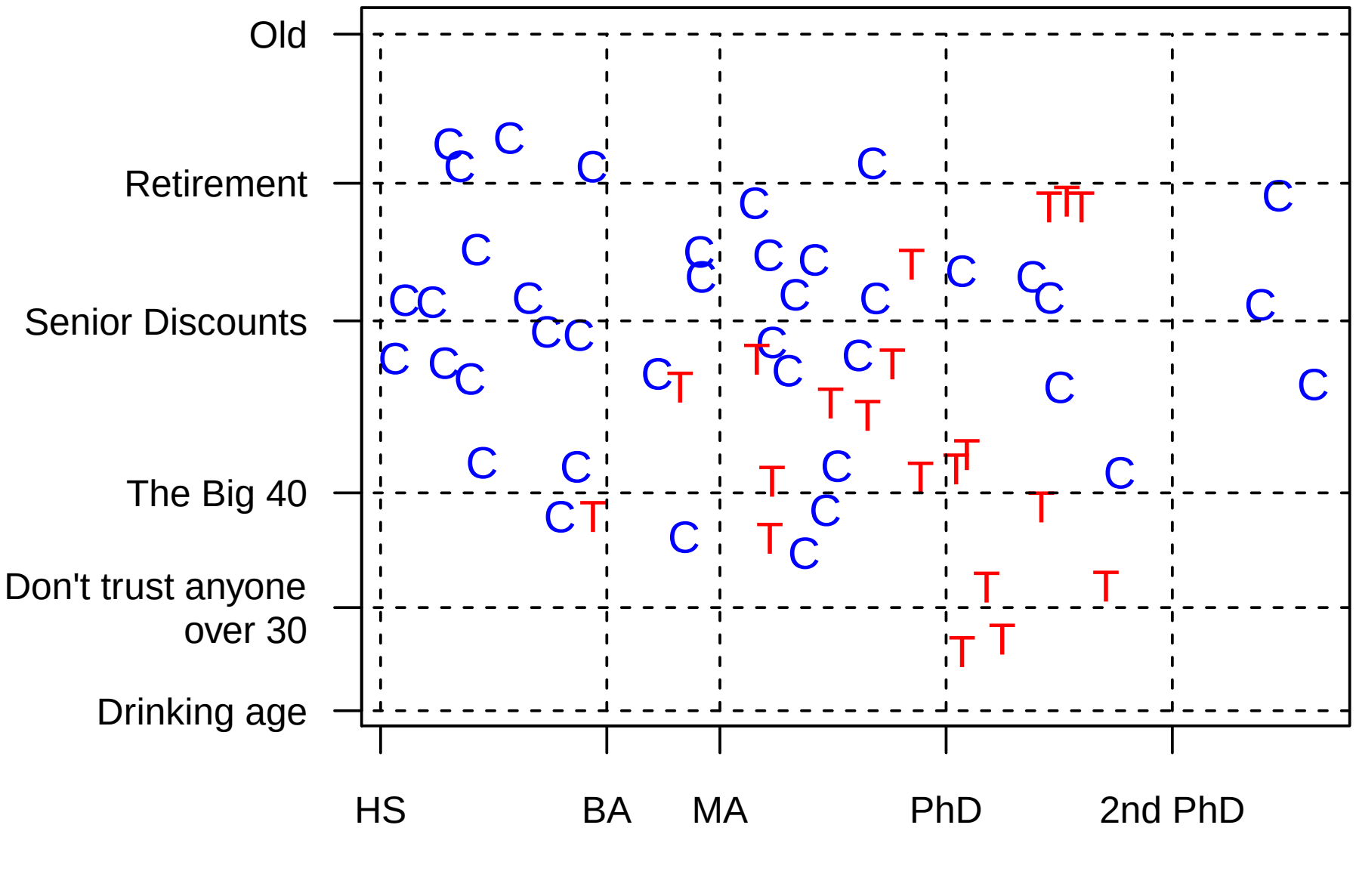

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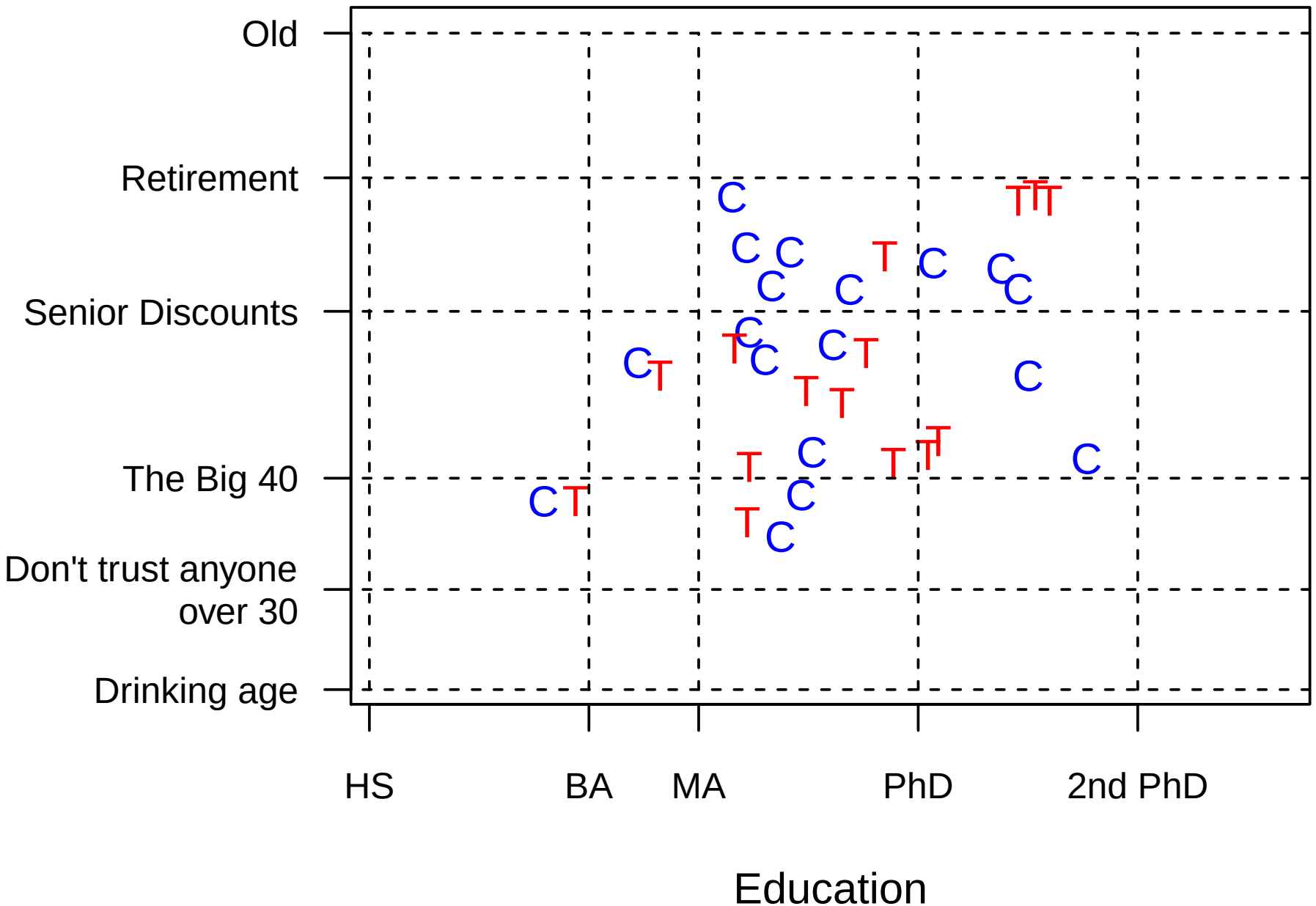

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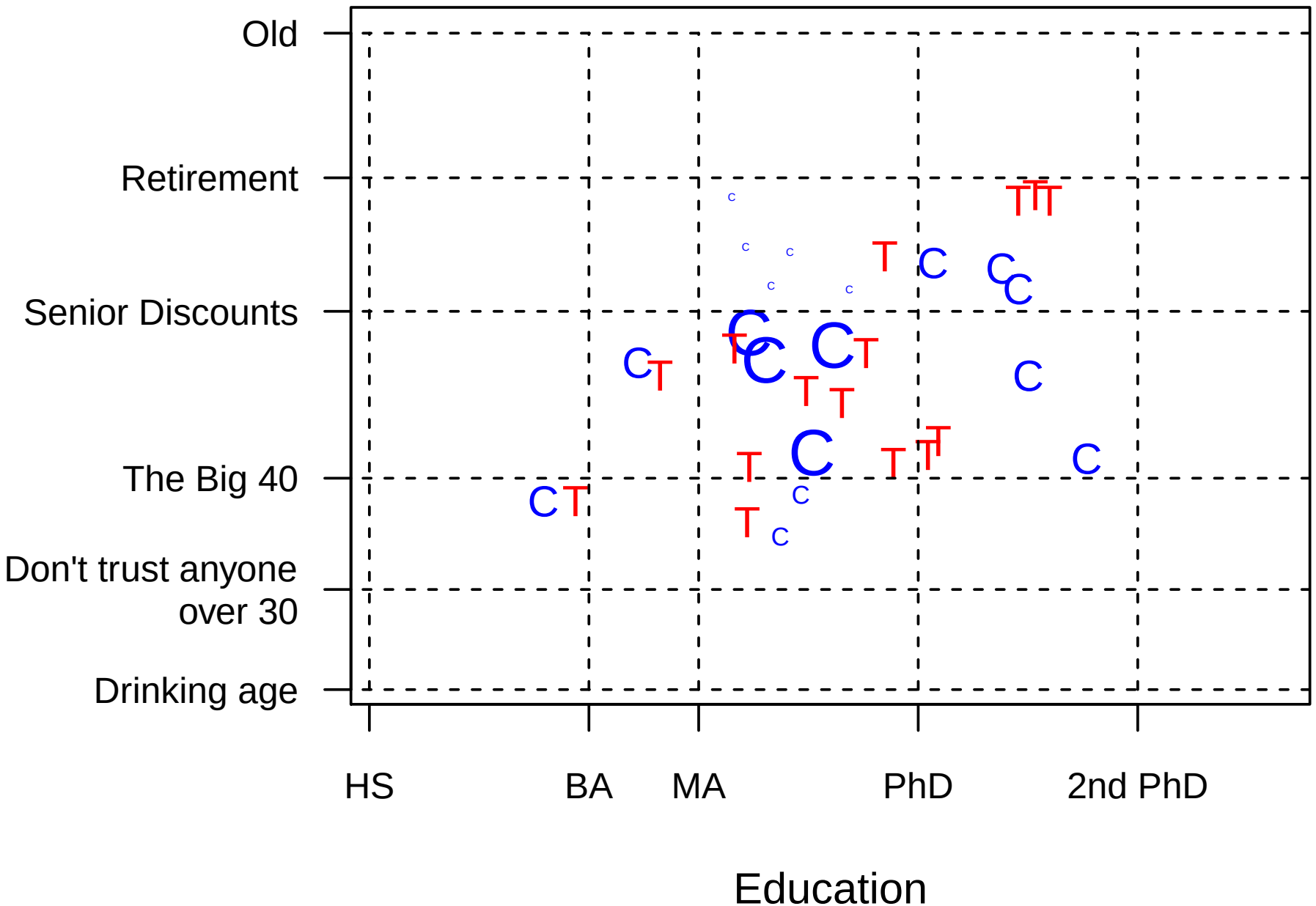

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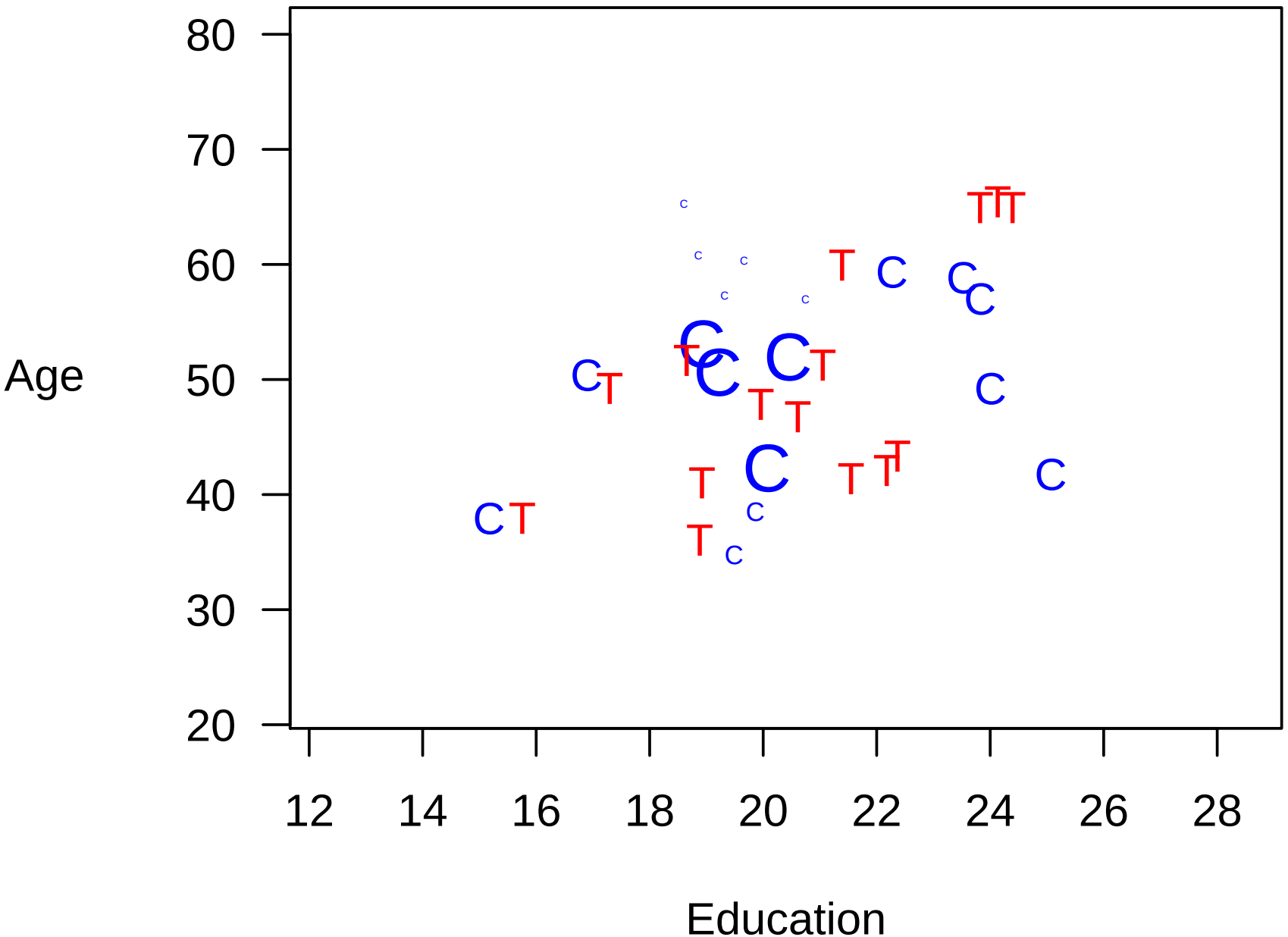

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Job Training Data: Frontier and Causal Estimates

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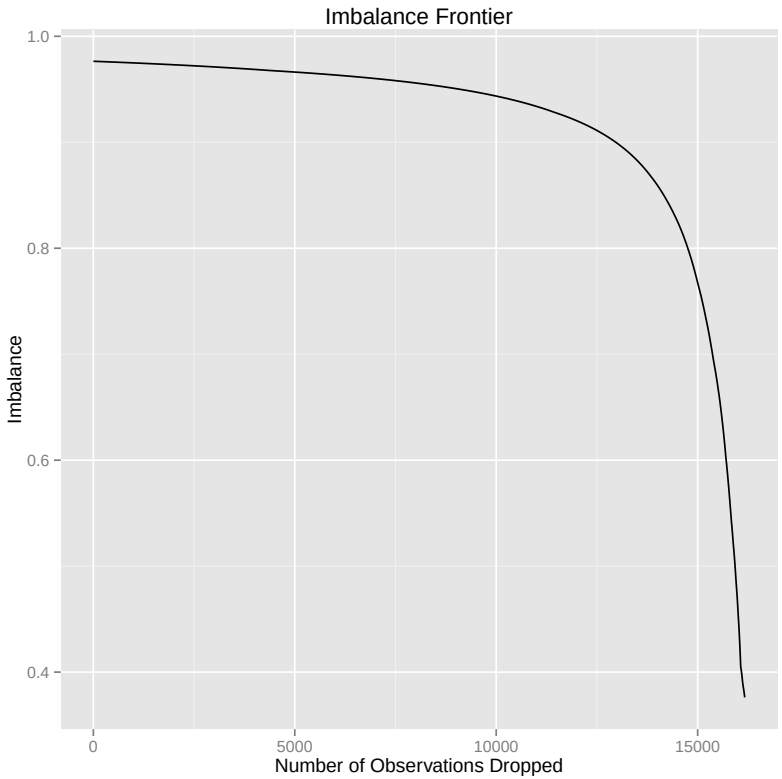

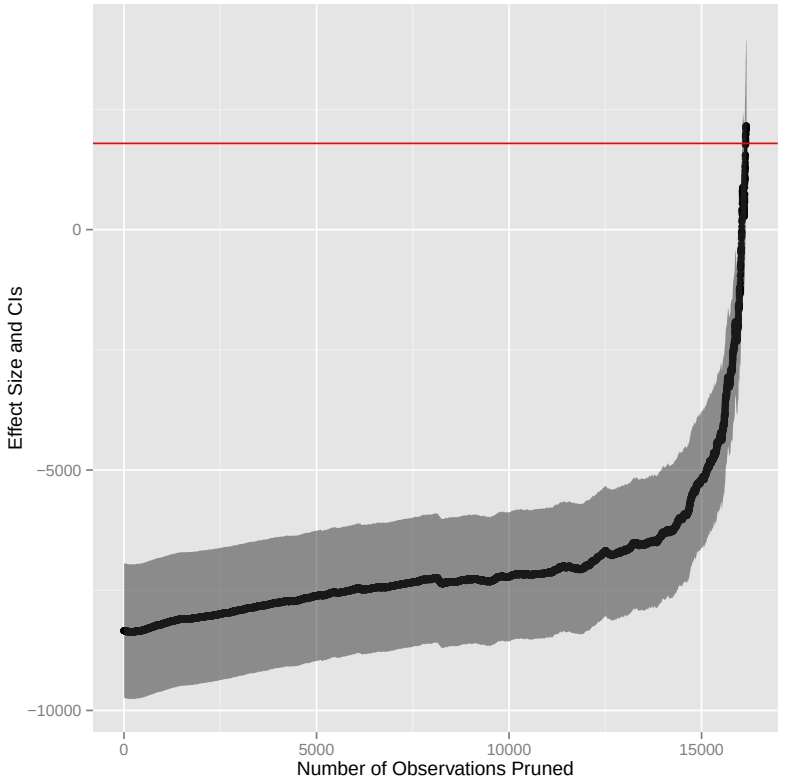

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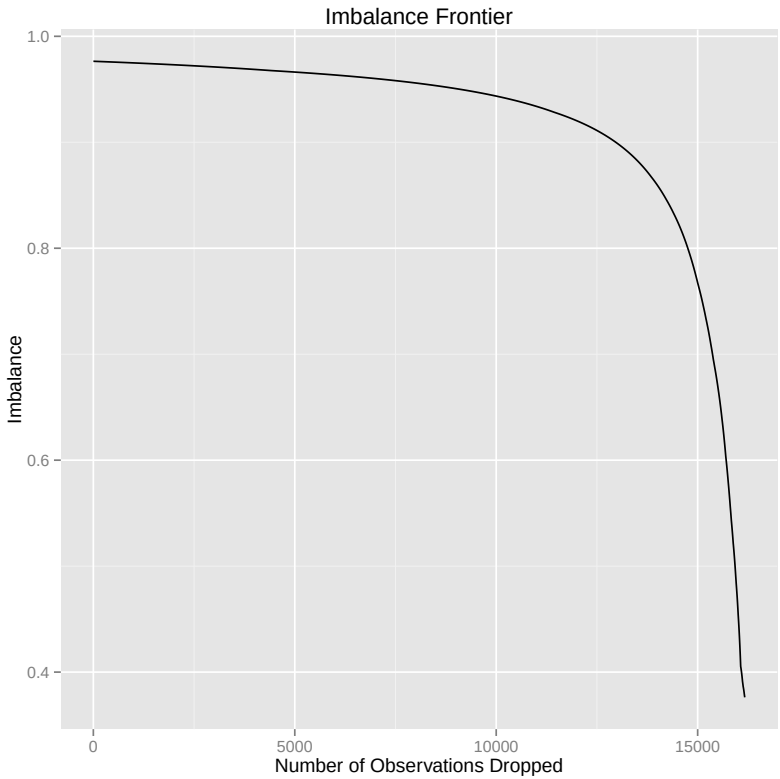

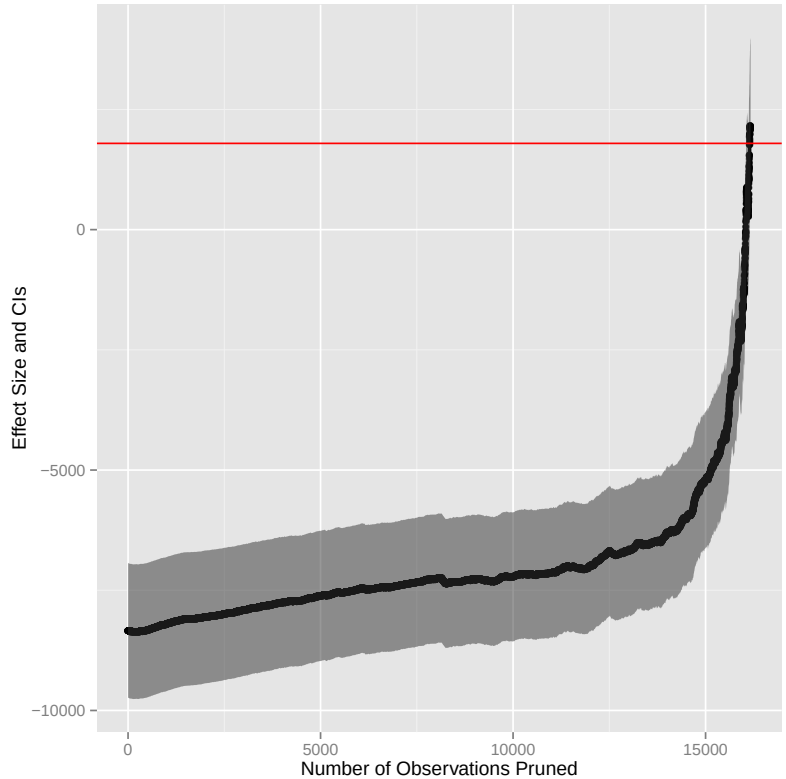

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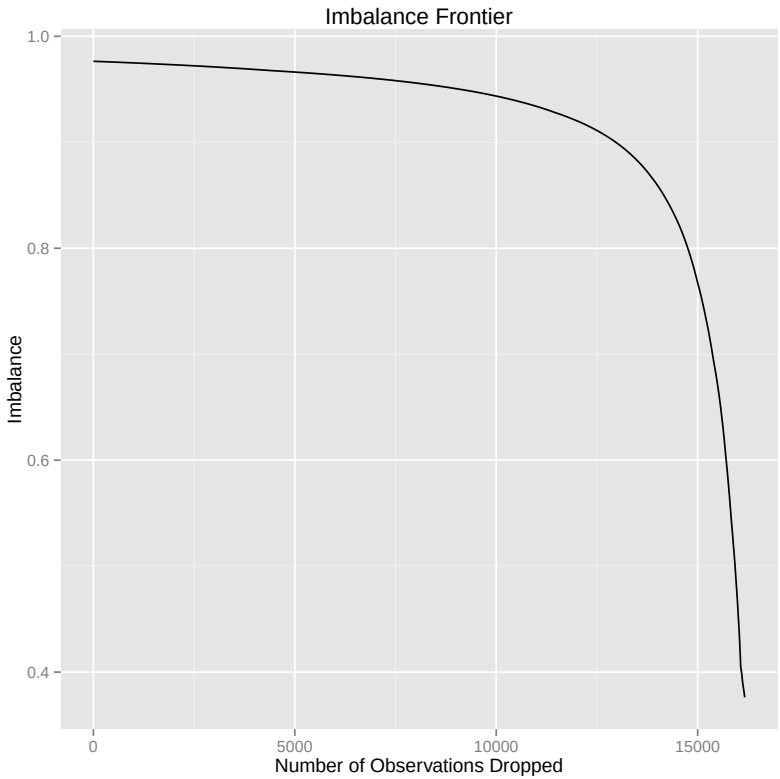

