Quantitative Social Science Methods, I, Lecture Notes: Missing Data

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  - Robust: separate missingness and analysis models

Readings

`j.mp/MisMeas`

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- Gary King; James Honaker; Anne ~~Joseph~~ O'Connell; Kenneth Scheve. "Analyzing Incomplete Political Science Data: An Alternative Algorithm for Multiple Imputation" APSR, 2001.

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- Amelia II: A Program for Missing Data

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- Quiz: If some values don't exist, what analysis would you do?

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   - Adding variables to predict income can change NI to MAR

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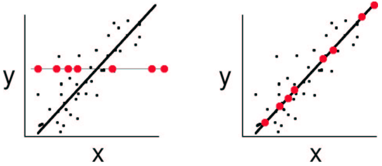

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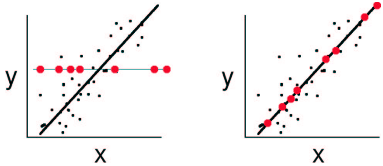

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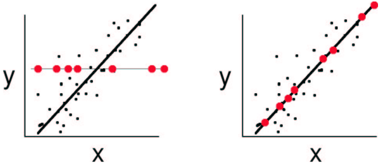

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- y-hat + ϵ Regression Imputation: Ignores estimation uncertainty; impossible with scattered missingness; getting there

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- With MAR: $P(D_{obs}|\theta)$ specialized; no help with X missingness
- Example: Censored Exponential Model (you've seen)
- Without MAR: missingness model is NI, can't be dropped
- Little known about M: Specifying $P(M|D_{obs}, \gamma)$ hard
- NI models: (Heckman, many others) often don't do well when can be evaluated

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- Complicated computationally: $D_{i,obs}$ has different elements observed for each i; each observation is informative about different pieces of (μ,Σ)

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- For social science survey data (mostly ordinal scales): this is a reasonable choice for imputation, even if not for analysis

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  - Equivalent to drawing from $\tilde{y}_i = X\tilde{\beta} + \tilde{\epsilon}_i$ (estimation, fundamental uncertainty)

Overview

Missingness Assumptions

Application Specific Methods

Multiple Imputation

Computational Algorithms

What Can Go Wrong

Time Series, Cross-Sectional Imputations

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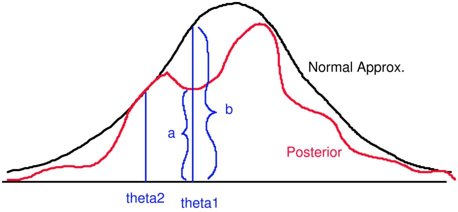

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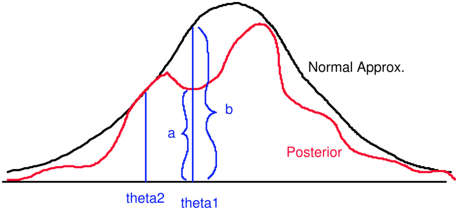

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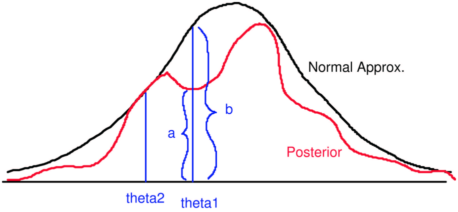

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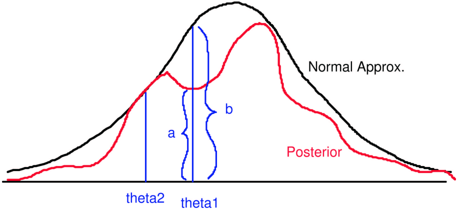

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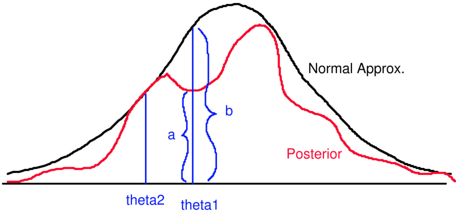

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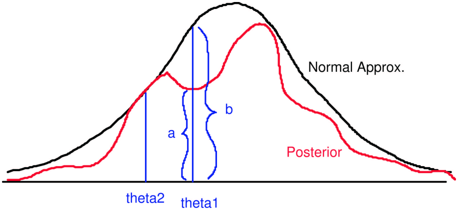

