Advanced Quantitative Research Methodology, Lecture Notes: Missing Data[1]

Gary King

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April 26, 2013

Readings

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- King, Gary; James Honaker; Ann Joseph; Kenneth Scheve. 2001. "Analyzing Incomplete Political Science Data: An Alternative Algorithm for Multiple Imputation," *APSR* 95, 1 (March, 2001): 49-69.

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- Amelia II: A Program for Missing Data

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- Amelia II: A Program for Missing Data
- http://gking.harvard.edu/amelia

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  - use a special method to combine the results

Missingness Notation

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$\rightsquigarrow$ Missing elements must exist (what's your view on the National Helium Reserve?)

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- Reasons for the odd terminology are historical.

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   - Adding variables to predict income can change NI to MAR

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Listwise Deletion Estimator Perform listwise deletion on $\{y, X_1, X_2\}$, and then estimate β_1 as b_1^L, the coefficient on X_1 when regressing y on X_1 and X_2.

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$$\text{M}SE(b_1^L) - \text{MSE}(b_1^O) = \begin{cases} > 0 & \text{when omitting the variable is better}\\ < 0 & \text{when listwise deletion is better}\end{cases}$$

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   - Larger for authors who work harder to avoid omitted variable bias

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5. **Result:** The point estimate in the average political science article is about an additional standard error farther away from the truth because of listwise deletion (as compared to omitting X_2 entirely).
6. **Conclusion**: Listwise deletion is often as bad a problem as the much better known omitted variable bias — in the best case scenerio (MCAR)

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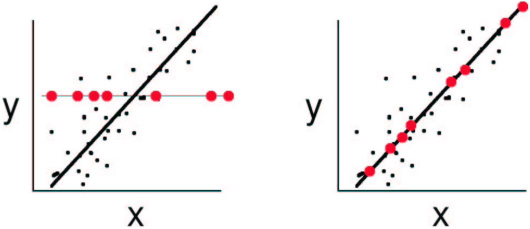
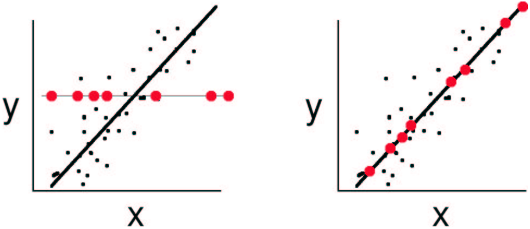
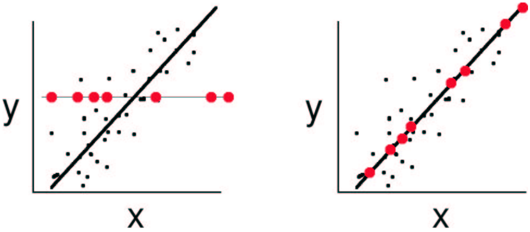
6. Mean substitution (attenuates estimated relationships)



y
x
y
x



7. y-hat regression imputation, (optimistic: scatter when observed, perfectly linear when unobserved; SEs too small)



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7. NI models (Heckman, many others) haven't always done well when truth is known

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3. Run whatever statistical method you would have with no missing data for each completed data set

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5. Easier by simulation: draw $1/m$ sims from each data set of the QOI, combine (i.e., concatenate into a larger set of simulations), and make inferences as usual.

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7. For social science survey data, which mostly contain ordinal scales, this is a reasonable choice for imputation, even though it may not be a good choice for analysis.

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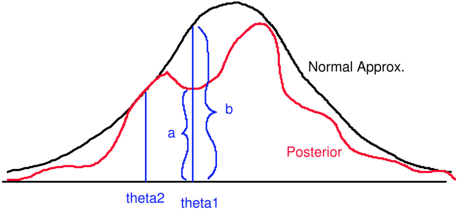

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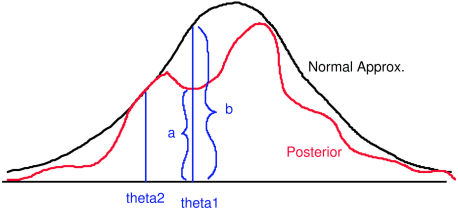

Keep $\tilde{\theta}_1$ with probability $\propto a/b$ (the importance ratio). Keep $\tilde{\theta}_2$ with probability 1.