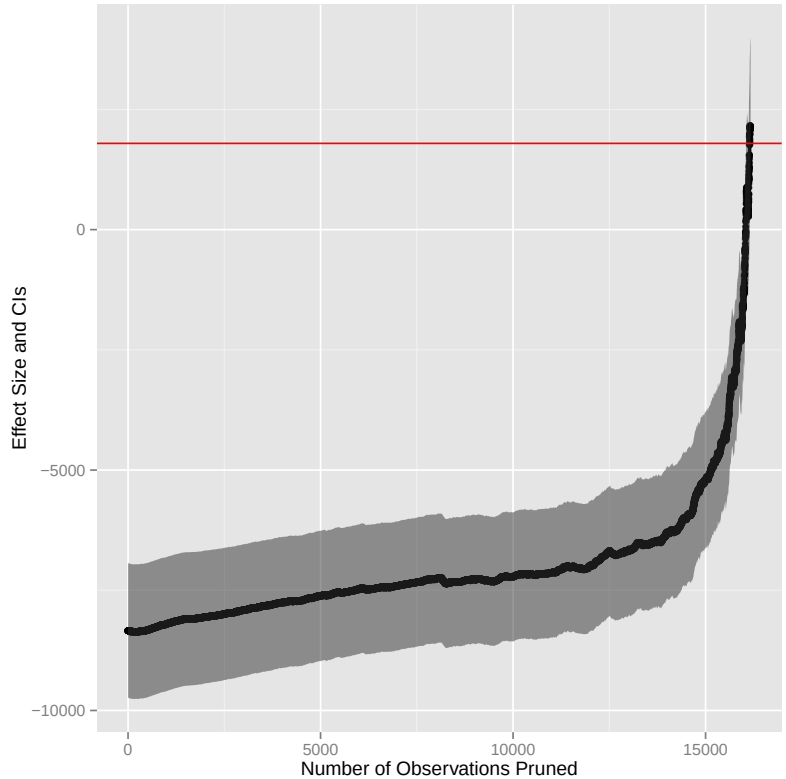

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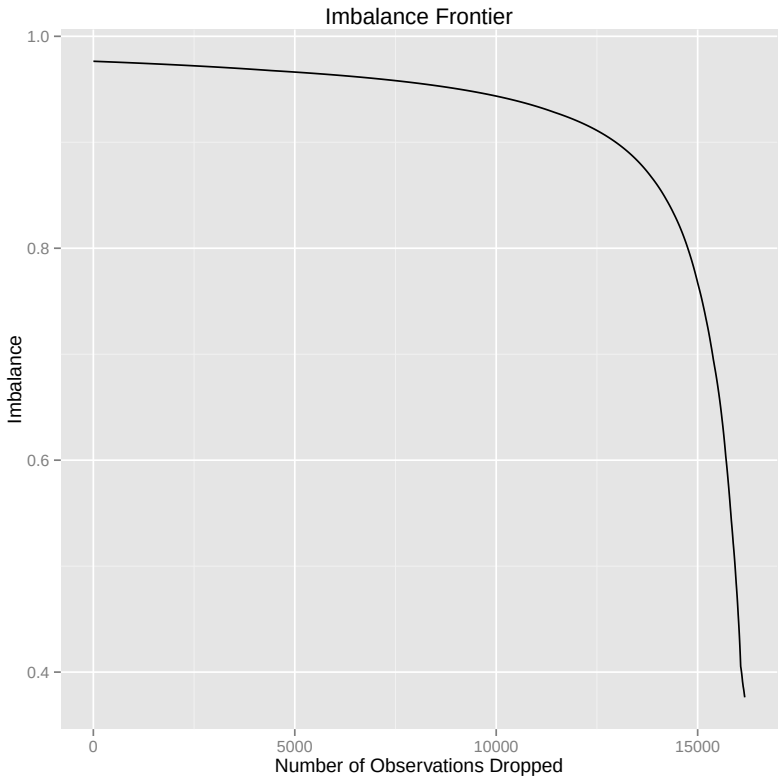

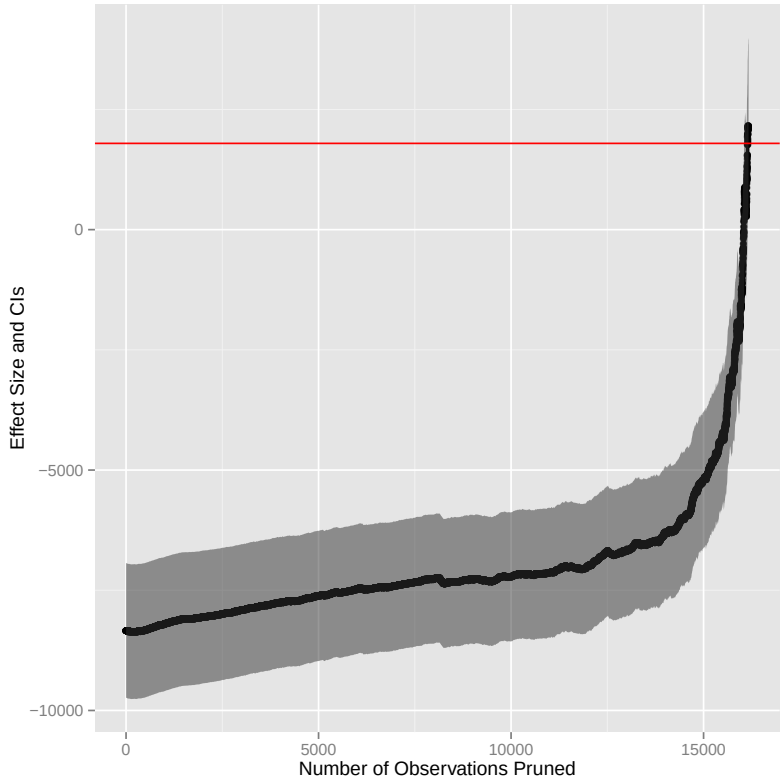

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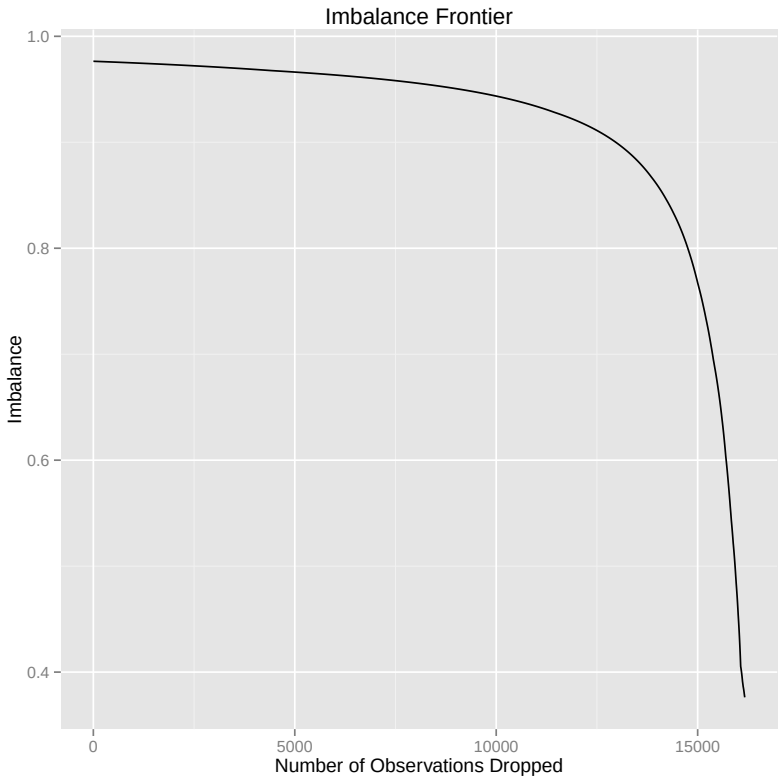

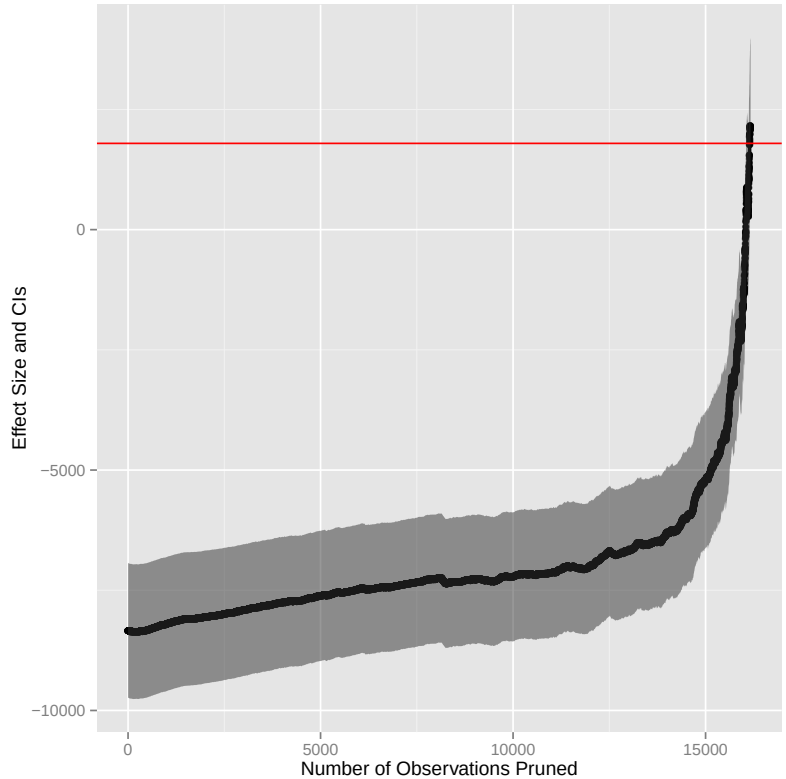

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- Estimates converge to experiment after removing bias
- No mysteries: basis of inference clearly revealed

Aid Shocks Data: Frontier and Causal Estimates

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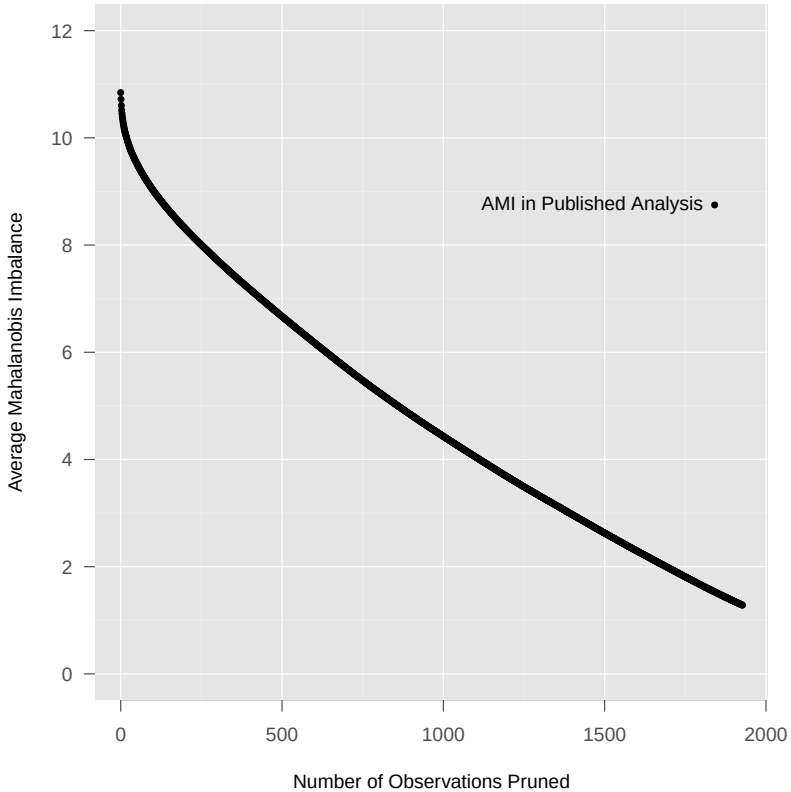

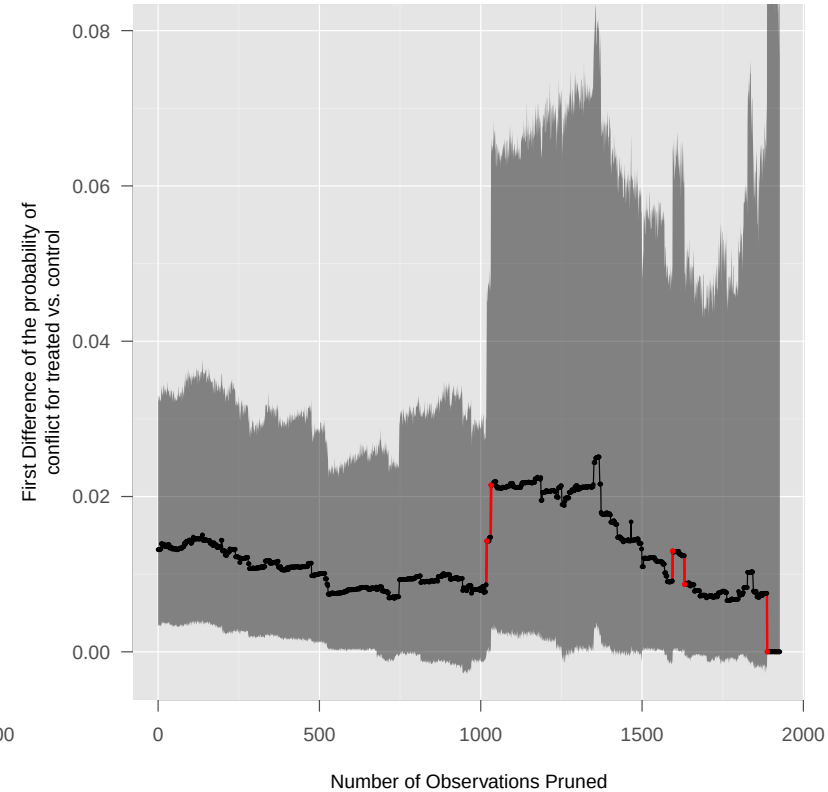

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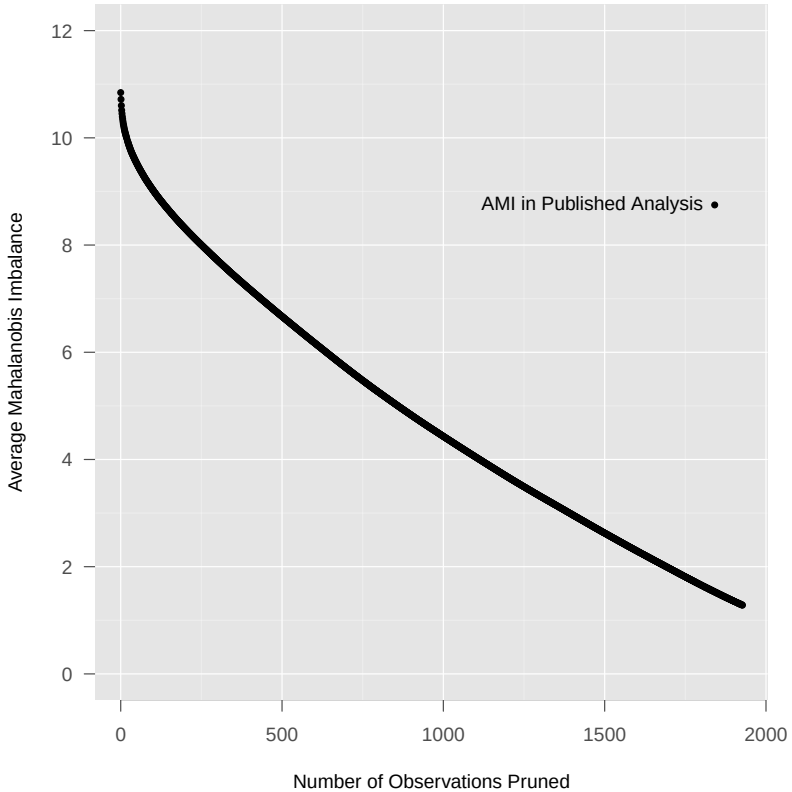

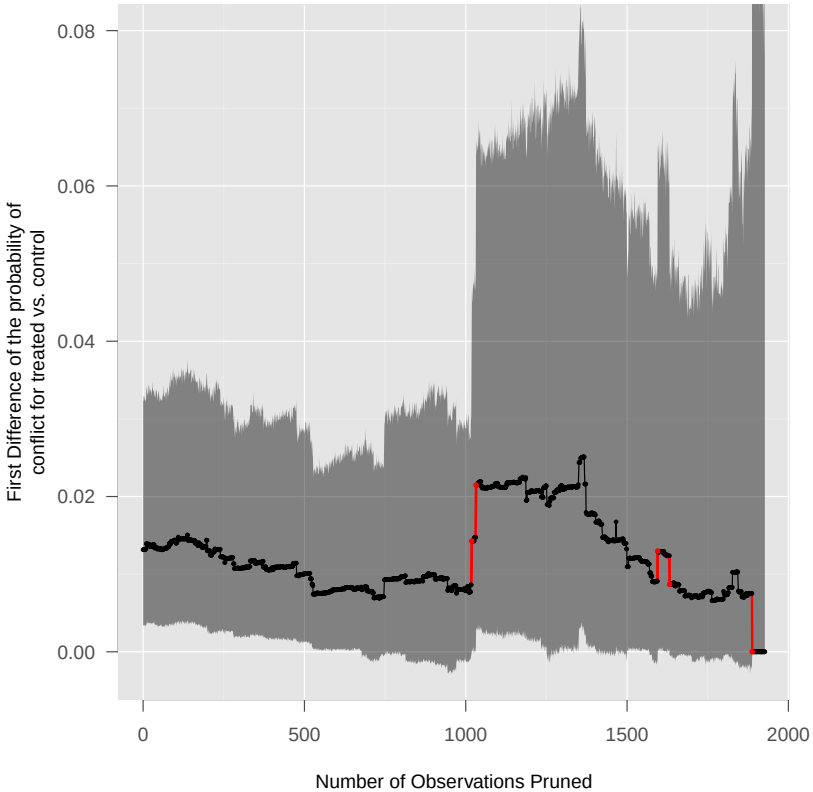

- Frontier is nearly linear (left)

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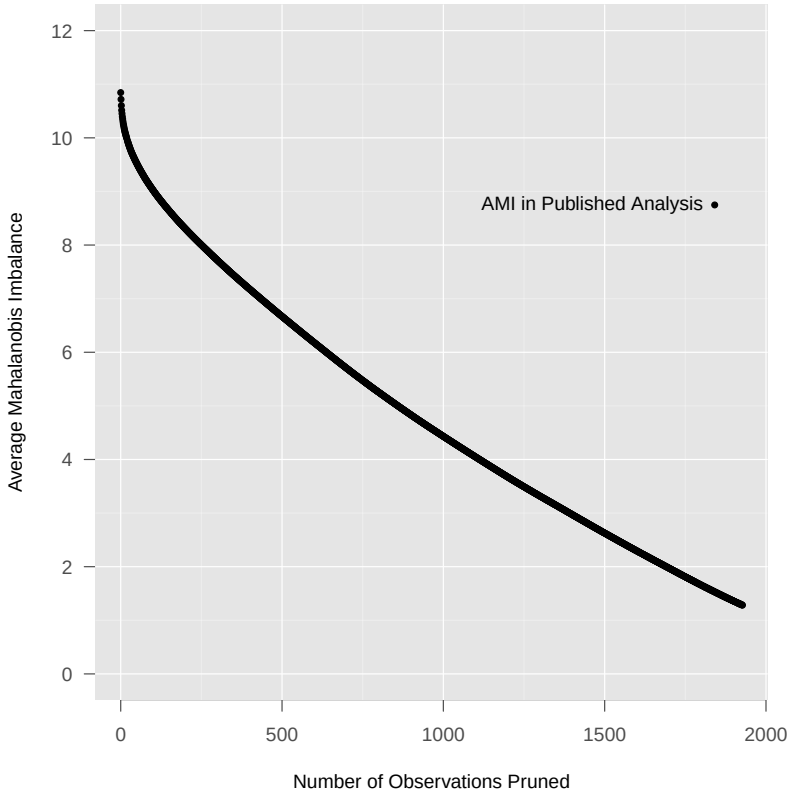

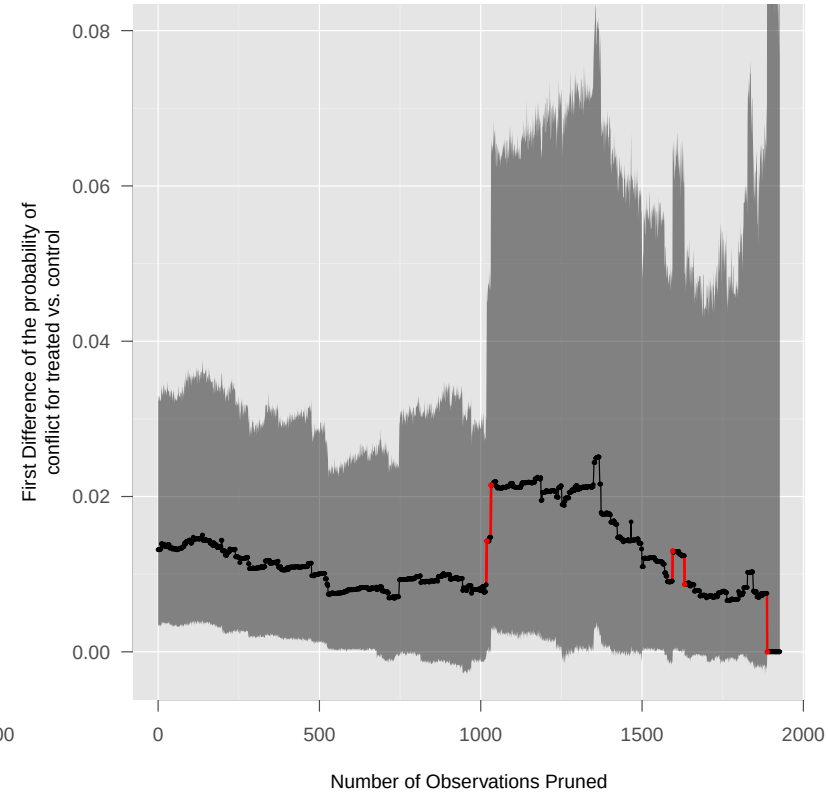

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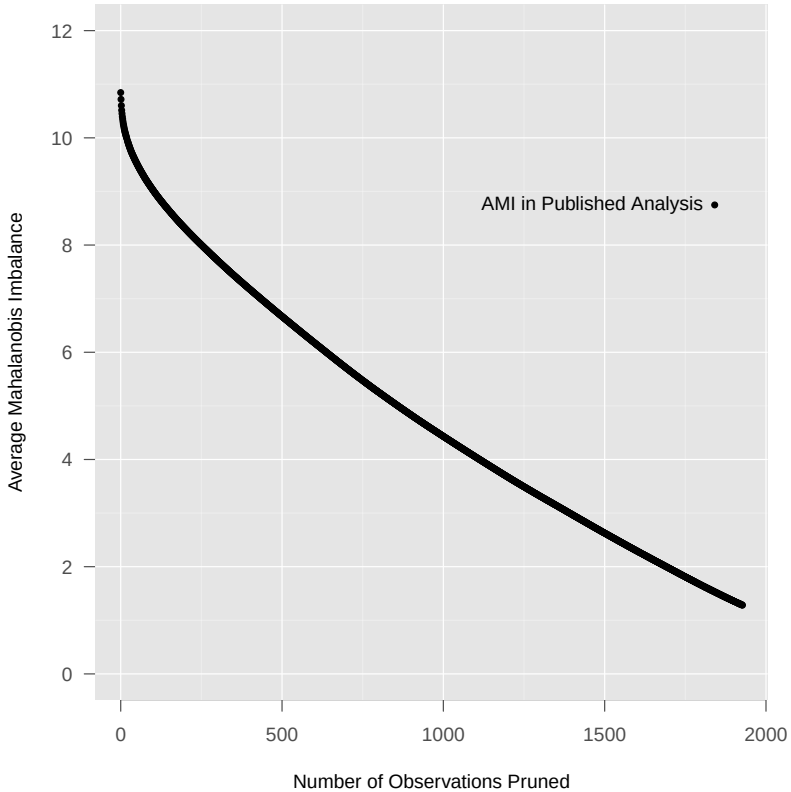