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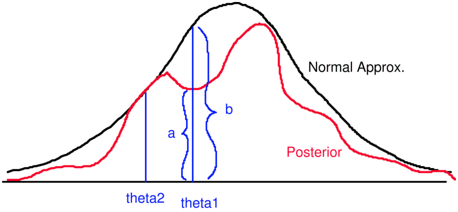

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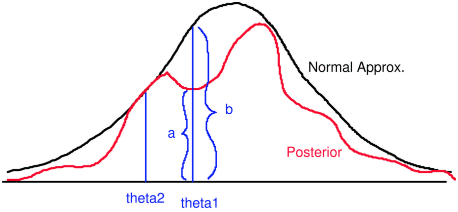

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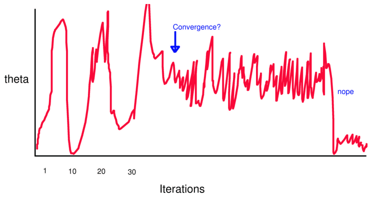

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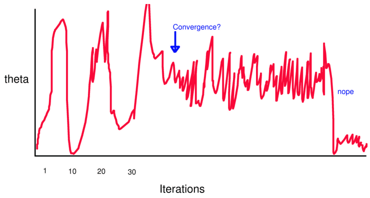

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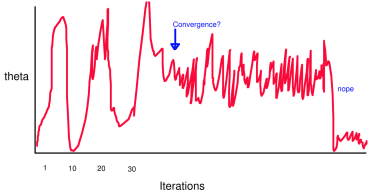

Multiple Imputation: Amelia Style

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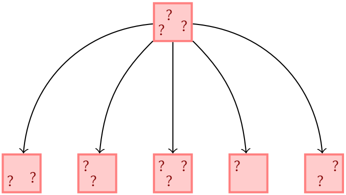

incomplete data

bootstrap

bootstrapped data

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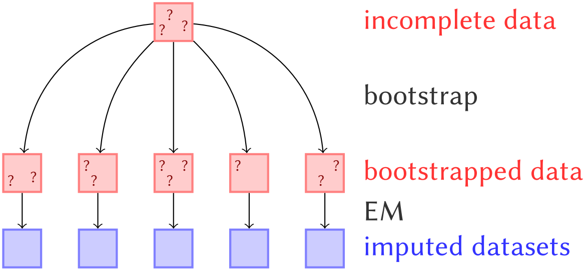

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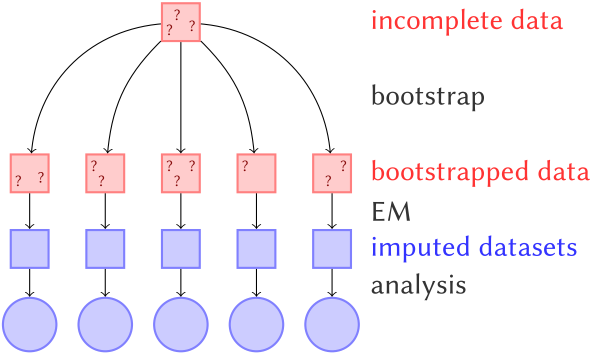

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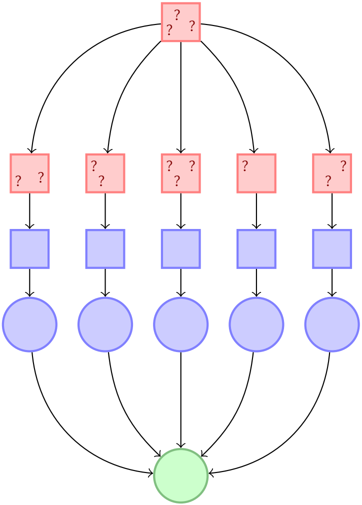


incomplete data

bootstrap

bootstrapped data

EM

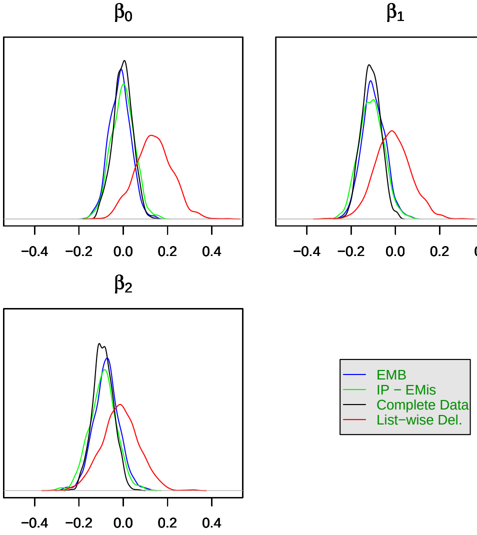
imputed datasets

analysis

combination

final results

Comparisons of Posterior Density Approximations



EMB: similar results, much faster, far easier to program

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- Answer to both: draws from the joint posterior are put back into the data; not changing the joint distribution