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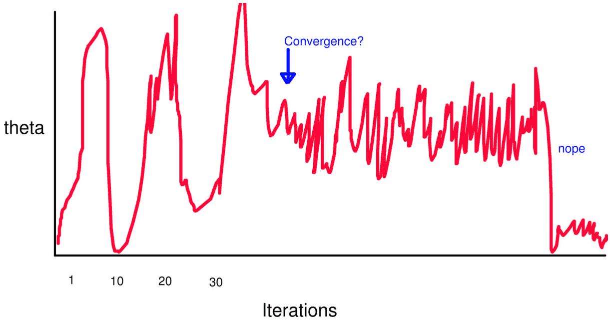
    (e) Hard to know when its over. Convergence is asymptotic in iterations. Plot traces are hard to interpret, and the worst-converging parameter controls the system.

MCMC Convergence Issues



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incomplete data

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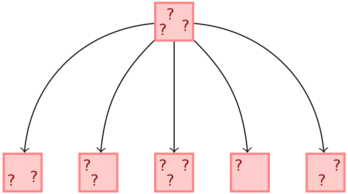

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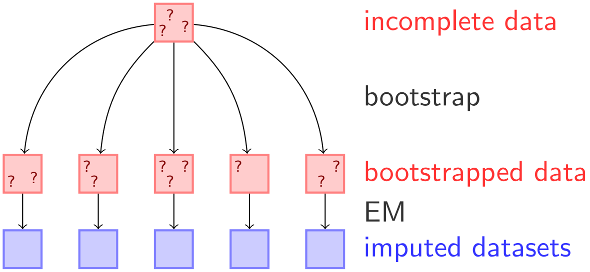

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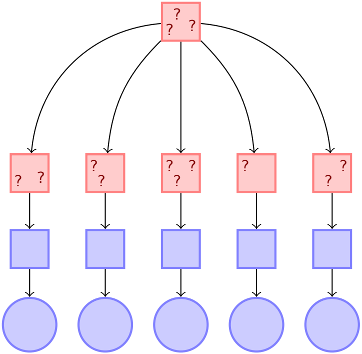

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EM

imputed datasets

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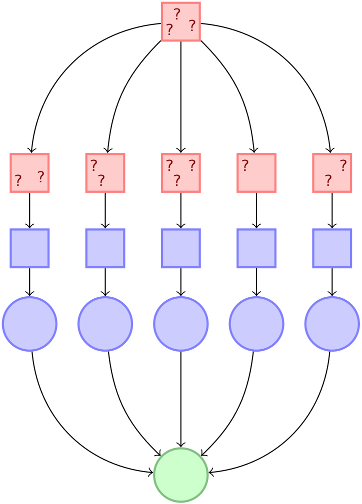

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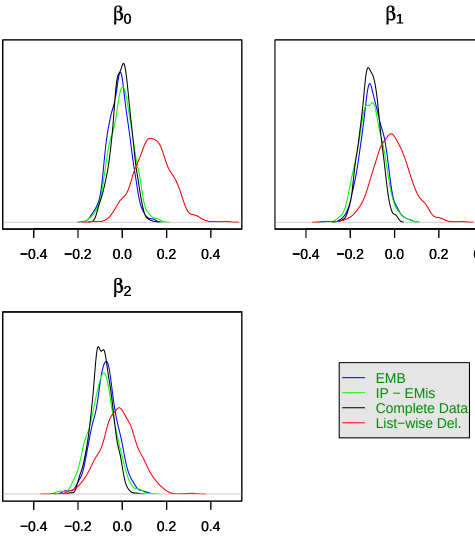
final results

Comparisons of Posterior Density Approximations



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- Code ordinal variables as close to interval as possible.

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  - Answer to both: the draws are from the joint posterior and put back into the data. Nothing is being changed.