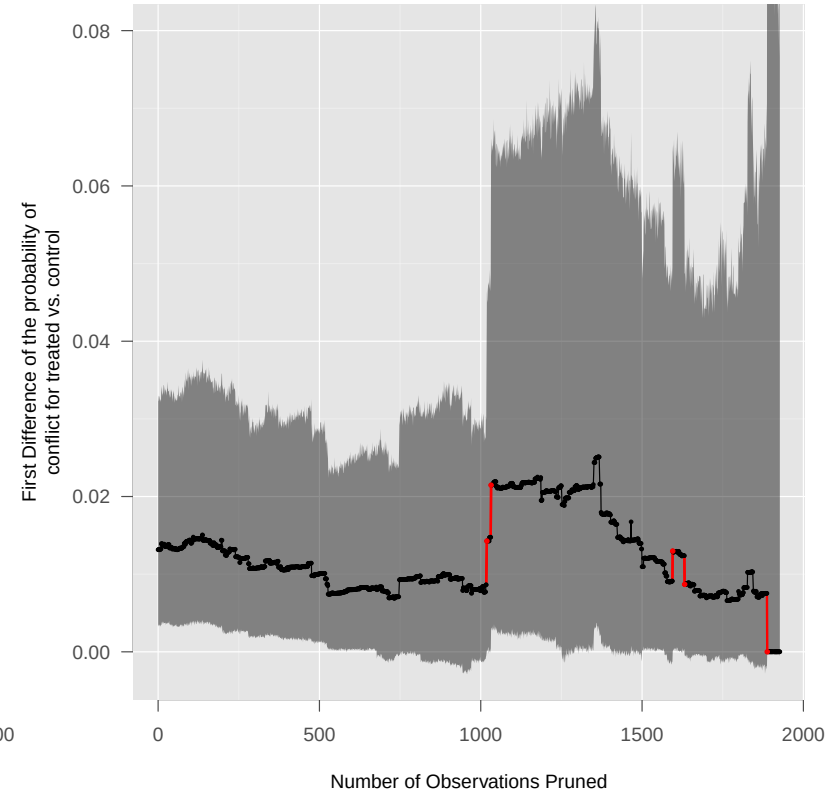

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- More difficult inferential task

Aids Shocks: Change in Quantity of Interest

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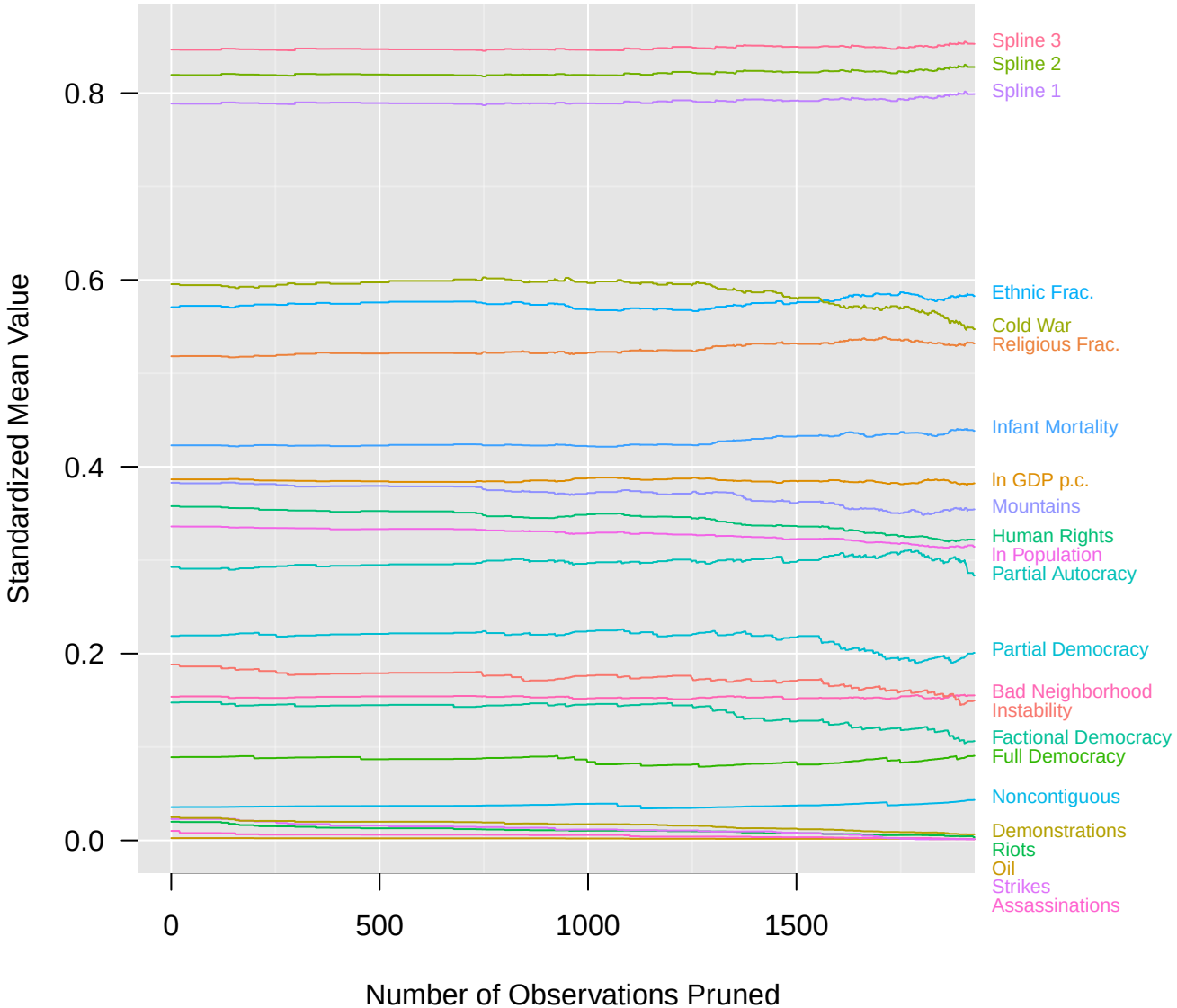

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- High leverage points
- Cases with few substitutes
- Not model dependence (which matching helps with), but data dependence

Constructing the FSATT Mahalanobis Frontier

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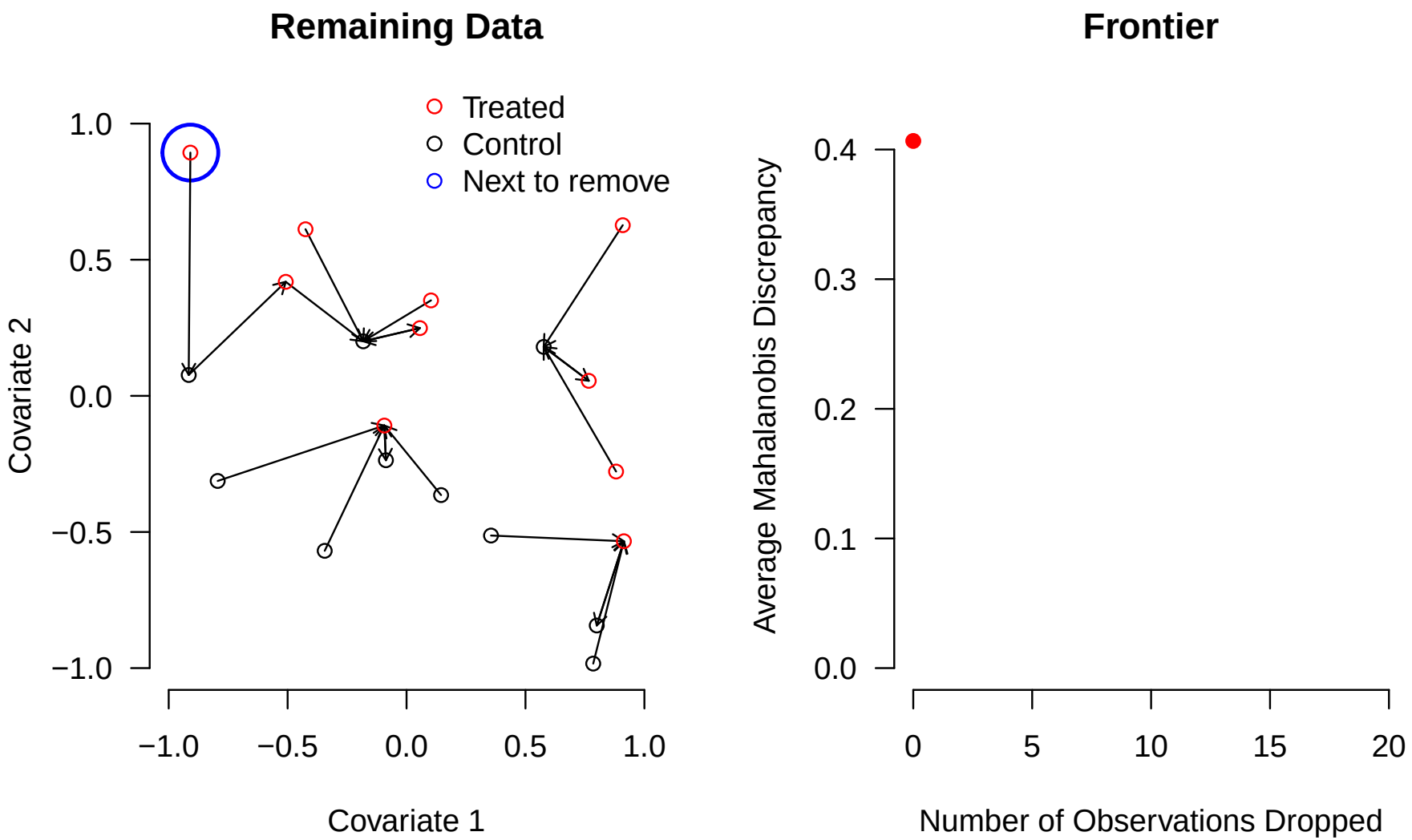

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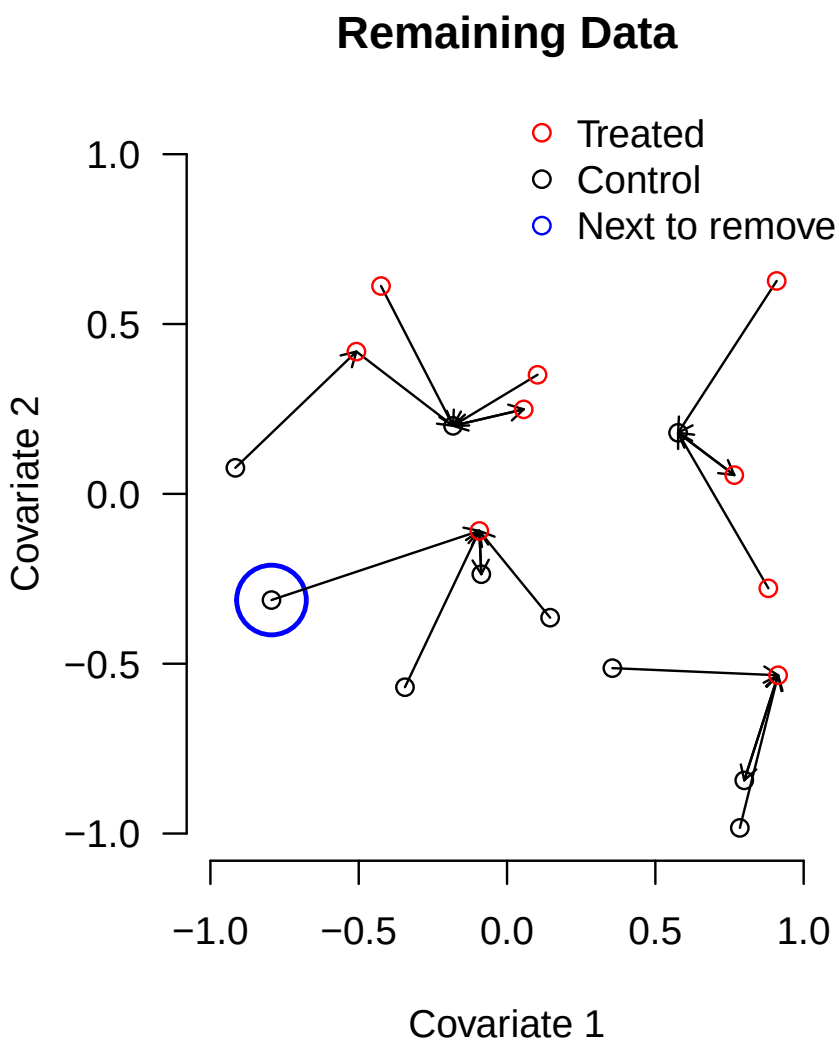

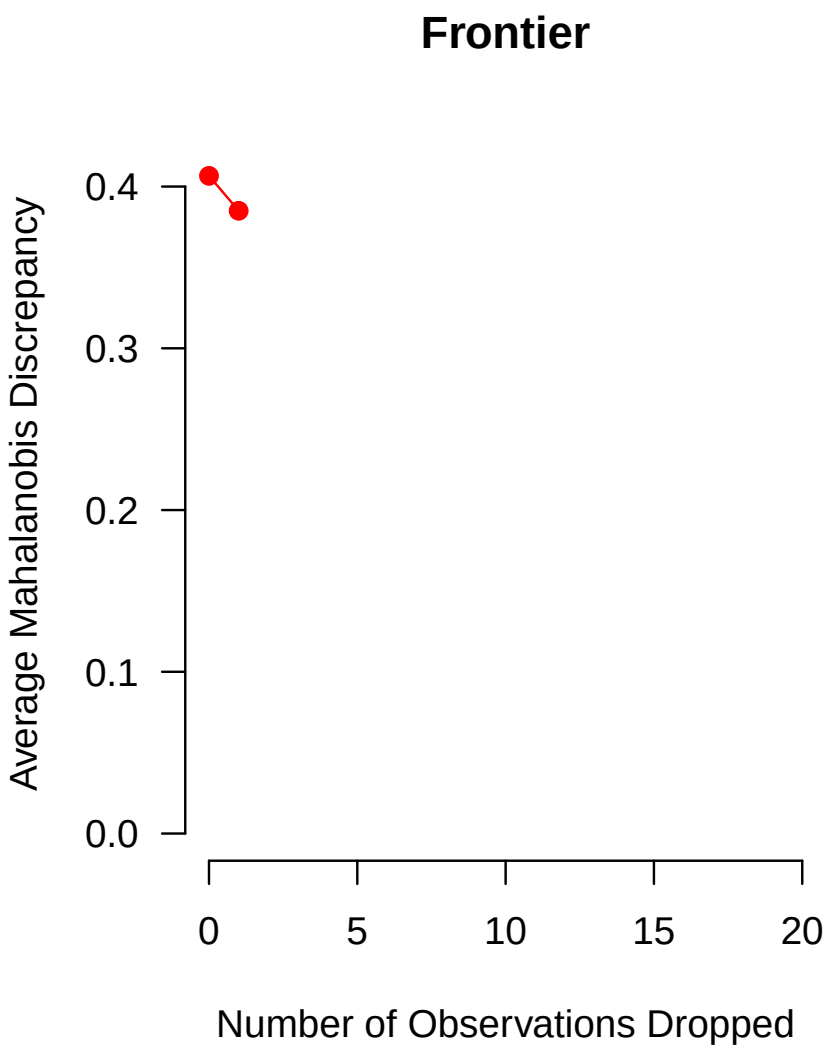

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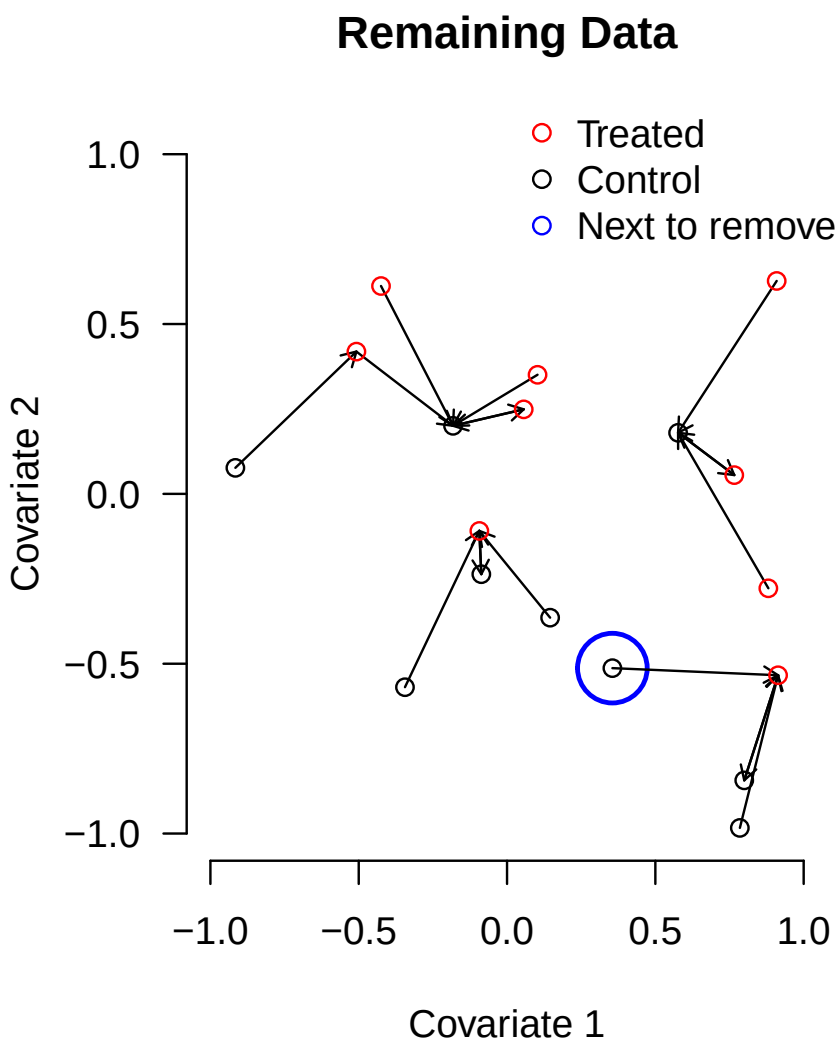

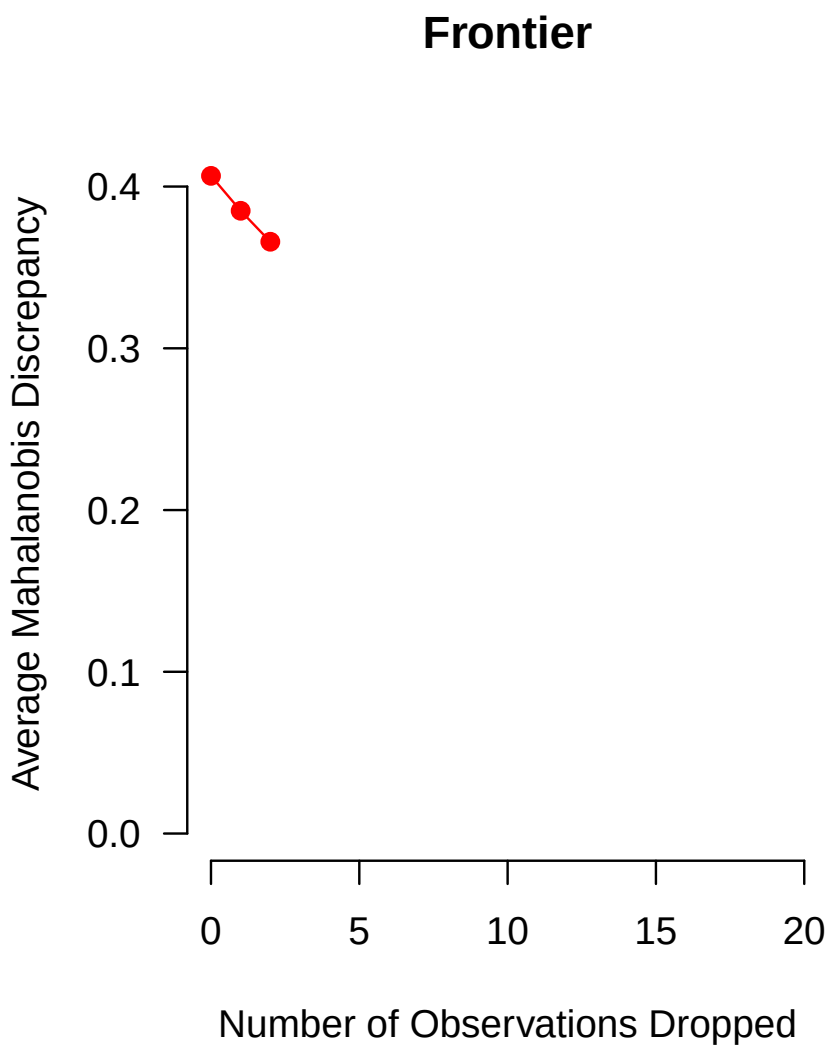

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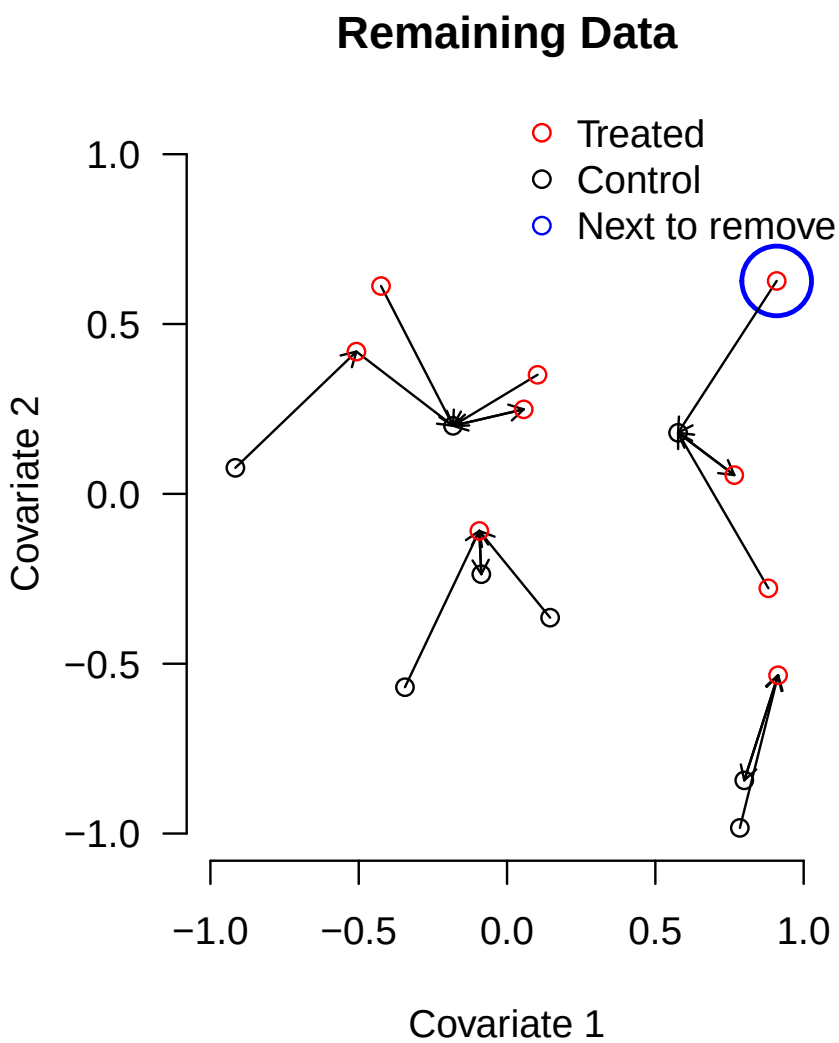