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I.e., you don't trust data to impute D_{mis} but trust it to analyze D_{obs}

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- Based on MI estimator: R's with negative retrospective economic evaluations more likely to have favorable views of Perot

MI in Time Series Cross-Section Data

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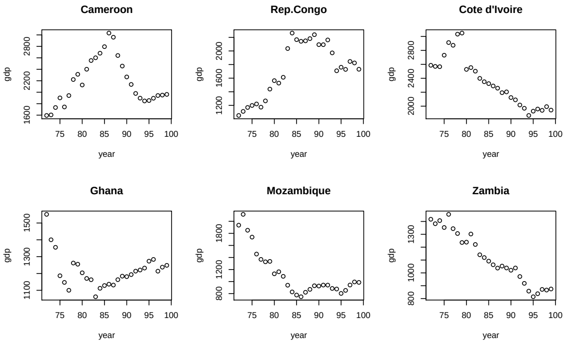

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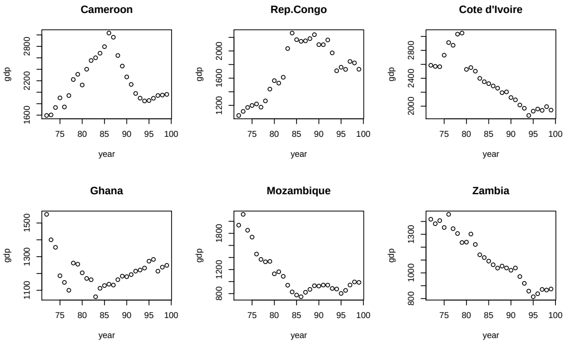
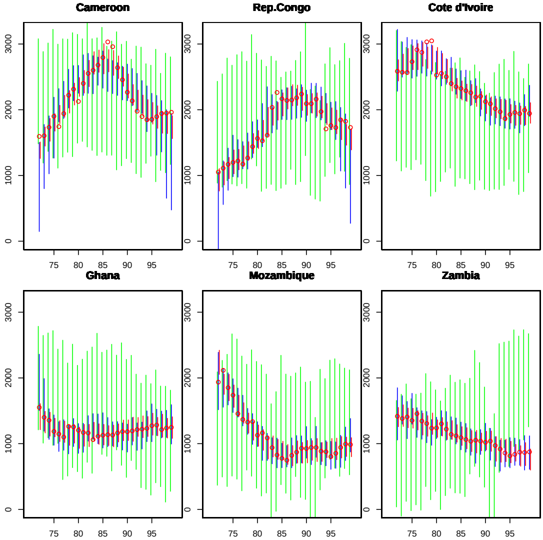


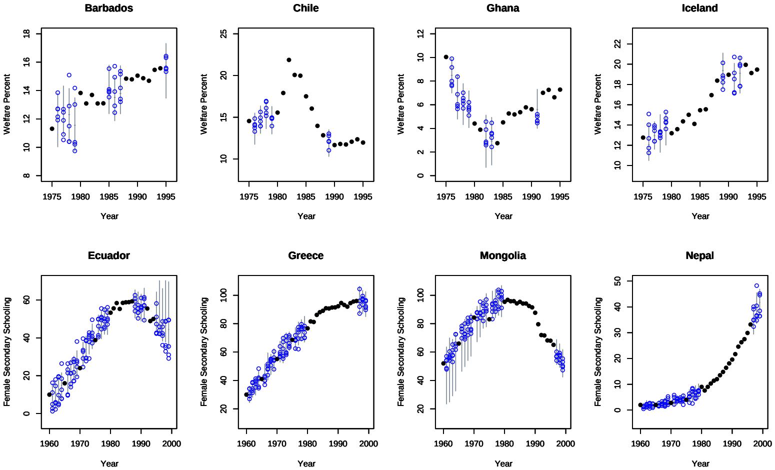
Include: (1) fixed effects, (2) time trends, and (3) priors for cells

Imputation one Observation at a time



Circles=true GDP; green=no time trends; blue=polynomials; red=LOESS

Replication of Baum and Lake: Imputation Model Fit



Black = observed. Blue circles = five imputations; Bars = 95% CIs