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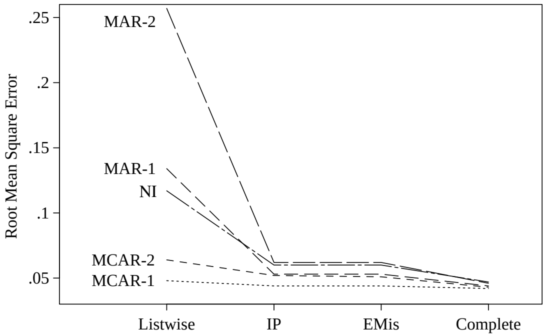
I.e., you don't trust data to impute D_{mis} but still trust it to analyze D_{obs}

Root Mean Square Error Comparisons



Each point is RMSE averaged over two regression coefficients in each of 100 simulated data sets. (IP and EMis have the same RMSE, which is lower than listwise deletion and higher than the complete data; its the same for EMB.)

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8. Include nonlinear terms: age^2

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so $(5-1) \times 1.65 = 6.6$, which is also a percentage of the range of Y.

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Multiple imputation	1.65
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so $(5-1)\times 1.65 = 6.6$, which is also a percentage of the range of Y.

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MI in Time Series Cross-Section Data

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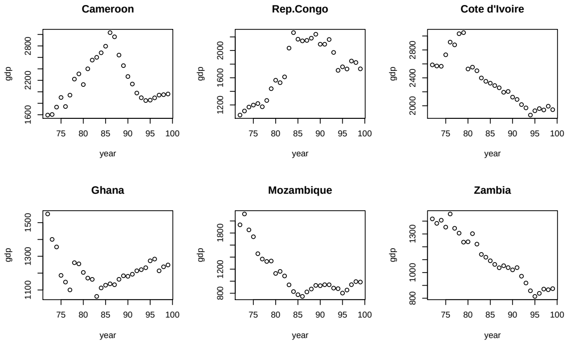

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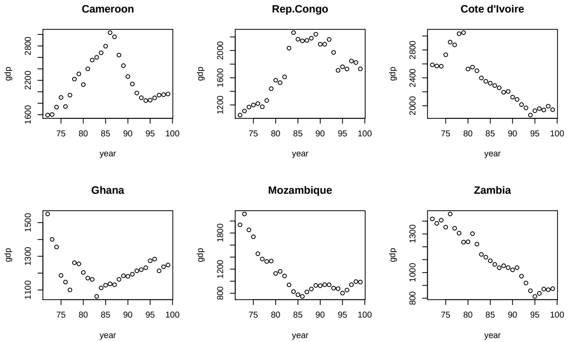

Include: (1) fixed effects, (2) time trends, and (3) priors for cells

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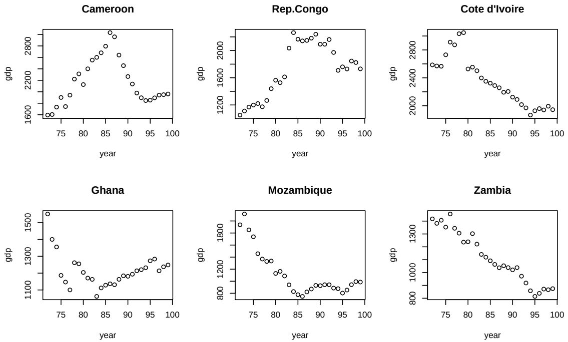

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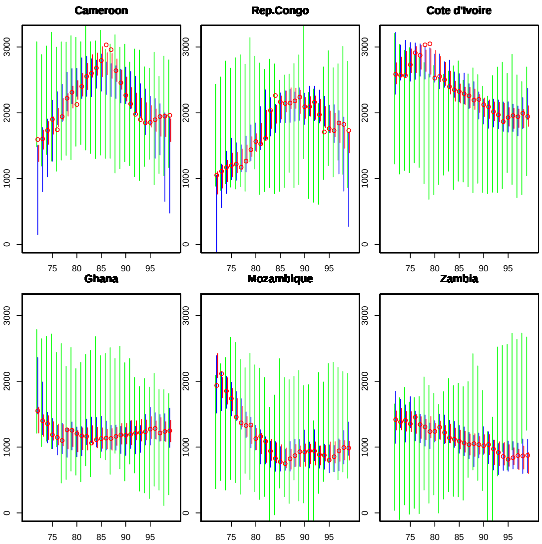
Read: James Honaker and Gary King, "What to do About Missing Values in Time Series Cross-Section Data,"