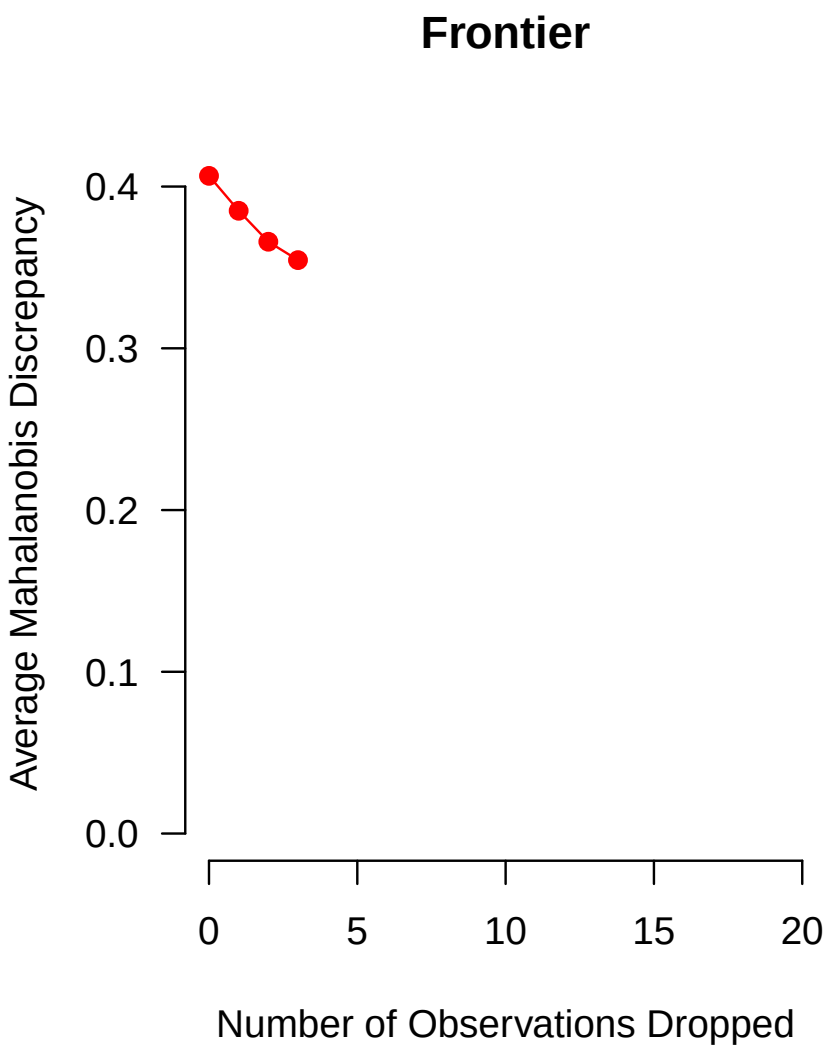

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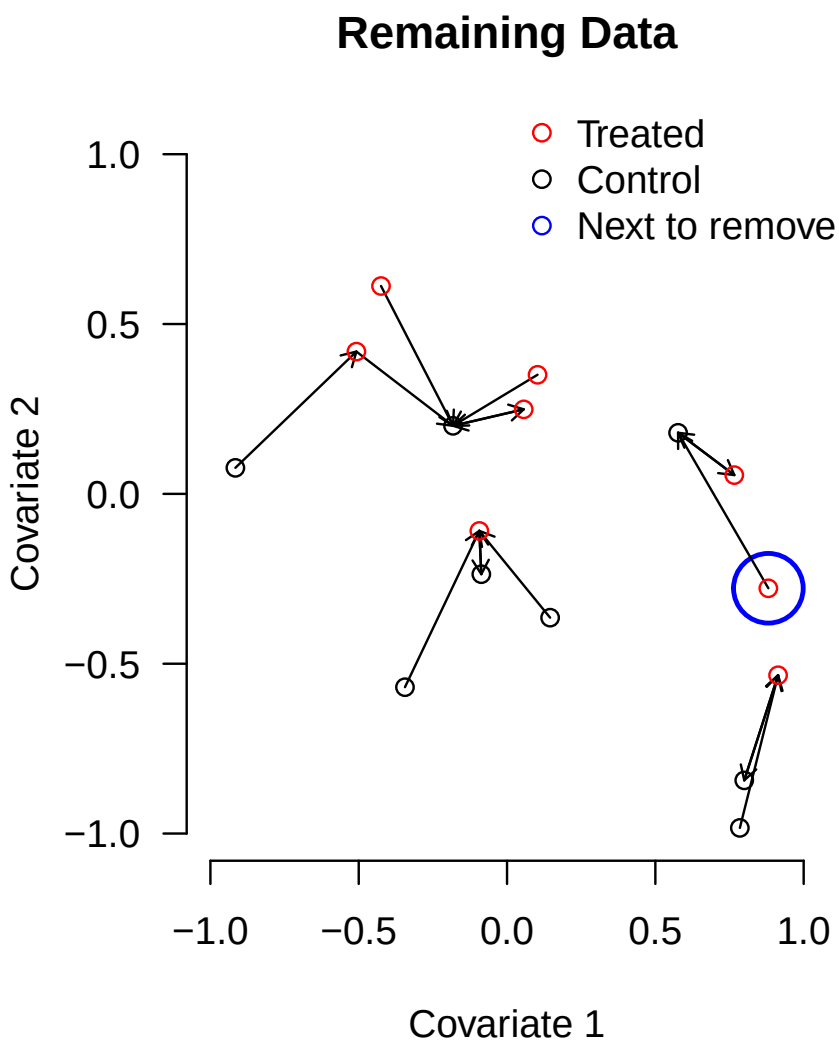

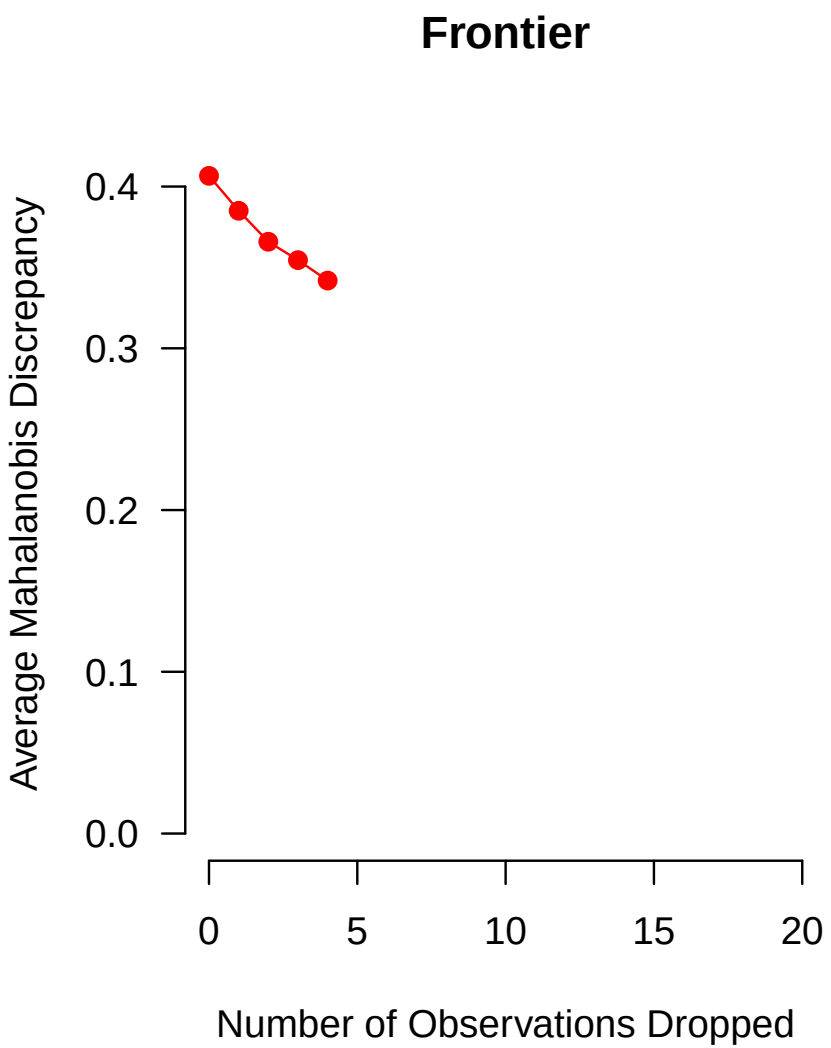

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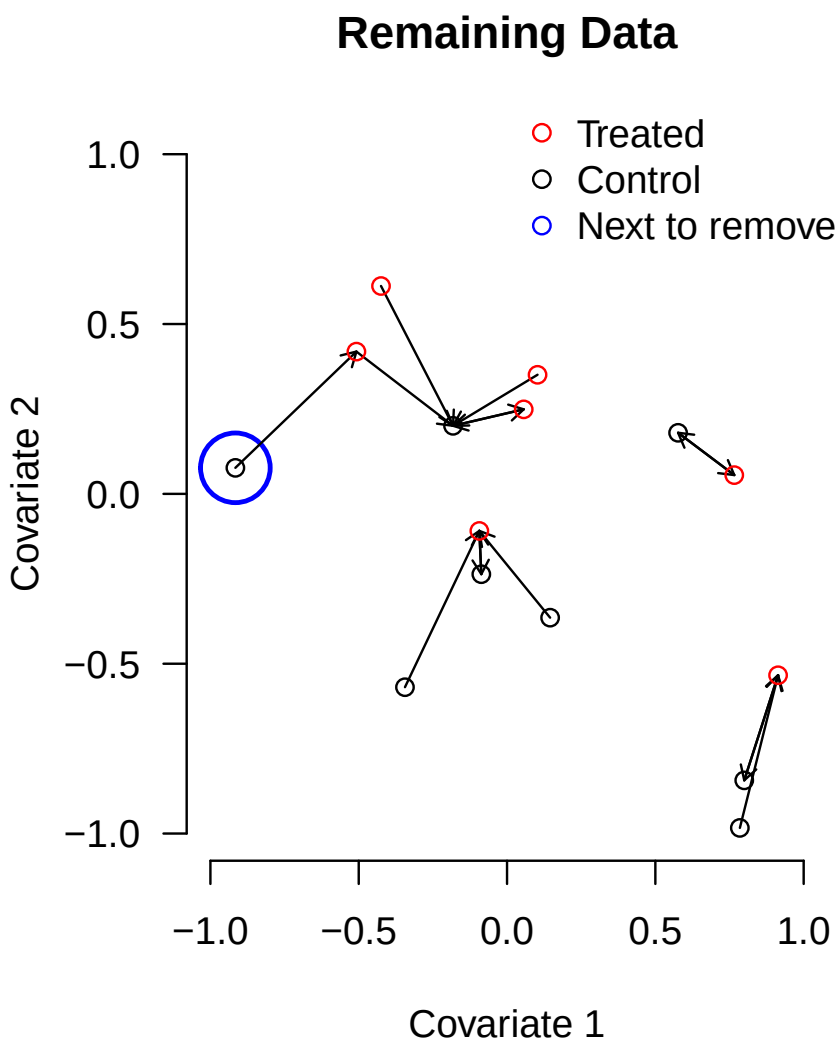

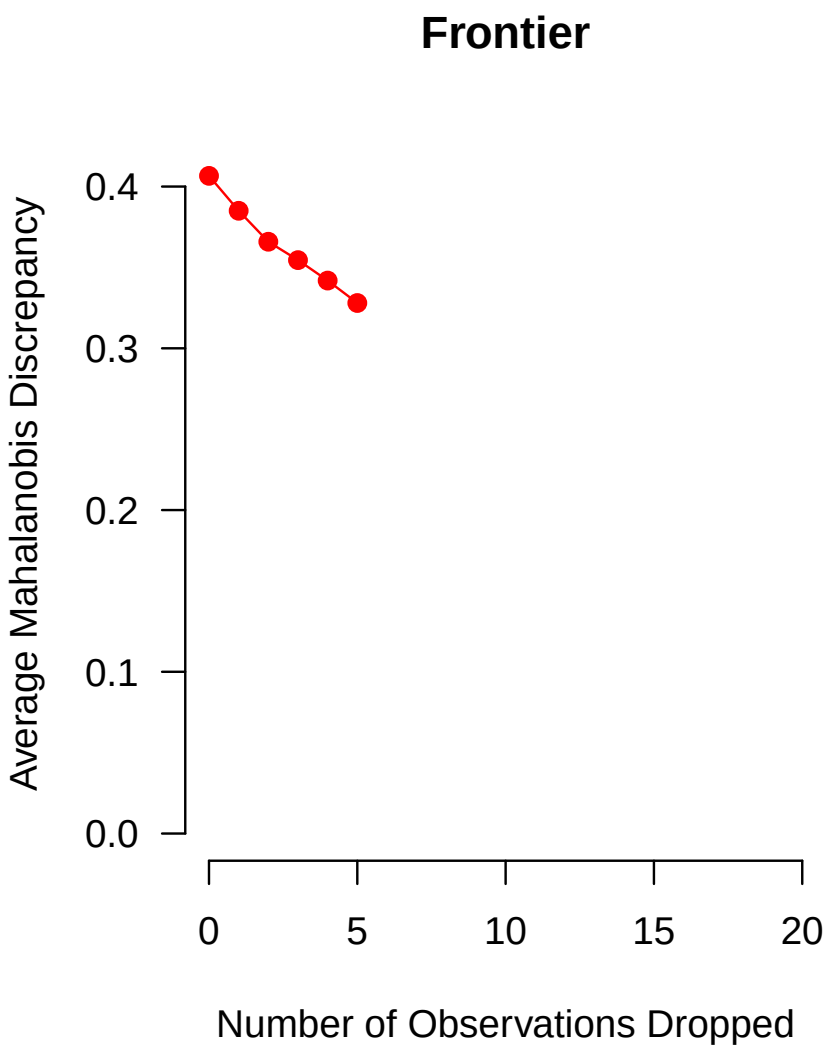

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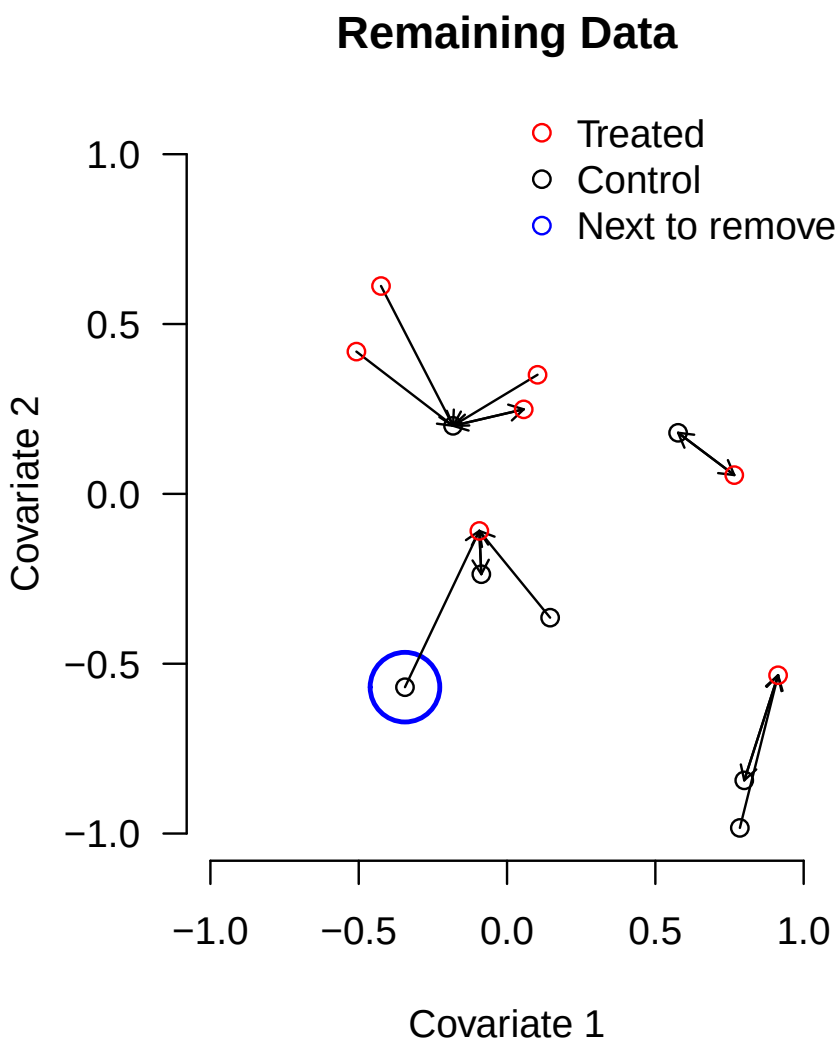

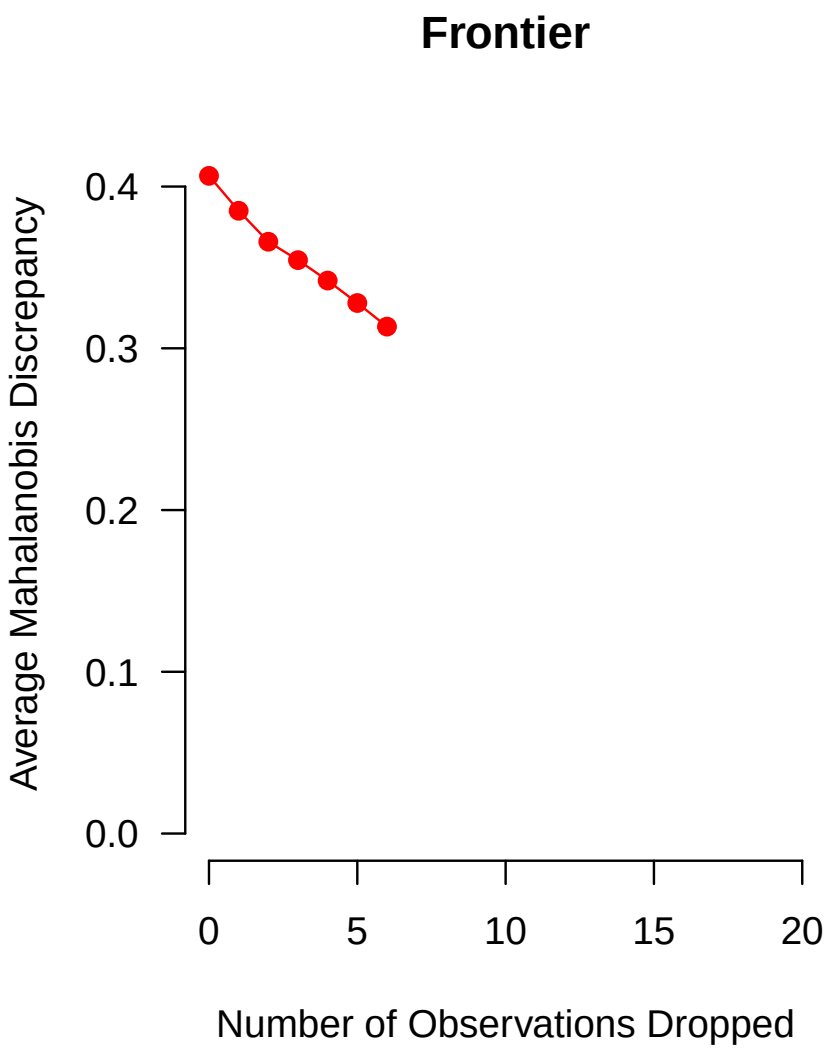

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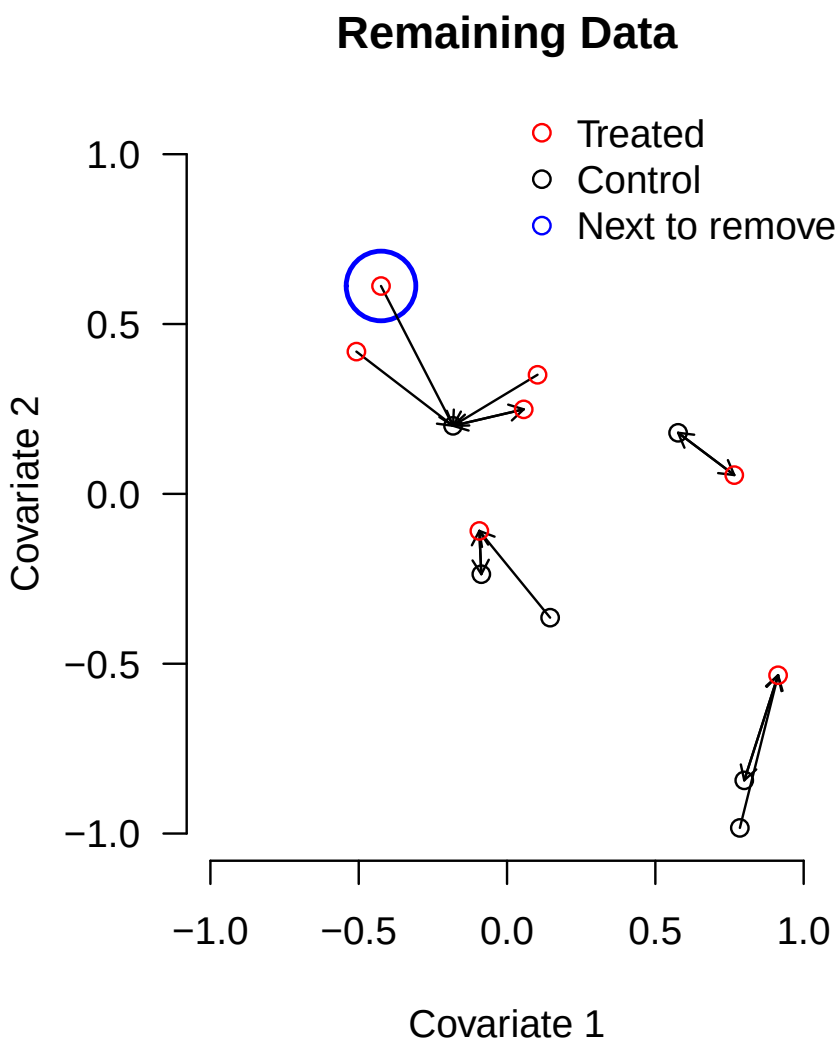

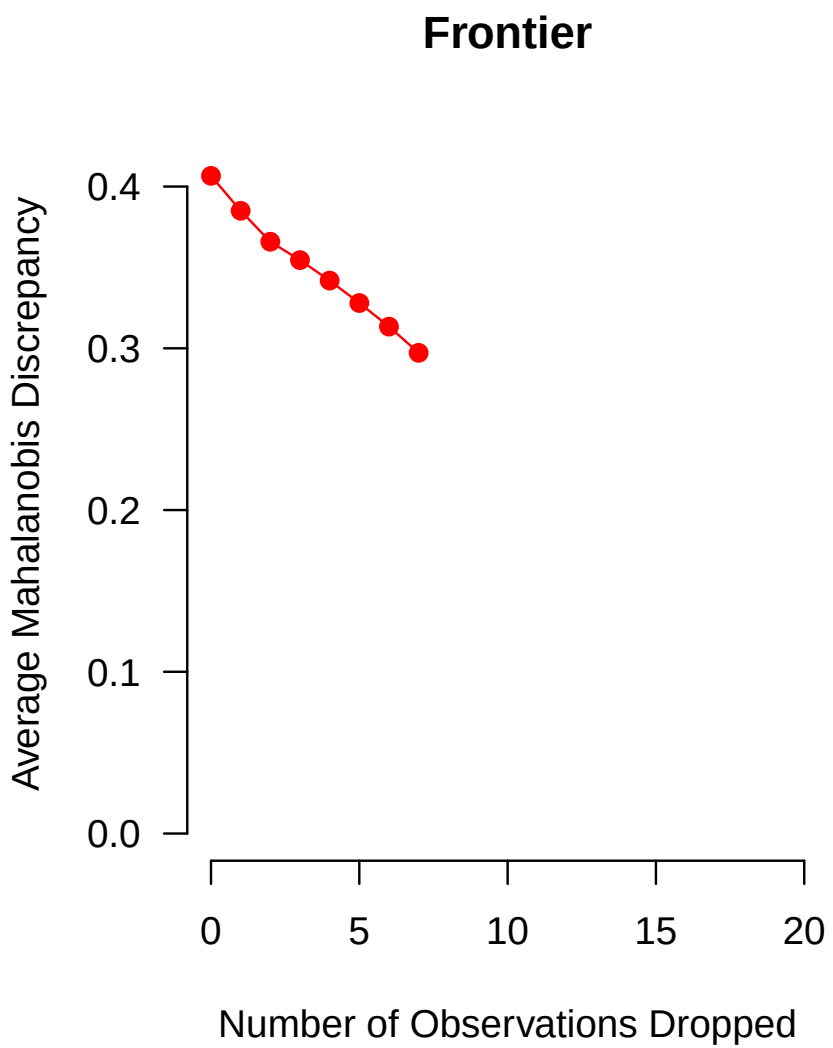

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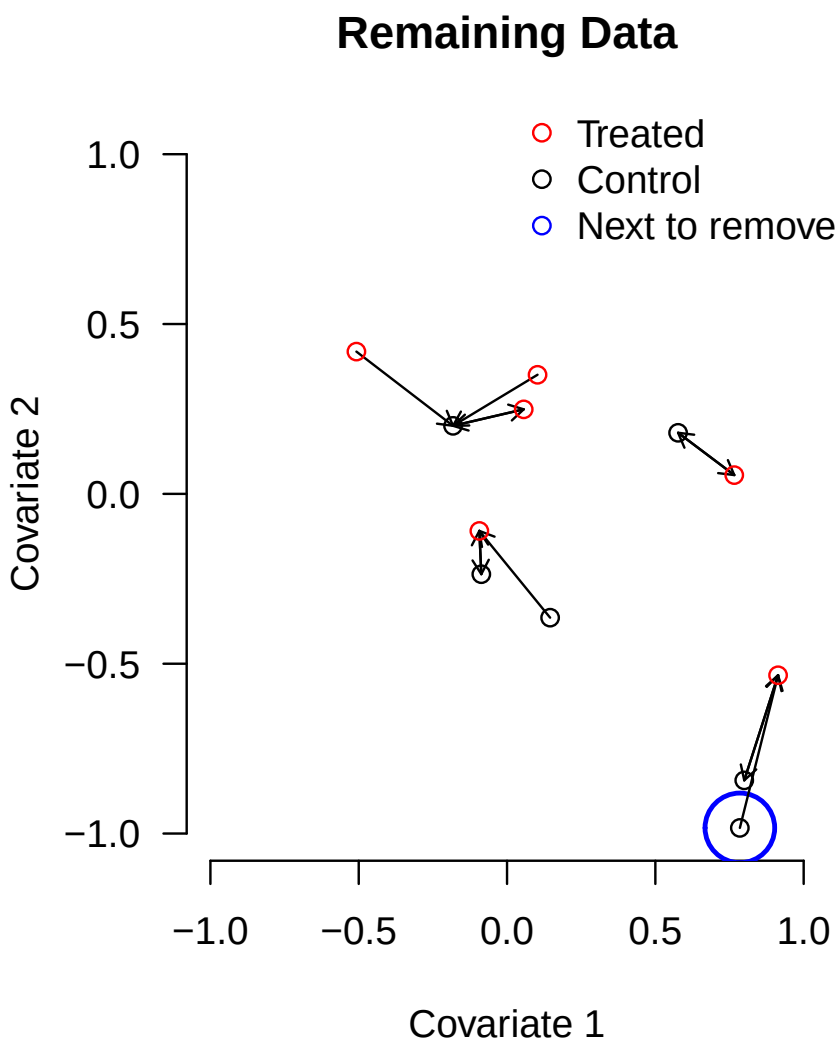

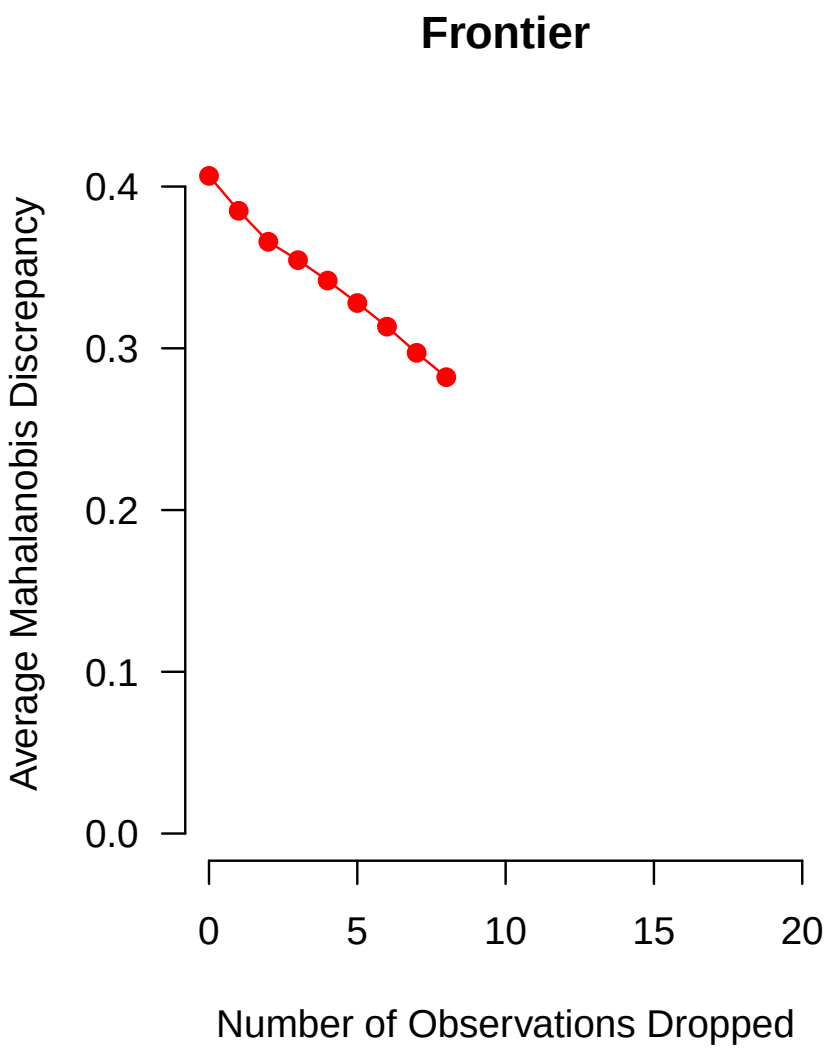

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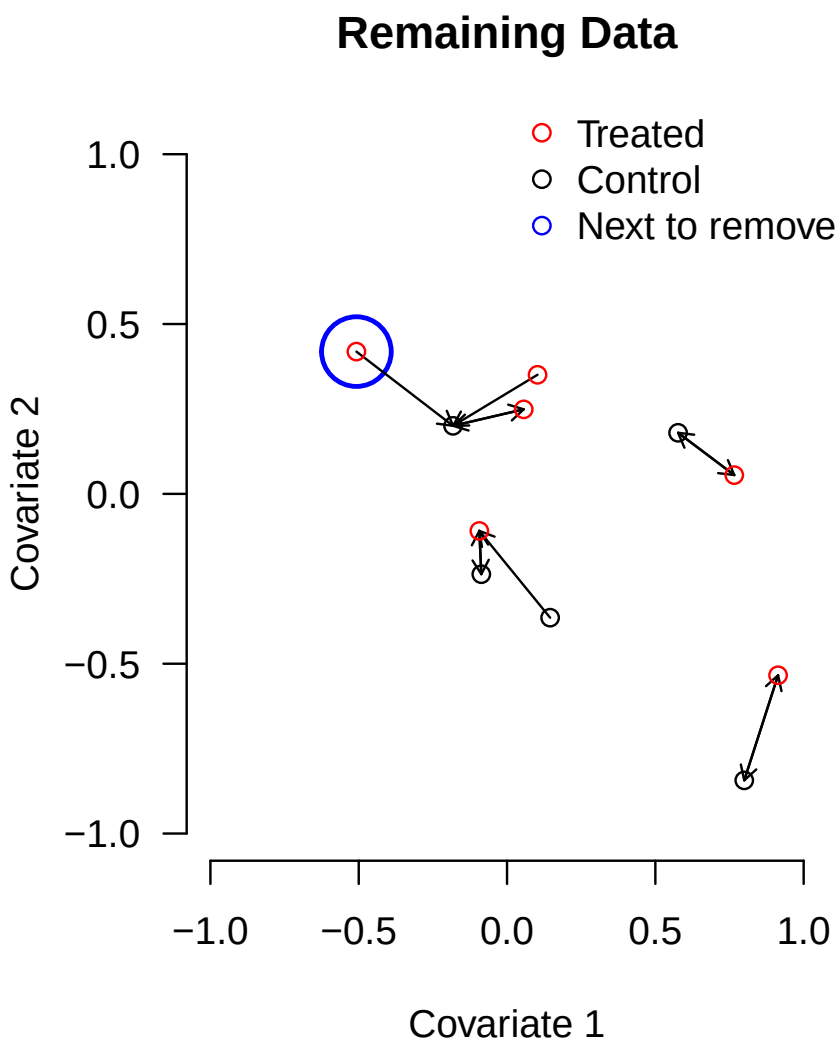

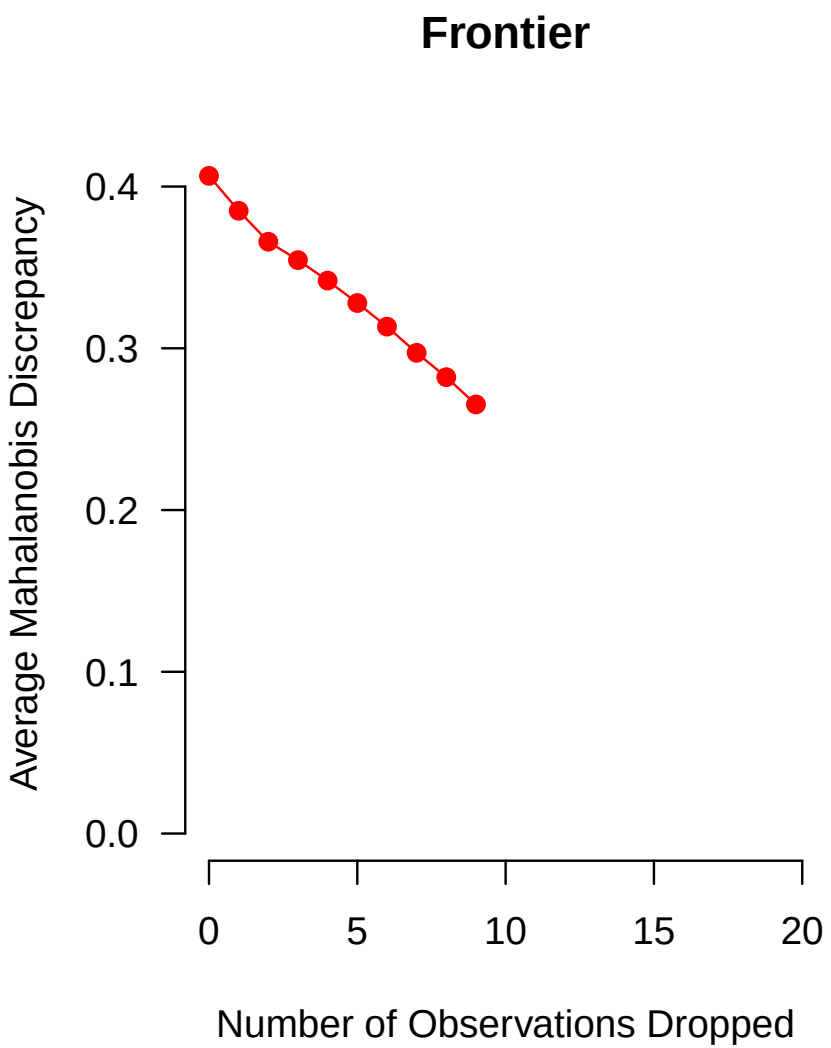

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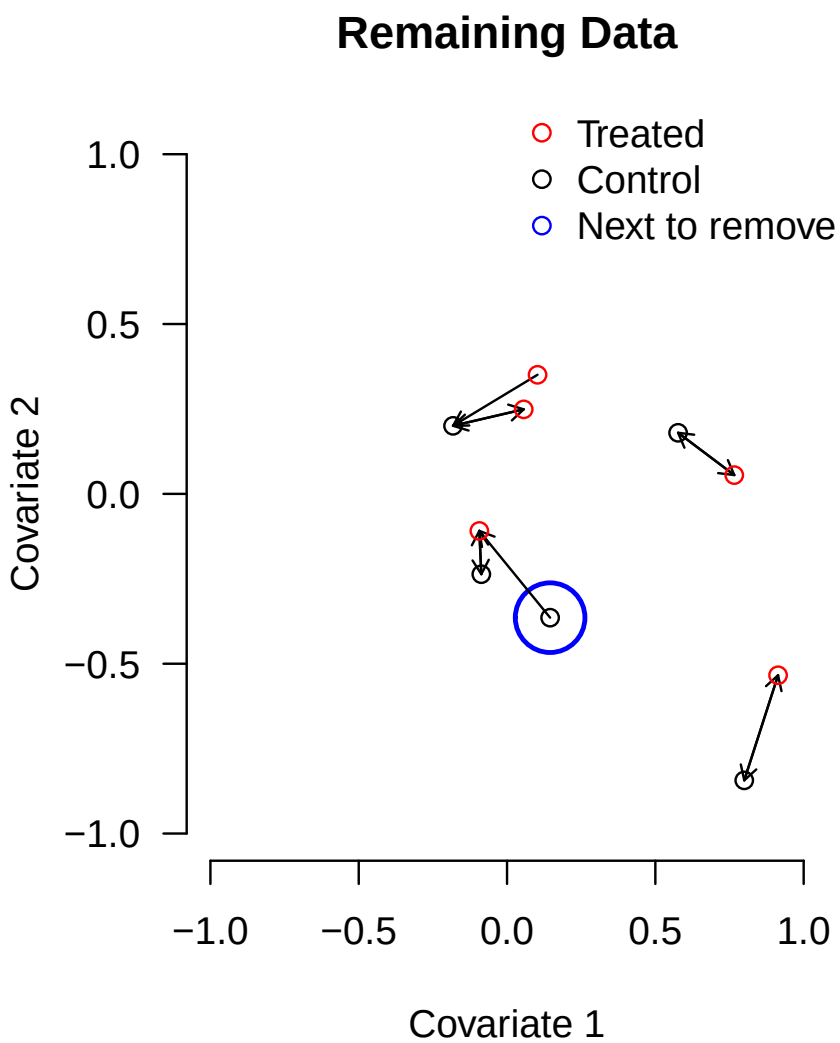

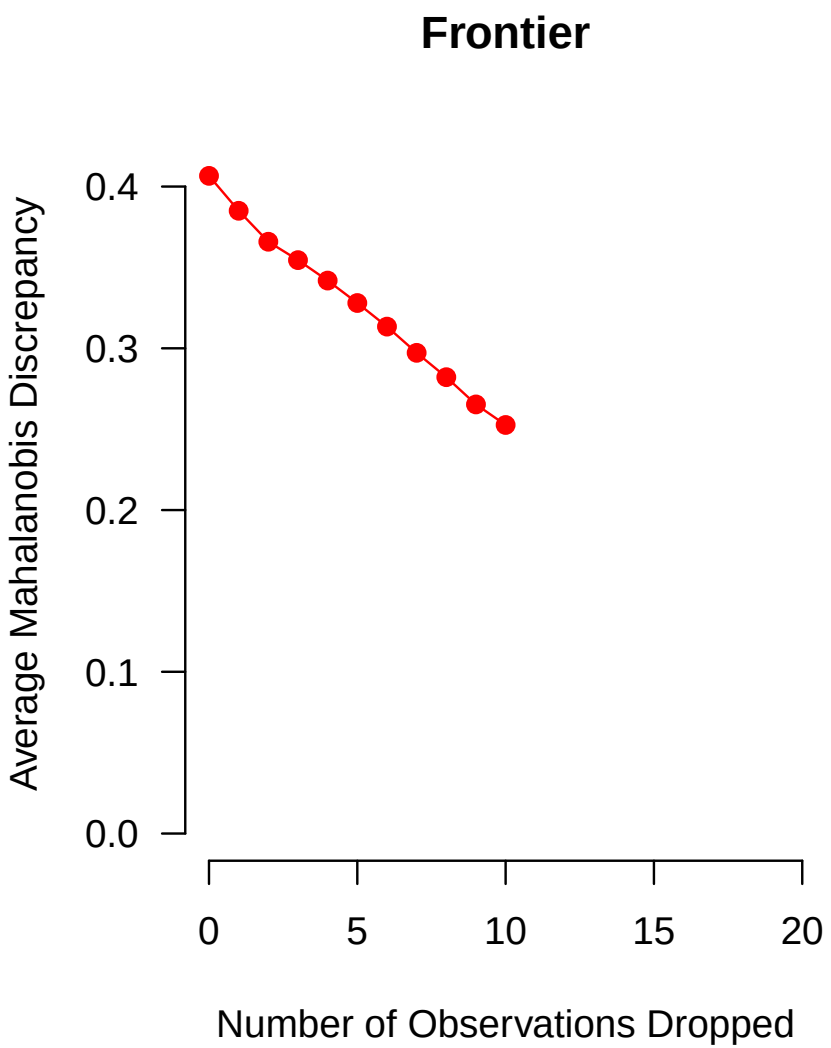

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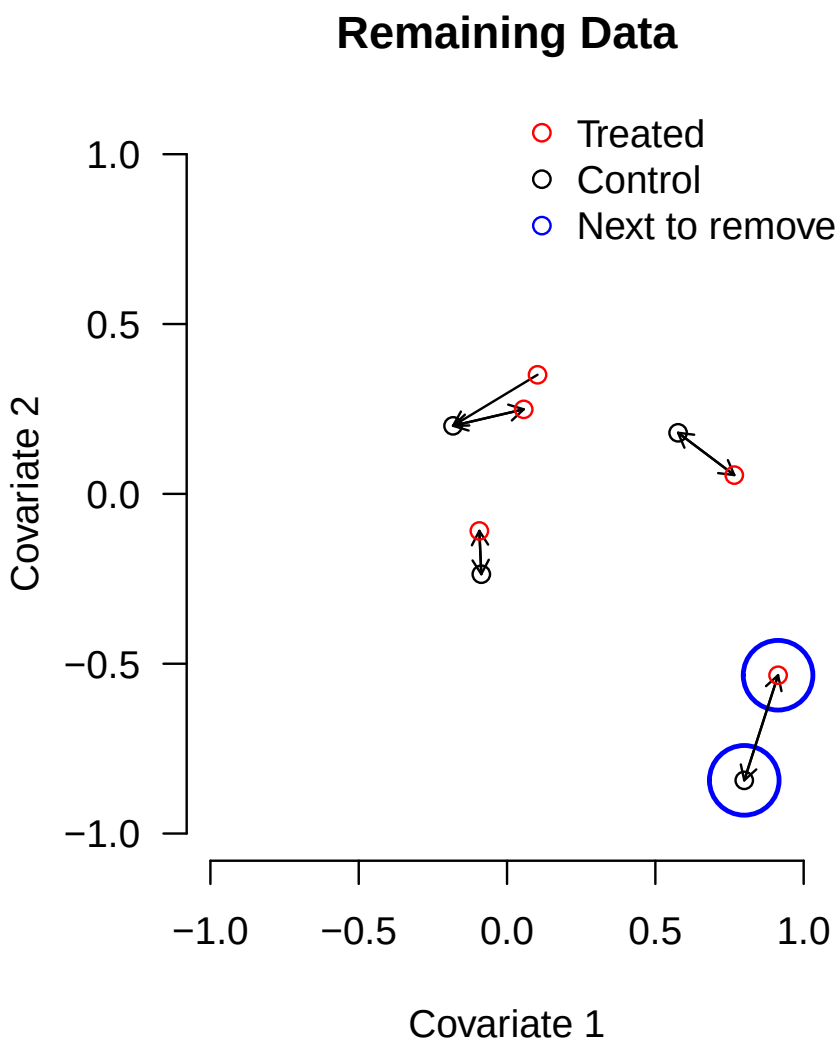

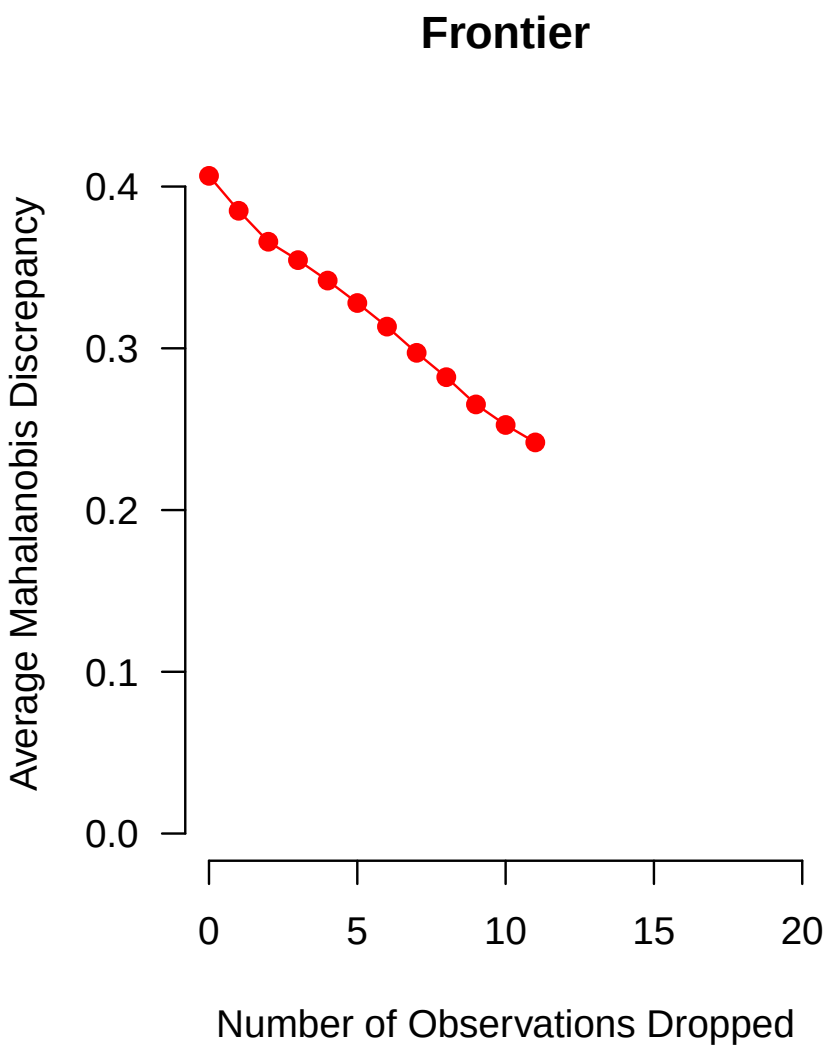

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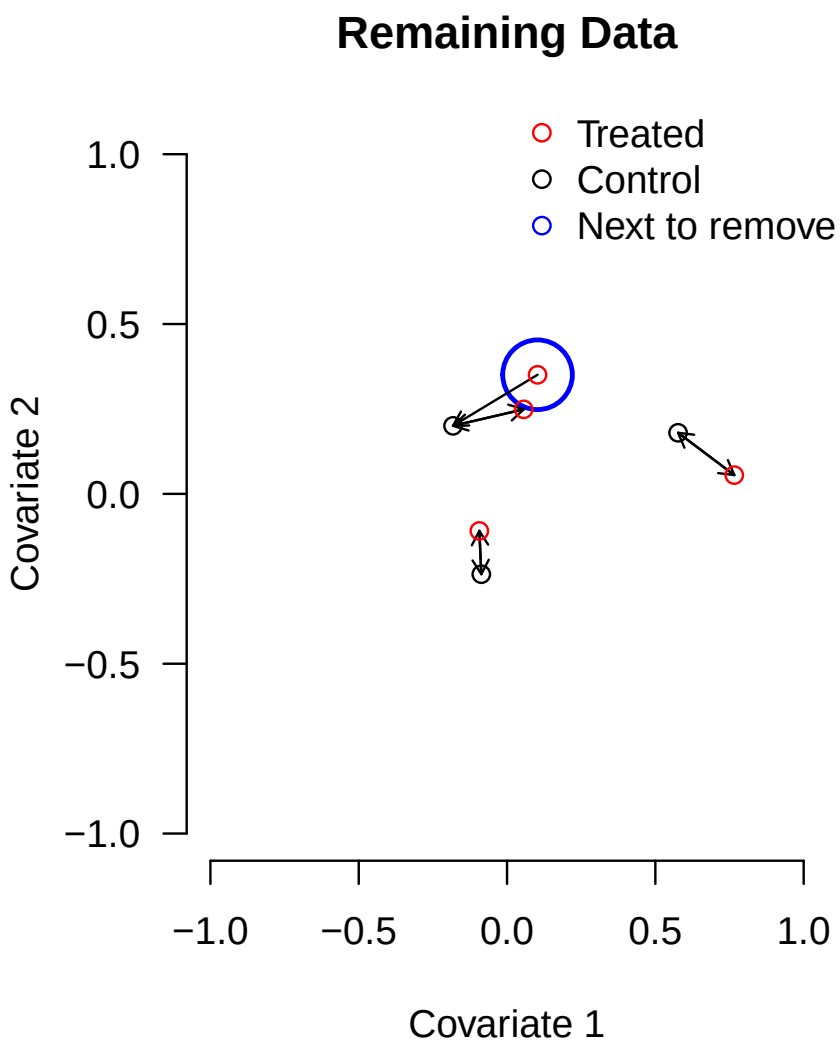