http://gking.harvard.edu/files/abs/pr-abs.shtml

Imputation one Observation at a time



Circles=true GDP; green=no time trends; blue=polynomials; red=LOESS

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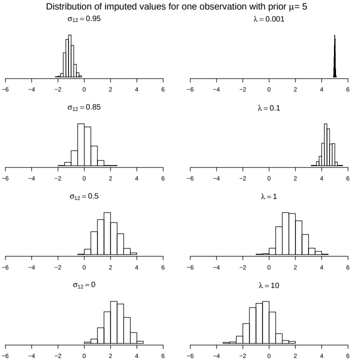
- Honaker and King show how to modify these "data augmentation priors" to put priors on missing values rather than on μ and σ (or β).

Posterior imputation: mean=0, prior mean=5



Left column: holds prior $N(5, \lambda)$ constant ($\lambda = 1$) and changes predictive strength (the covariance, σ_{12}).

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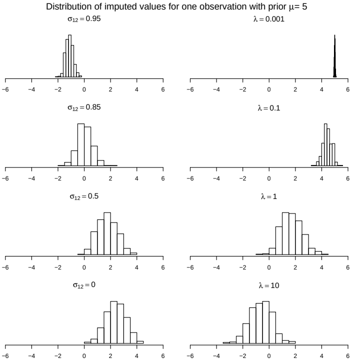
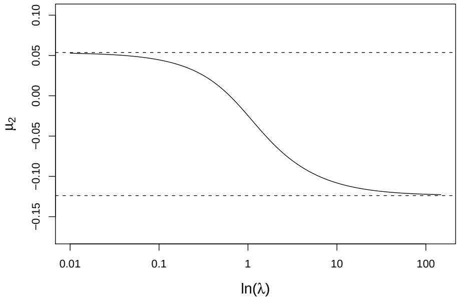


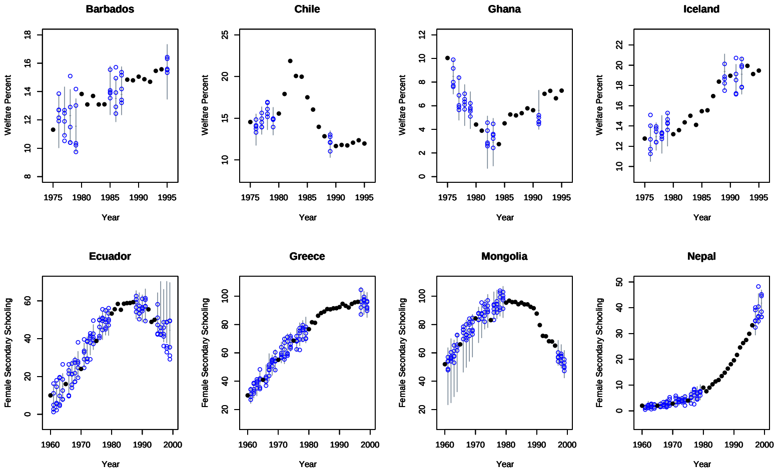
Left column: holds prior $N(5, \lambda)$ constant ($\lambda = 1$) and changes predictive strength (the covariance, σ_{12}).
Right column: holds predictive strength of data constant (at $\sigma_{12} = 0.5$) and changes the strength of the prior (λ).

Model Parameters Respond to Prior on a Cell Value



Prior: $p(x_{12}) = N(5, \lambda)$. The parameter approaches the theoretical limits (dashed lines), upper bound is what is generated when the missing value is filled in with the expectation; lower bound is the parameter when the model is estimated without priors. The overall movement is small.

Replication of Baum and Lake; Imputation Model Fit



Black = observed. Blue circles = five imputations; Bars = 95% CIs

	Listwise Deletion	Multiple Imputation
Life Expectancy		
Rich Democracies	−.072	.233
	(.179)	(.037)
Poor Democracies	−.082	.120
	(.040)	(.099)
N	1789	5627
Secondary Education		
Rich Democracies	.948	.948
	(.002)	(.019)
Poor Democracies	.373	.393
	(.094)	(.081)
N	1966	5627

Replication of Baum and Lake; the effect of being a democracy on life expectancy and on the percentage enrolled in secondary education (with p-values in parentheses).