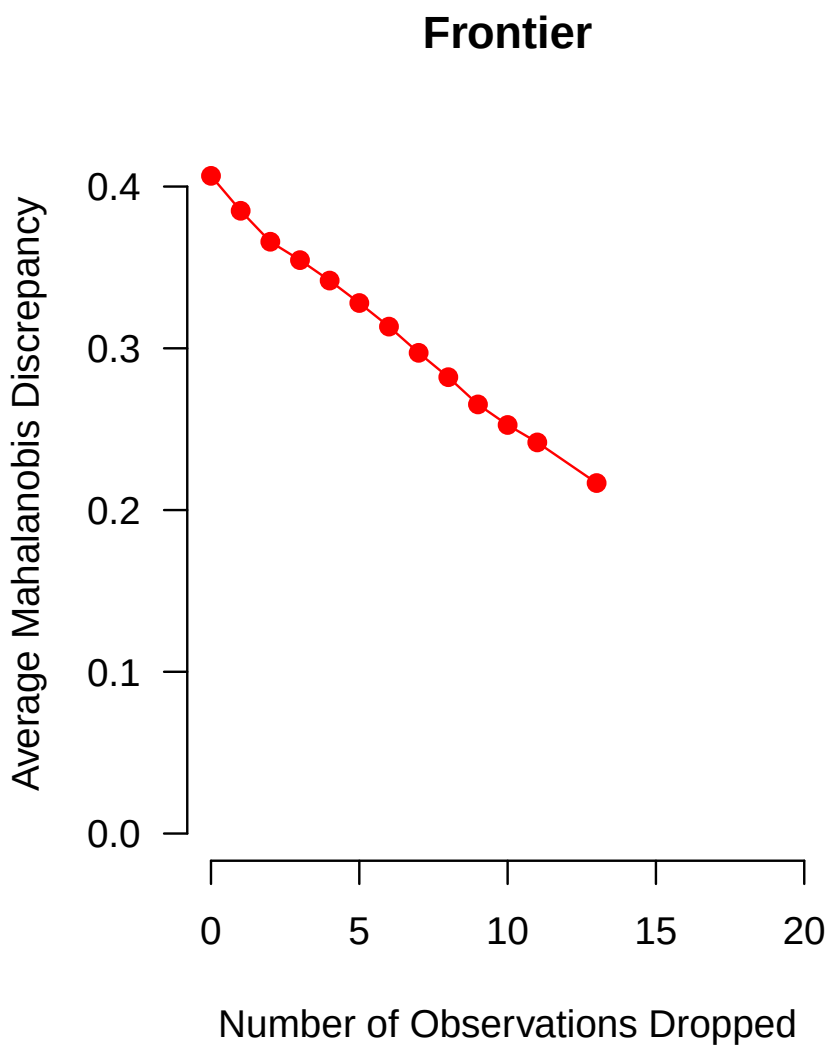

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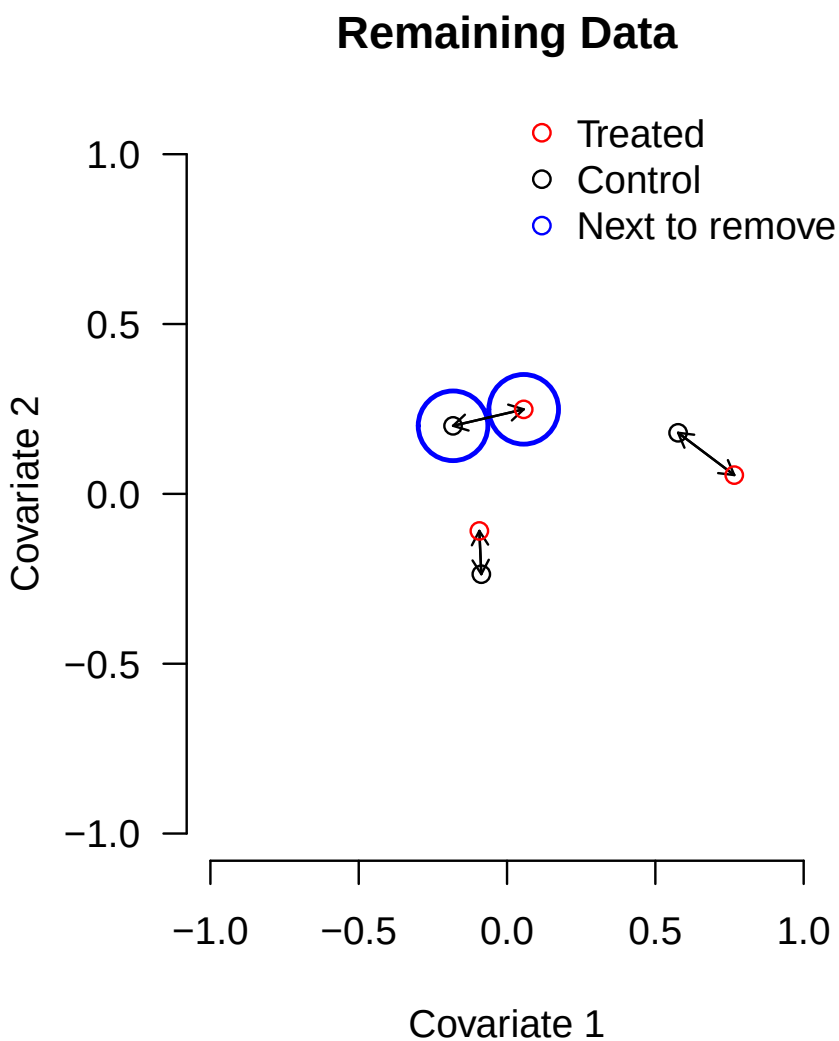

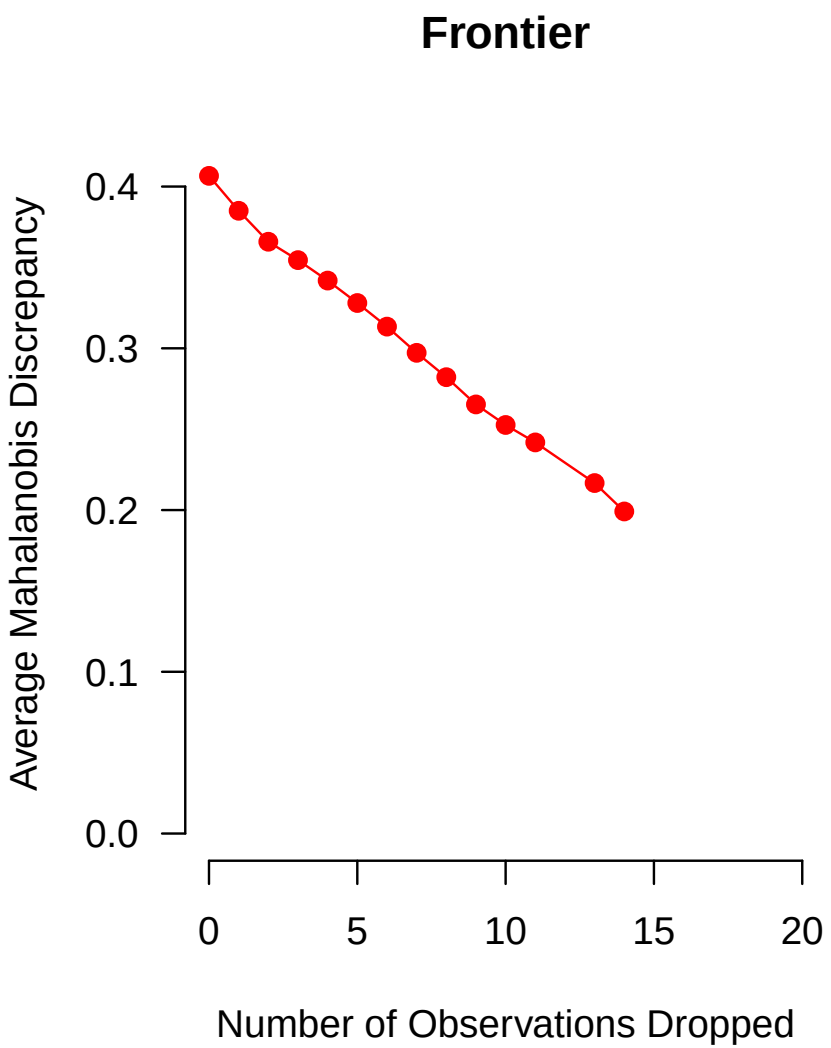

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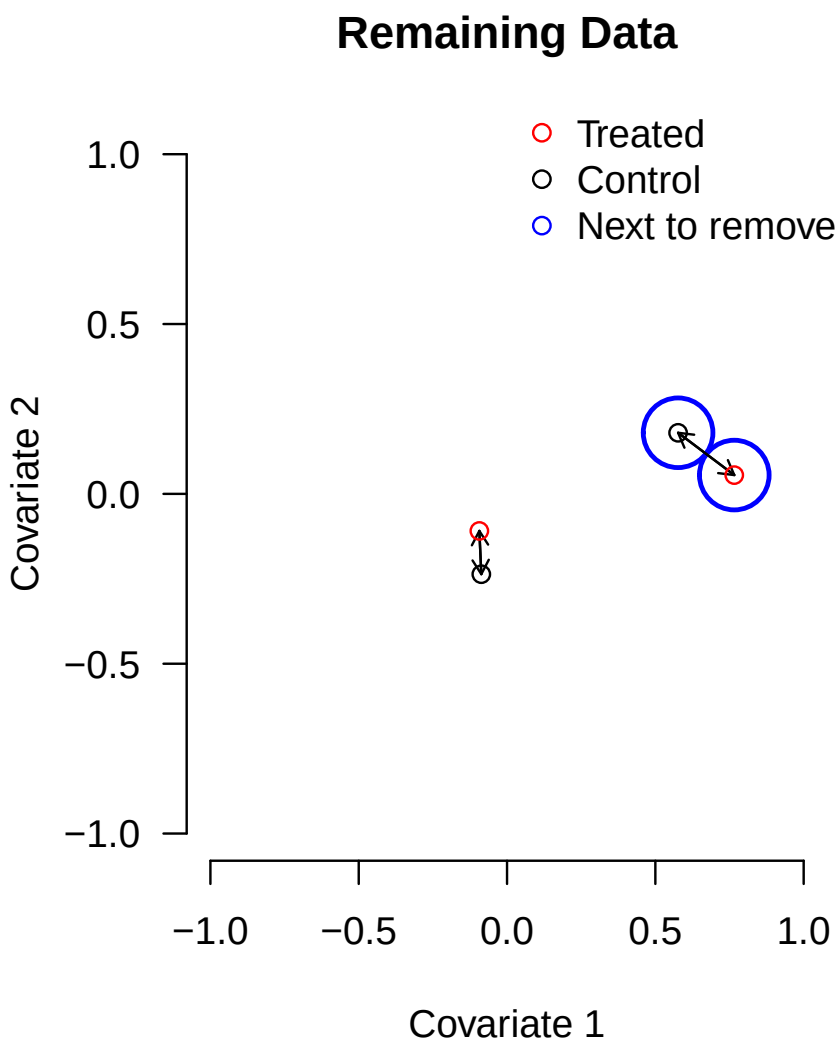

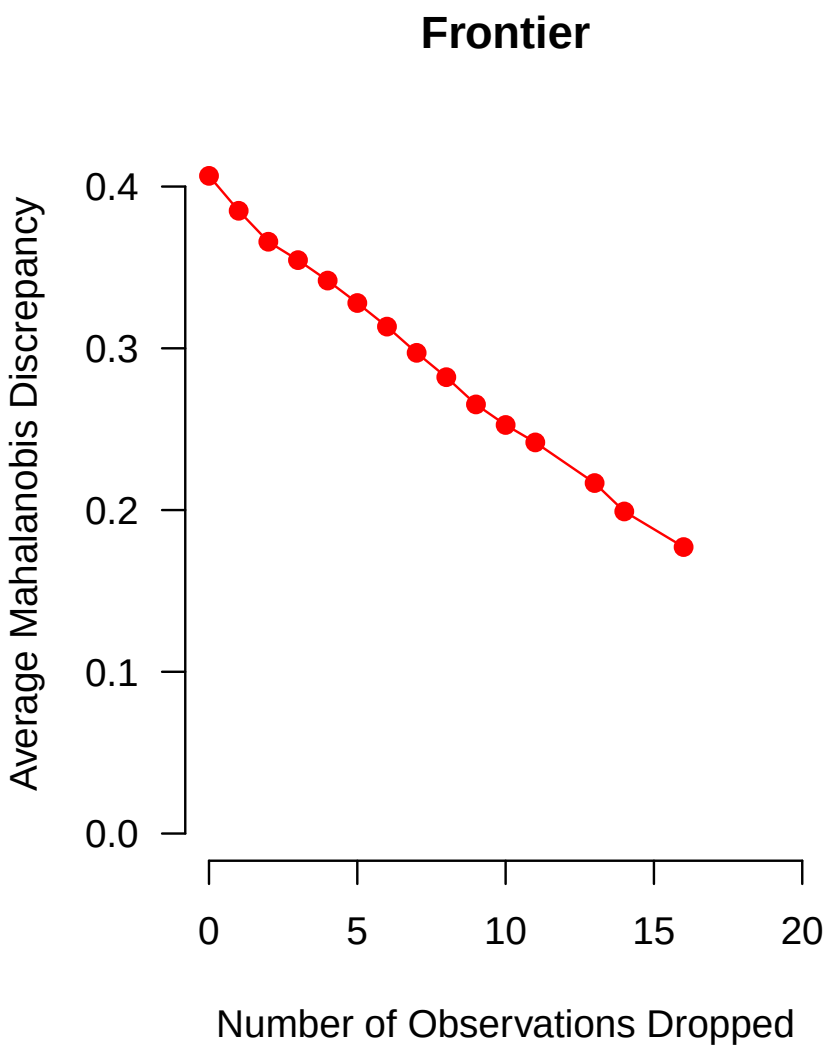

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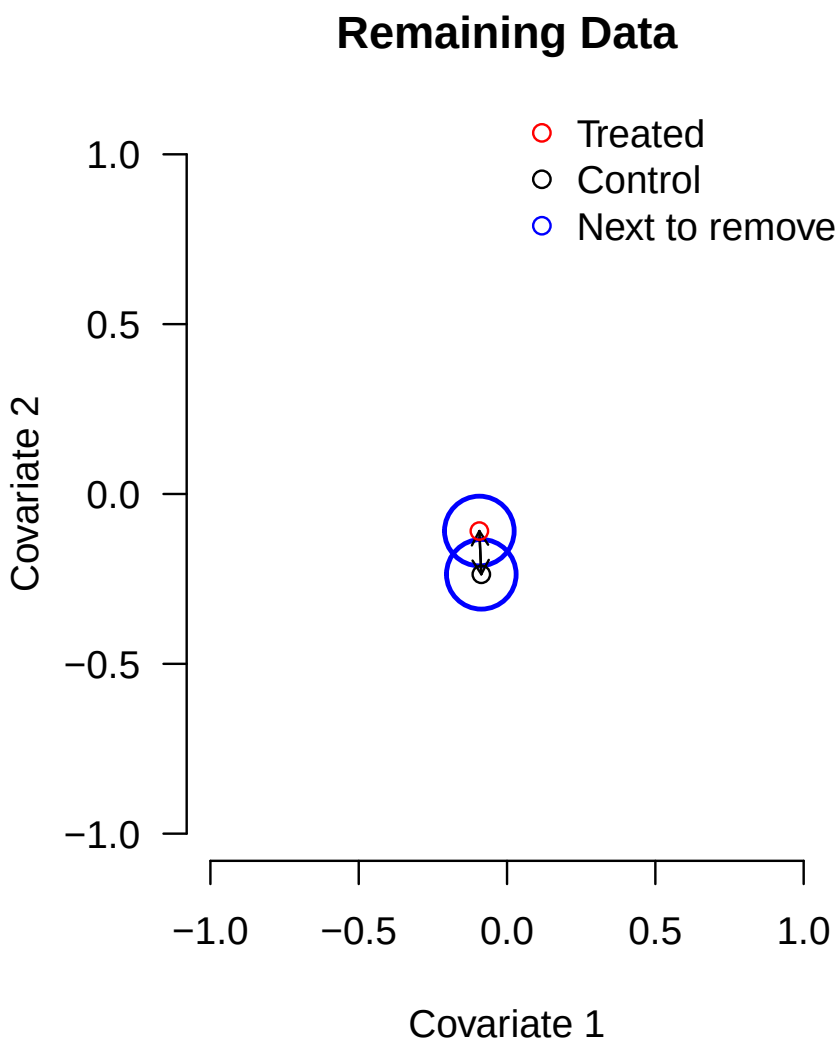

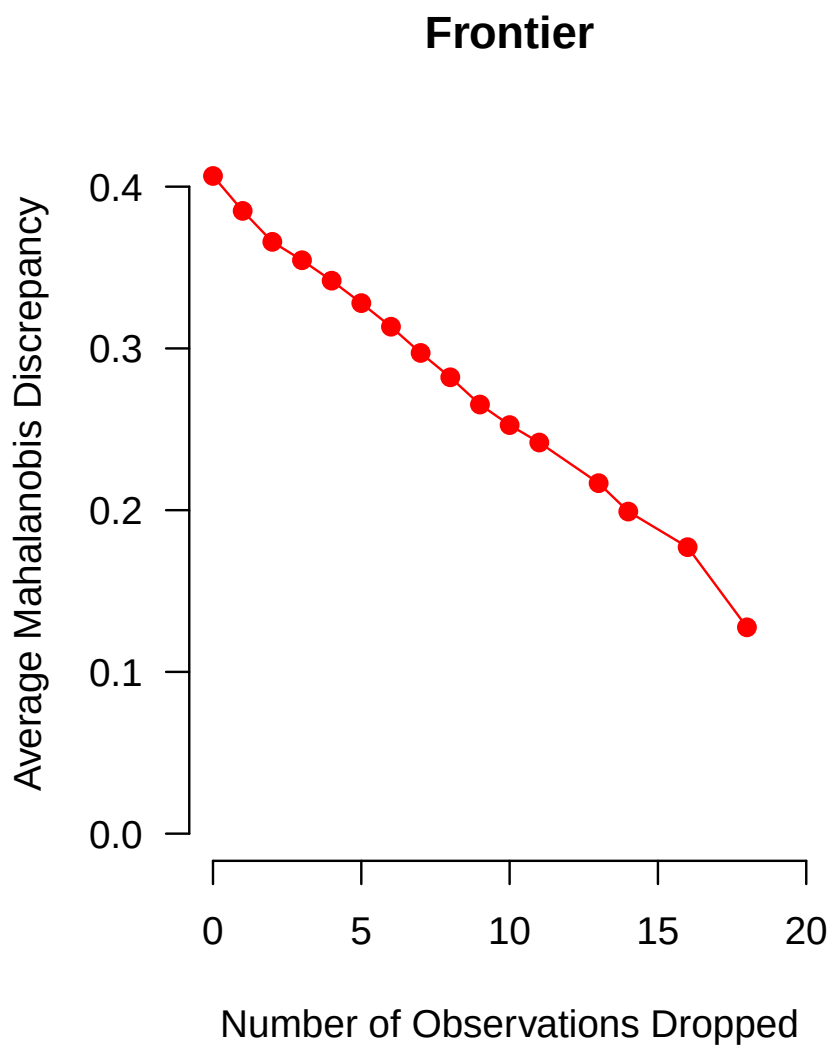

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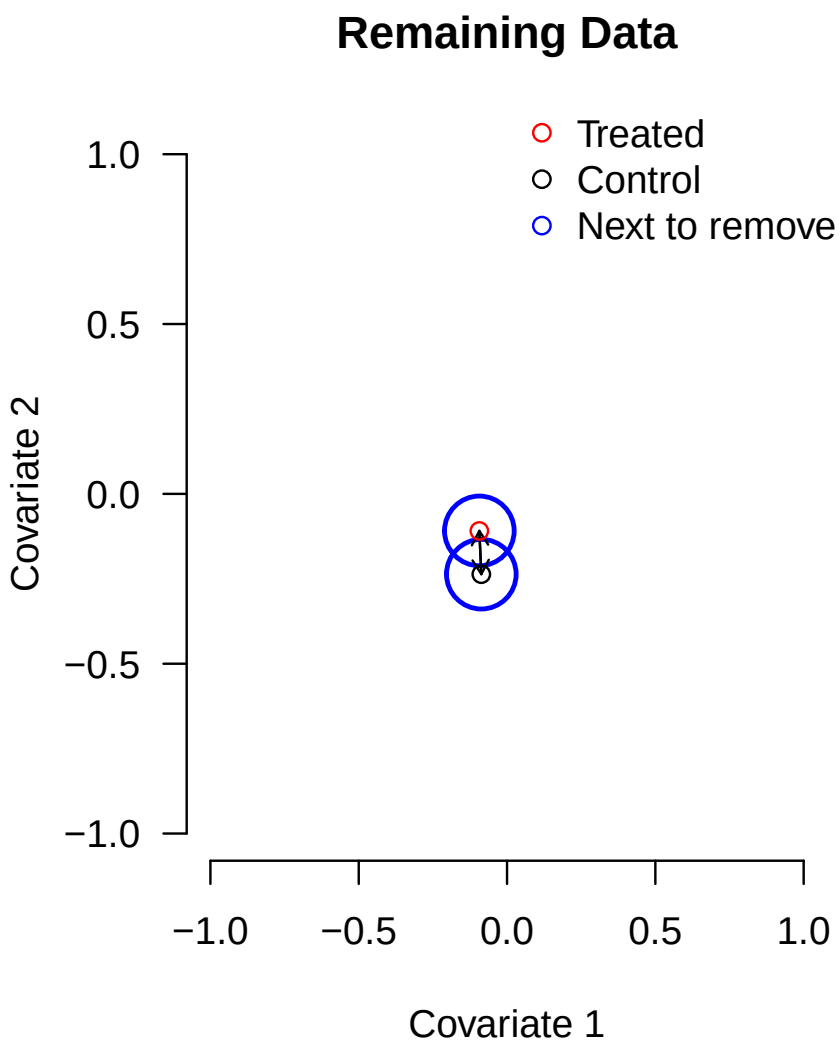

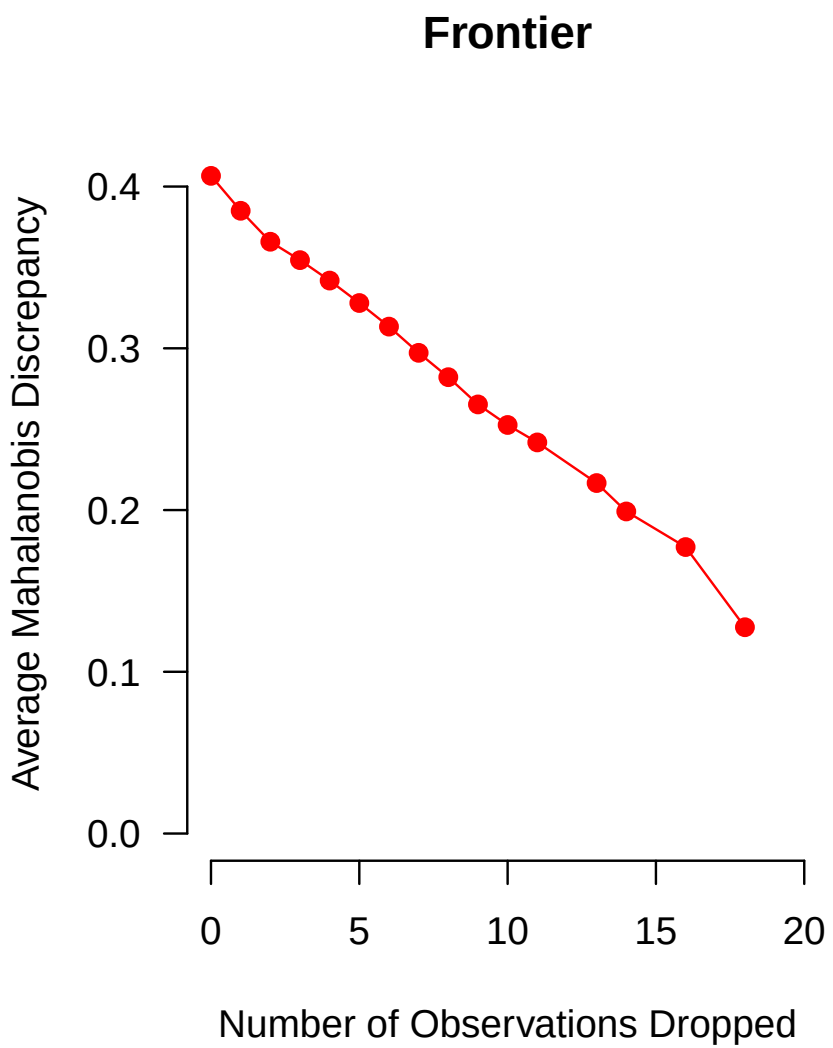

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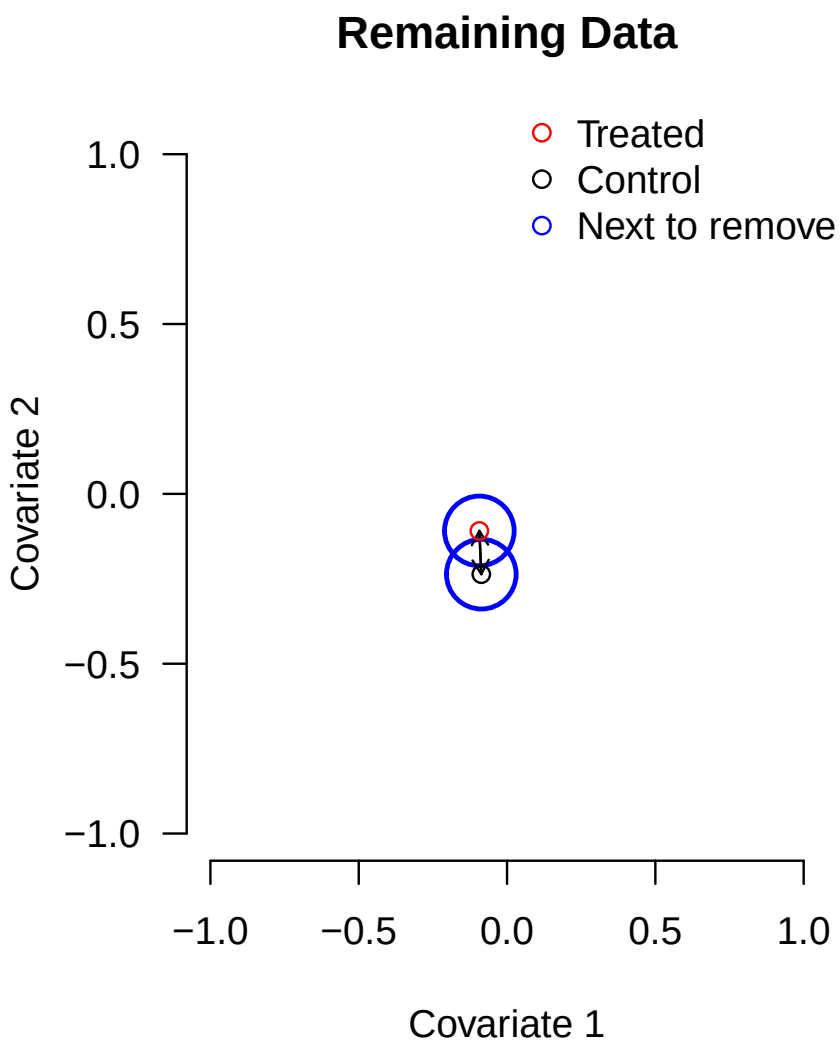

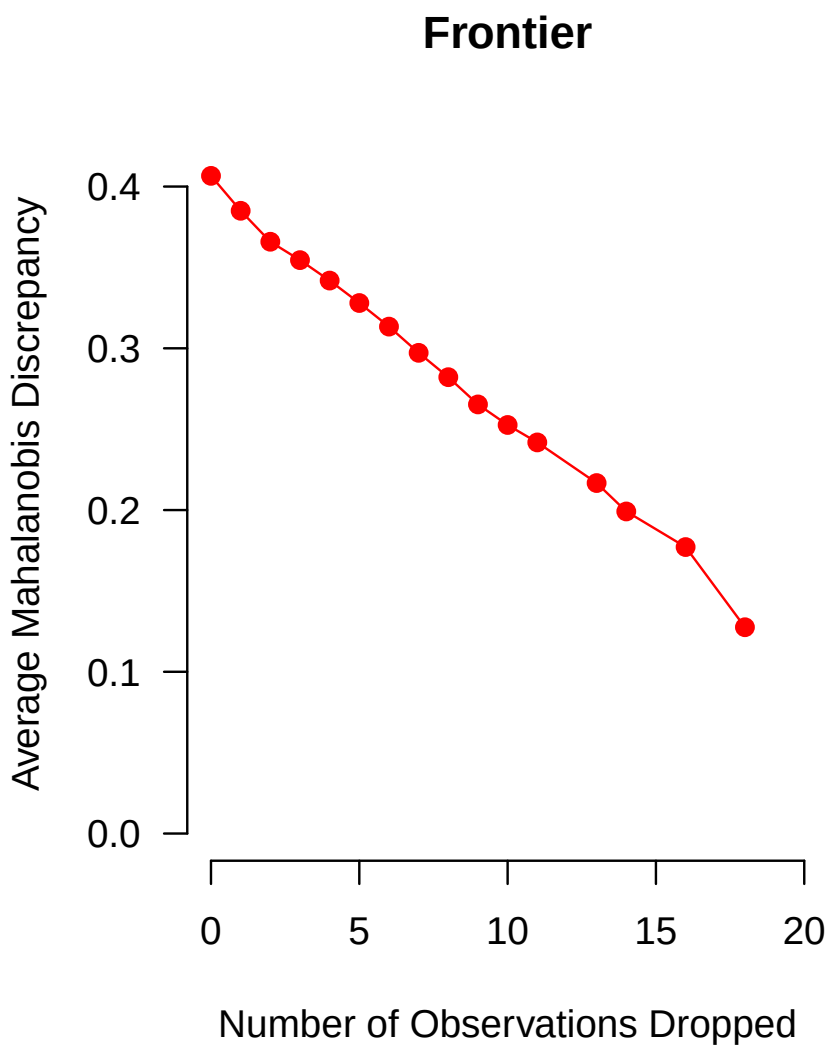

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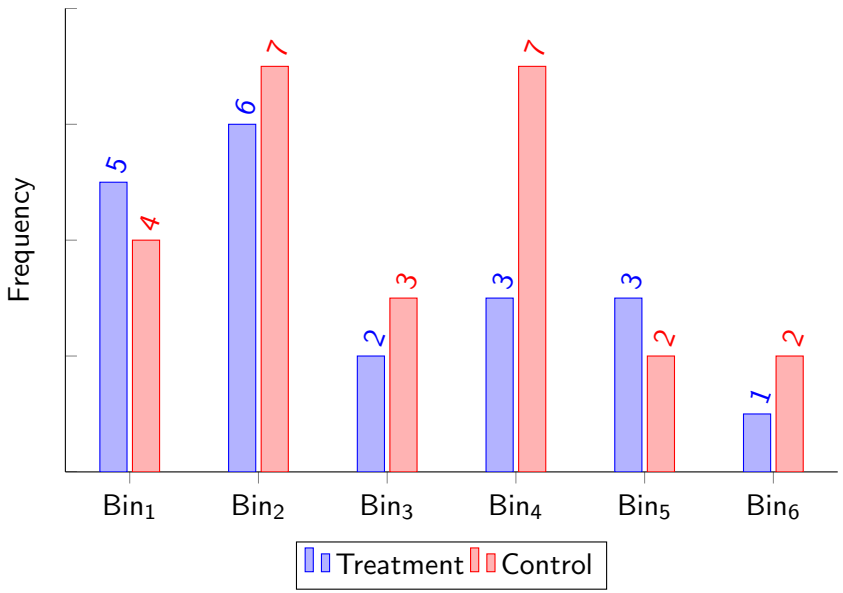
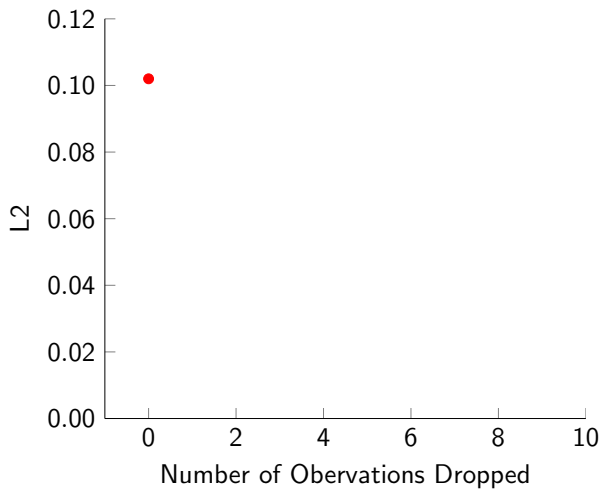
Constructing the L1/L2 SATT Frontier





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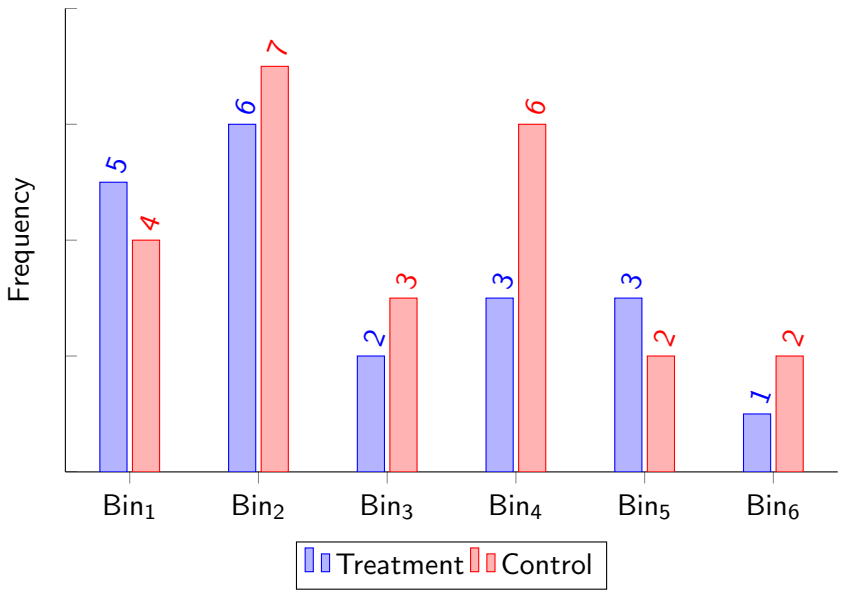

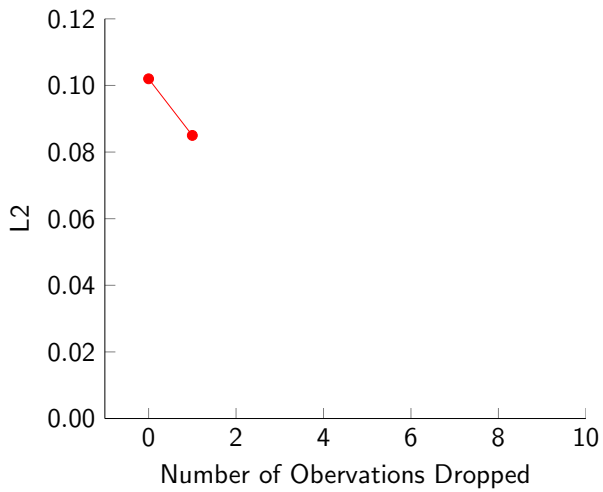

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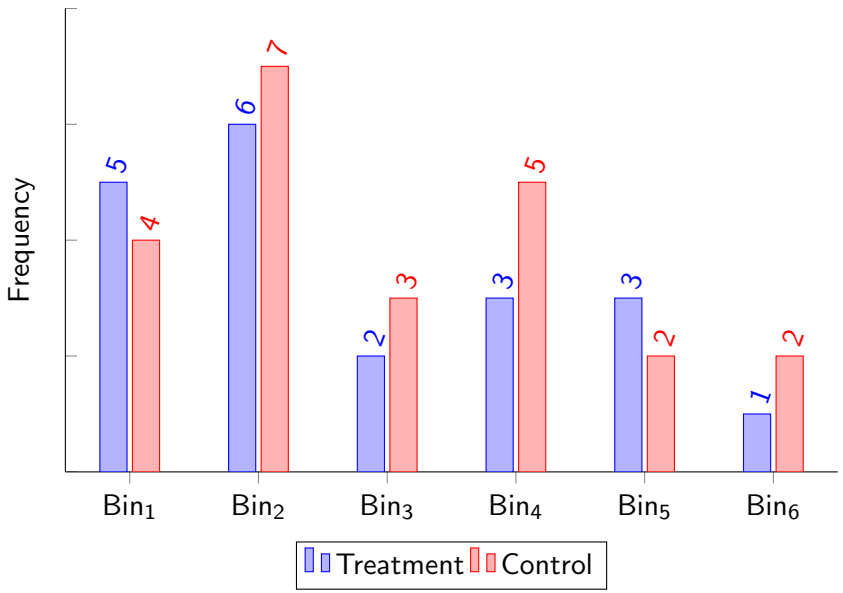

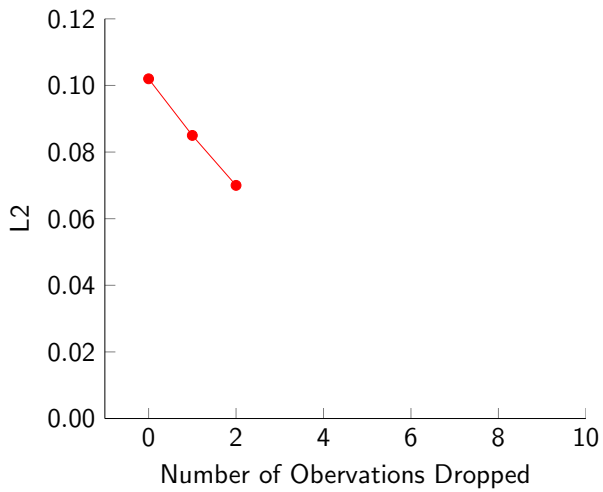

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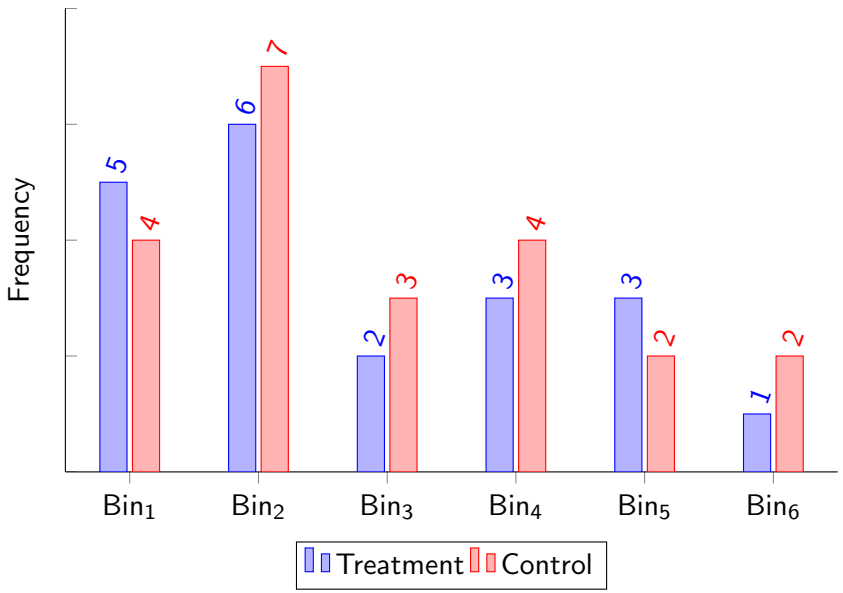

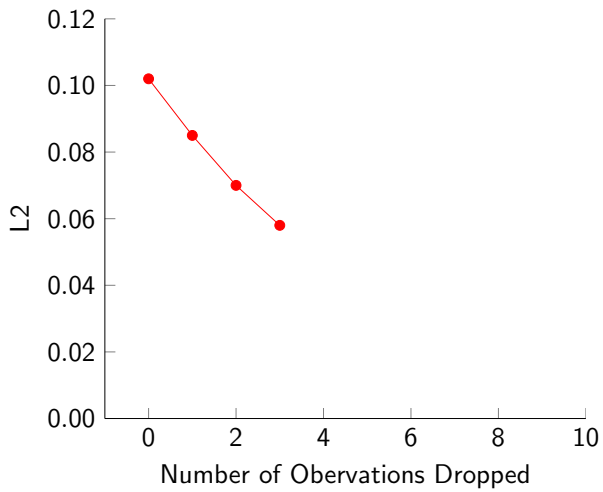

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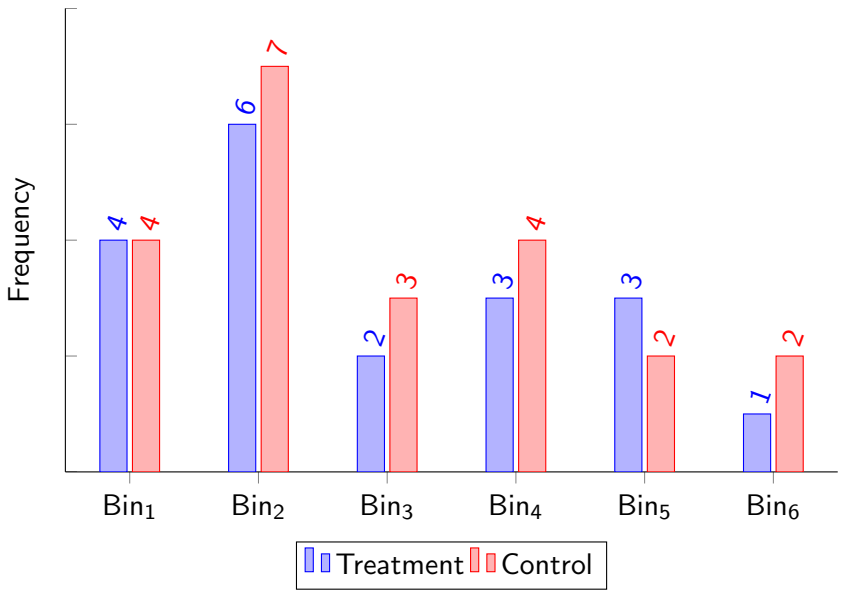

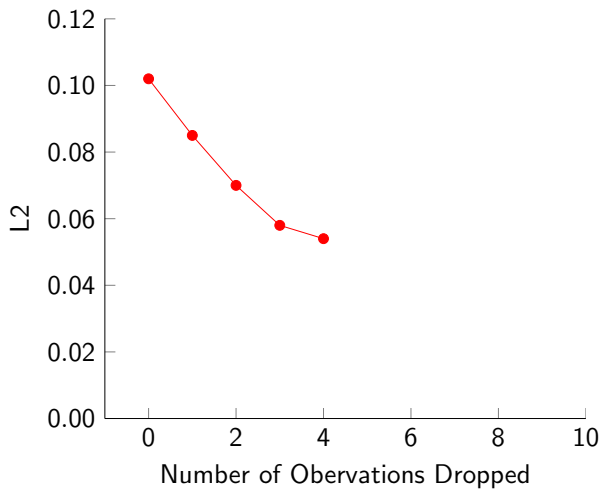

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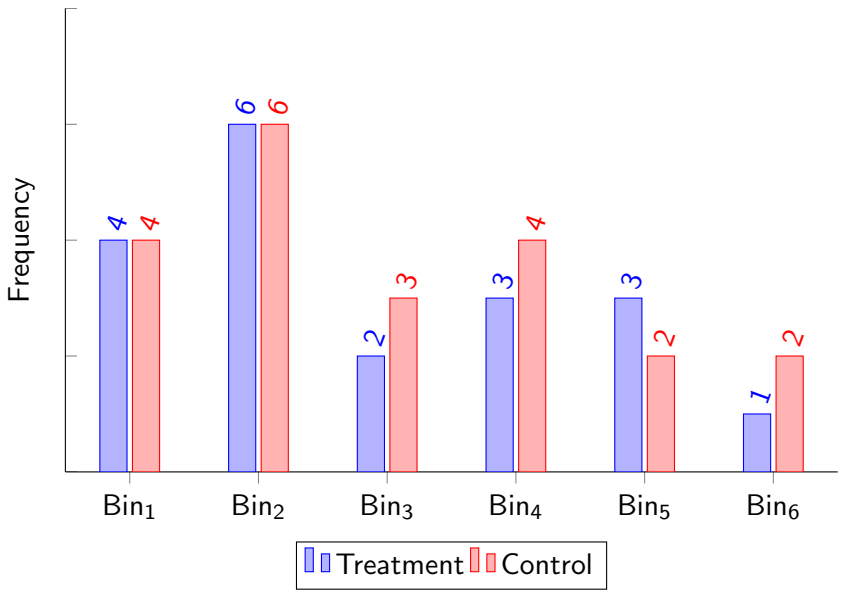

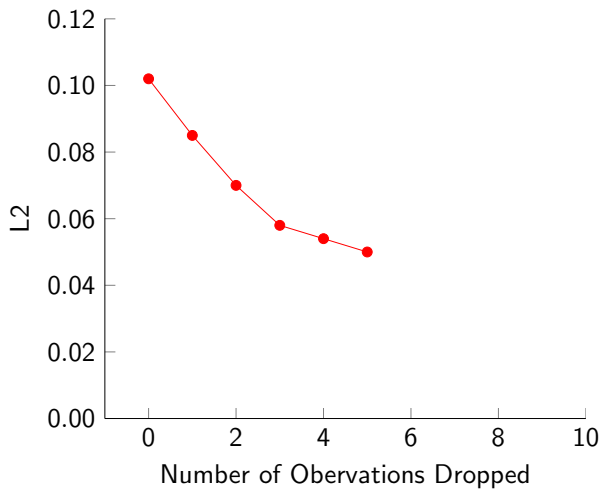

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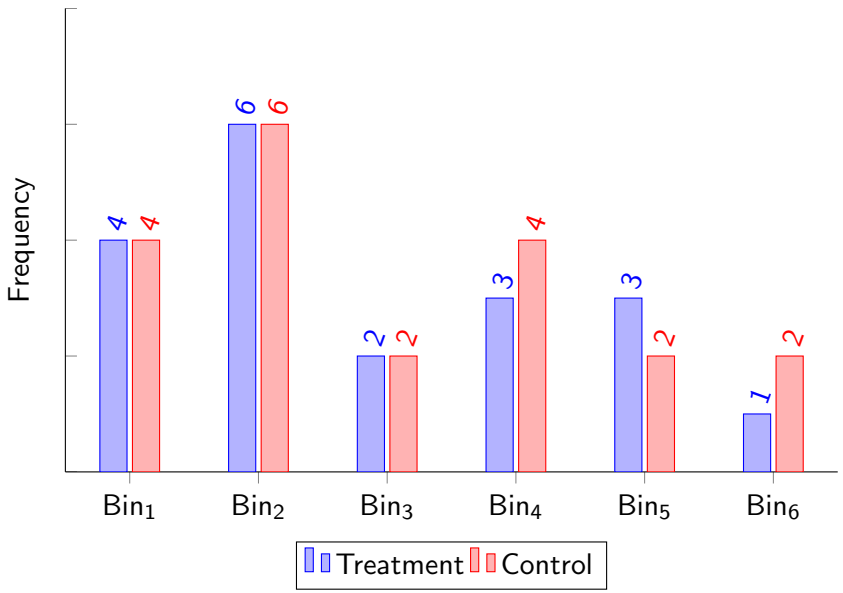

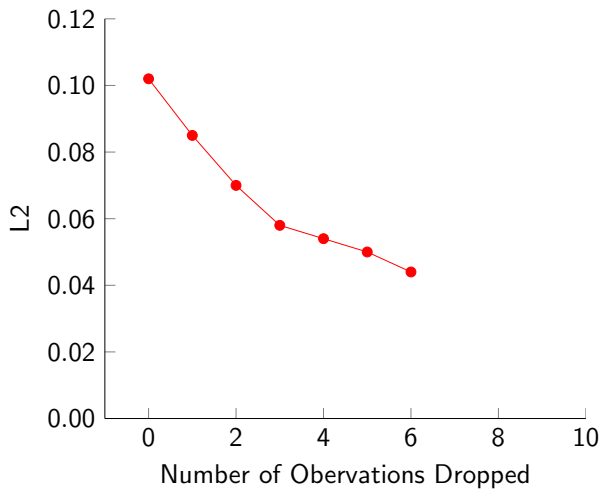

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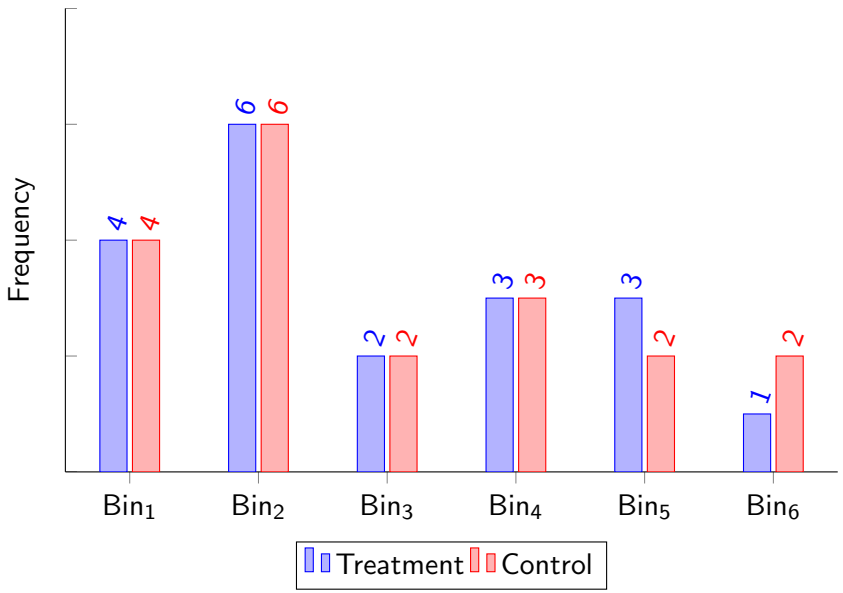

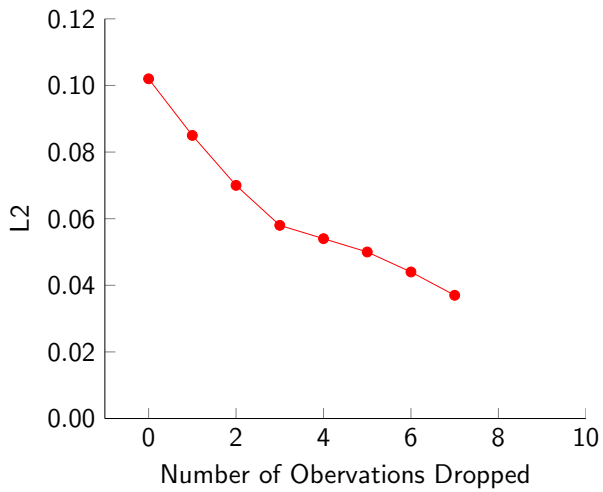

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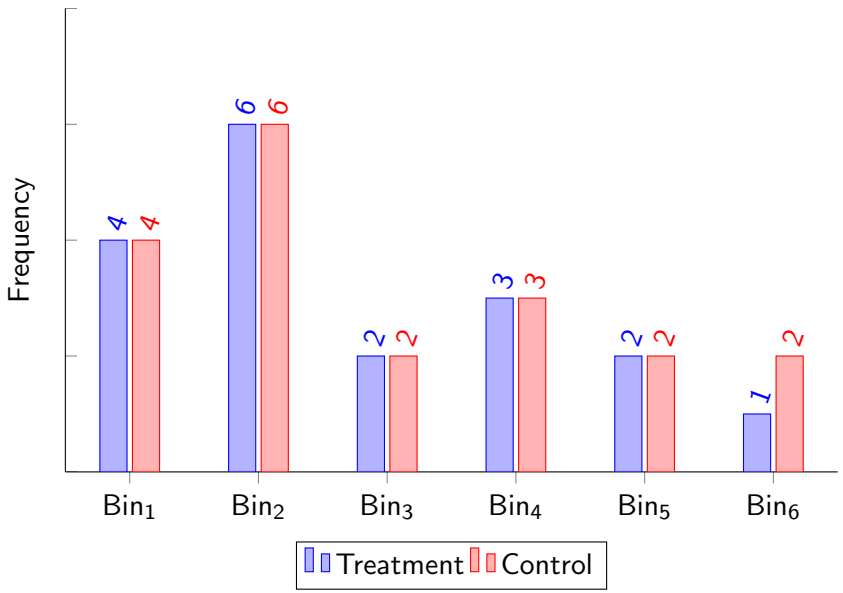

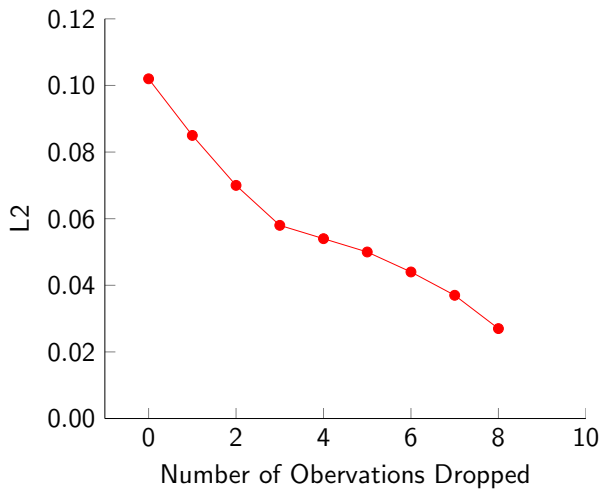

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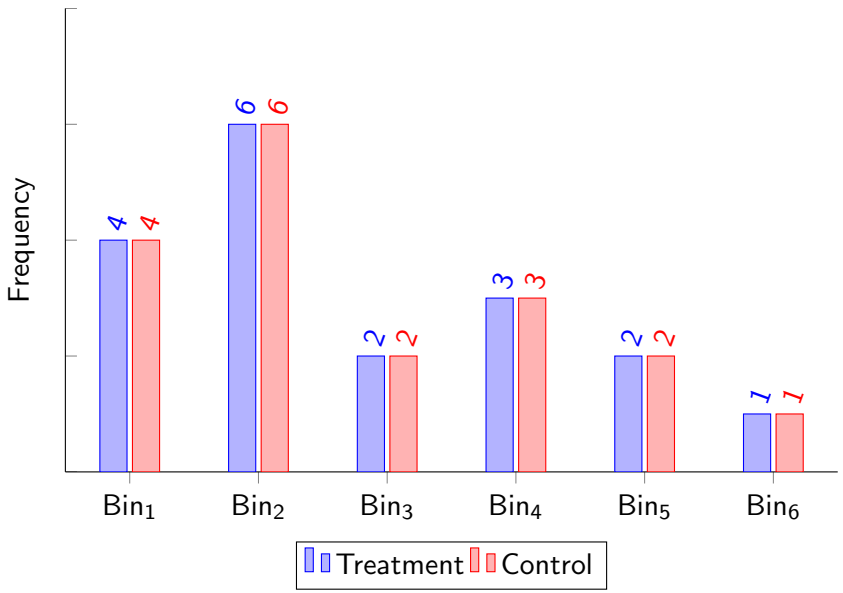

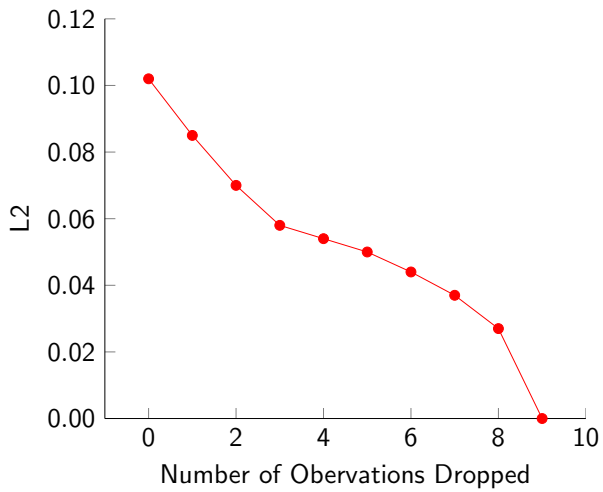

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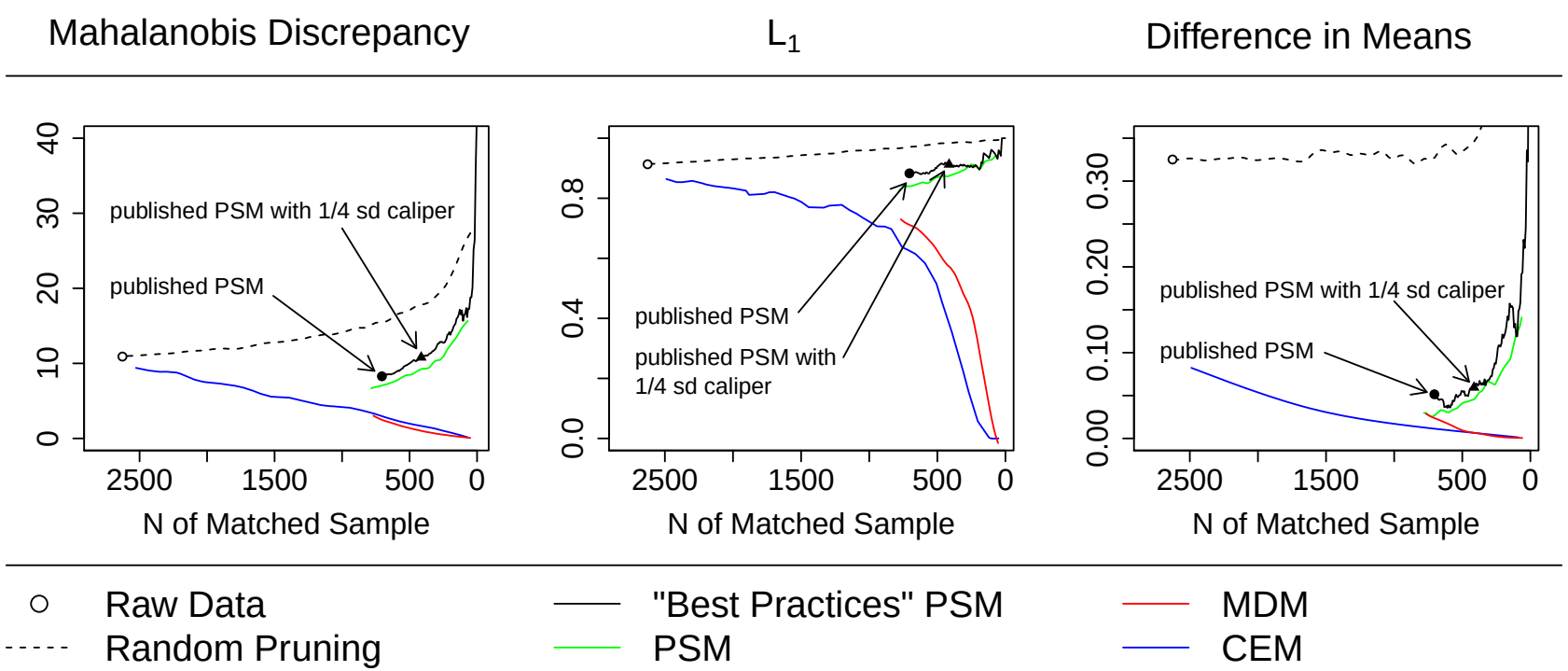
Problems with PSM: Foreign Aid Shocks

King, Nielsen, Coberley, Pope, and Wells (2012)

Imbalance Metric



Methods-specific frontiers (for methodological research only)

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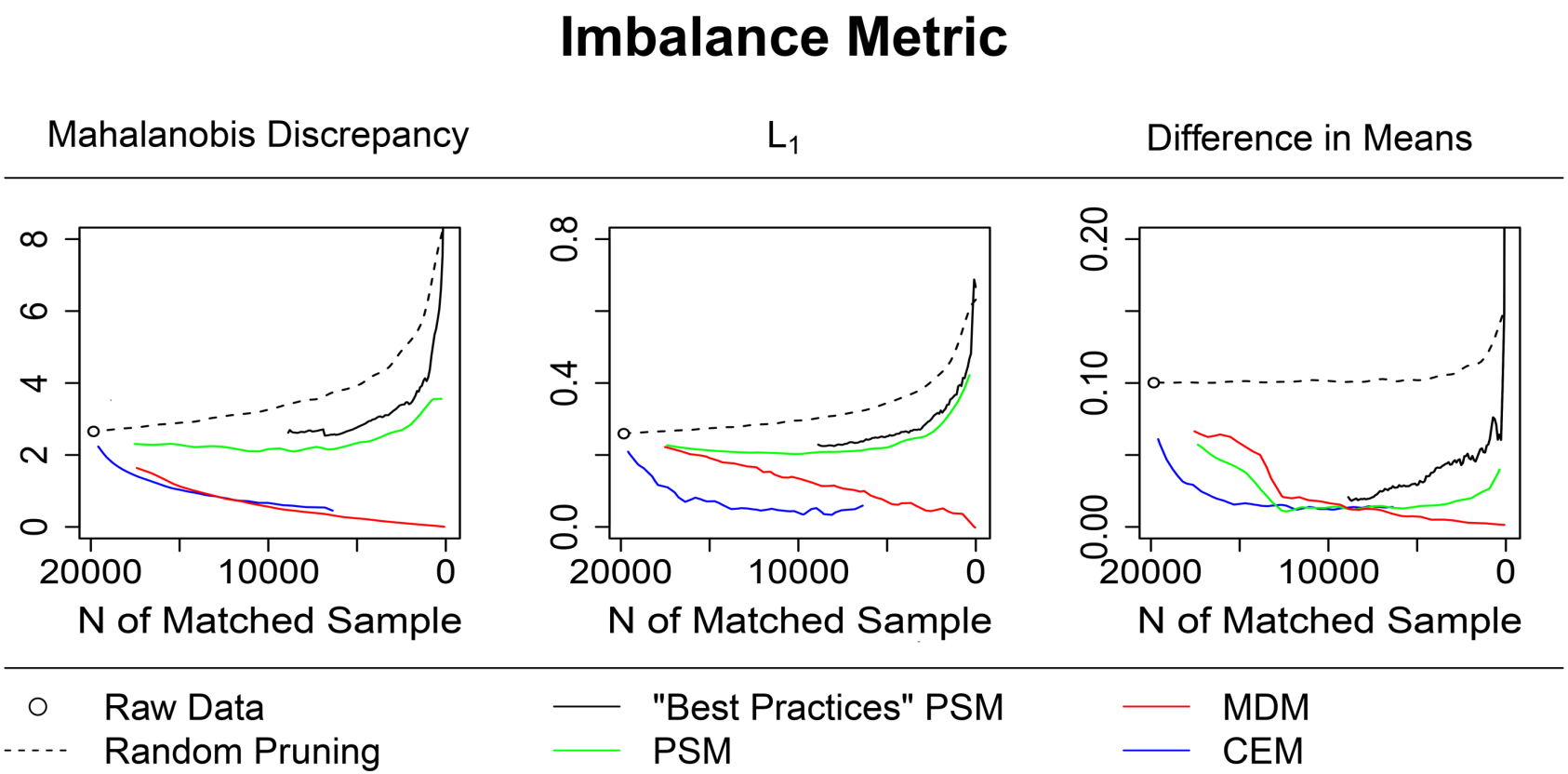

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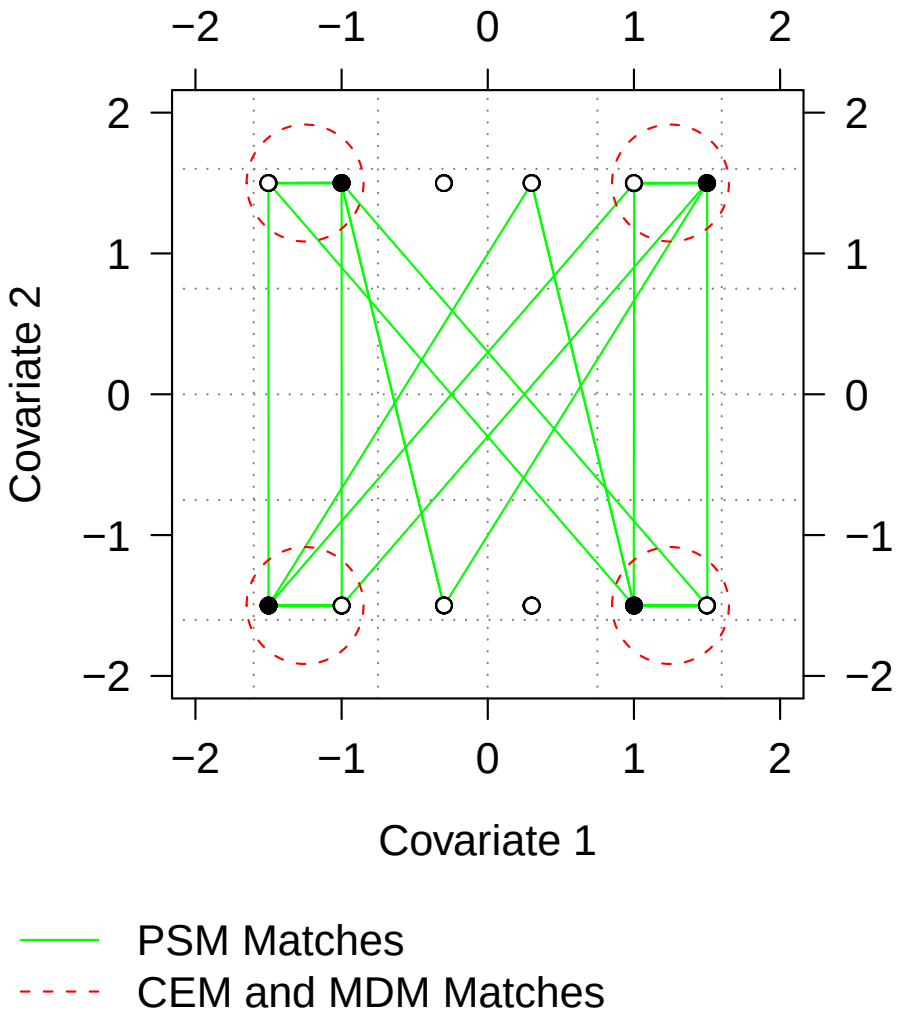
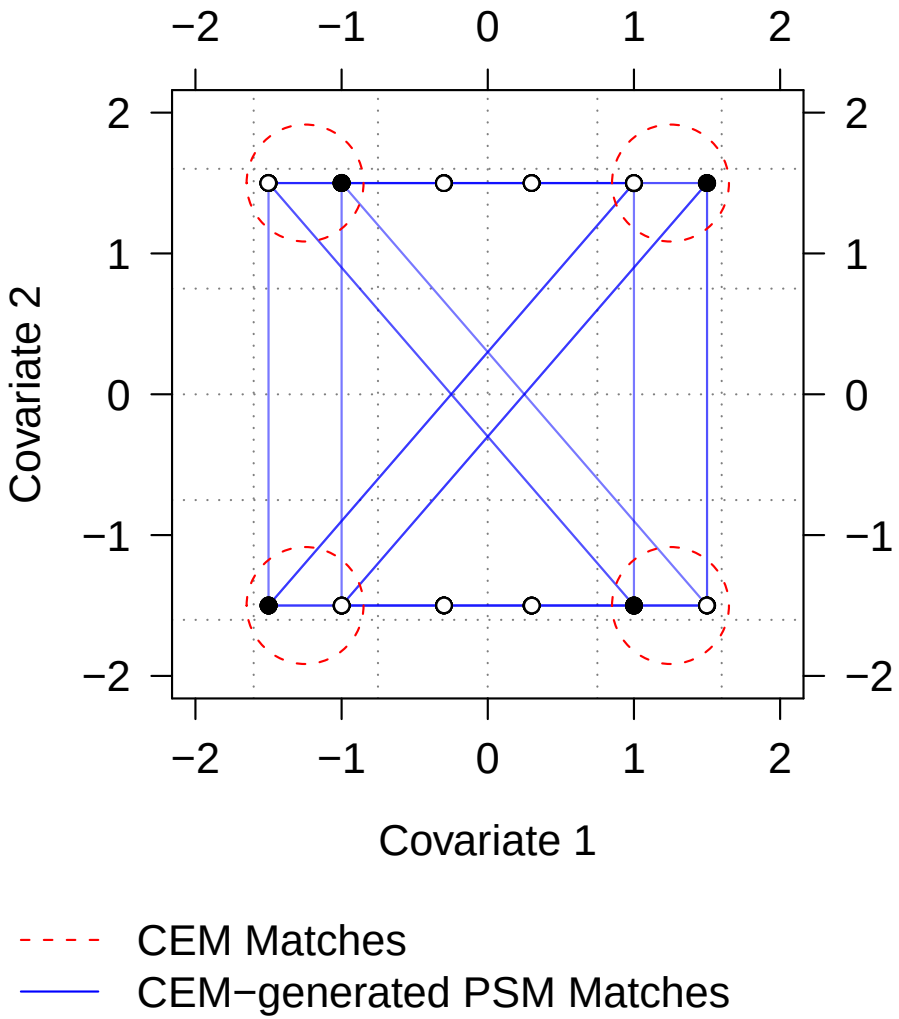
PSM Approximates Random Matching in Balanced Data



Destroying CEM with PSM's Two Step Approach



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- Simplification we will use: focus on TE for treated units; so $Y_i(0)$ is unobserved; $Y_i(1) = Y_i$

Implied Notation Assumptions

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Causal Quantities of Interest

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