Quantitative Social Science Methods, I, Lecture Notes: Research Designs for Causal Inference

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August 17, 2020

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Components of Causal Estimation Error

Research Designs

Issues in Ideal Designs

Reference

- Kosuke Imai, Gary King, and Elizabeth Stuart. Misunderstandings among Experimentalists and Observationalists: Balance Test Fallacies in Causal Inference *Journal of the Royal Statistical Society*, Series A, 171, Part 2 (2008): 1–22.
- `http://j.mp/MisExpObs`

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- (I_i, T_i, Y_i) are random; $Y_i(1)$ and $Y_i(0)$ are fixed.
- Quiz: How can Y_i be random when $Y_i(0)$ and $Y_i(1)$ are fixed?

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Error due to: Δ_S (sample selection), Δ_T (treatment imbalance), and each due to observed (X_i) and unobserved (U_i) covariates

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- Population average treatment effect on the treated

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- Quiz: How do they differ in randomized experiments?

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Complete stratified random sampling	$= 0$	$\overset{\text{avg}}{=} 0$		

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Complete stratified random sampling	$= 0$	$\stackrel{\text{avg}}{=} 0$		
Focus on SATE rather than PATE	$= 0$	$= 0$		

Effects of Design Components on Estimation Error

$\Delta = \Delta_S + \Delta_T = (\Delta_{S_X} + \Delta_{S_U}) + (\Delta_{T_X} + \Delta_{T_U})$

Design Choice	Δ_{S_X}	Δ_{S_U}	Δ_{T_X}	Δ_{T_U}
Random sampling	$\overset{\text{avg}}{=} 0$	$\overset{\text{avg}}{=} 0$		
Complete stratified random sampling	$= 0$	$\overset{\text{avg}}{=} 0$		
Focus on SATE rather than PATE	$= 0$	$= 0$		
Weighting for nonrandom sampling	$= 0$	$= ?$		

Effects of Design Components on Estimation Error

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Focus on SATE rather than PATE	$= 0$	$= 0$		
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Large sample size	$\longrightarrow ?$	$\longrightarrow ?$	$\longrightarrow ?$	$\longrightarrow ?$

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$\Delta = \Delta_S + \Delta_T = (\Delta_{S_X} + \Delta_{S_U}) + (\Delta_{T_X} + \Delta_{T_U})$

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Complete stratified random sampling	$= 0$	$\stackrel{\text{avg}}{=} 0$		
Focus on SATE rather than PATE	$= 0$	$= 0$		
Weighting for nonrandom sampling	$= 0$	$= ?$		
Large sample size	$\longrightarrow ?$	$\longrightarrow ?$	$\longrightarrow ?$	$\longrightarrow ?$
Random treatment assignment			$\stackrel{\text{avg}}{=} 0$	$\stackrel{\text{avg}}{=} 0$

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Complete stratified random sampling	$= 0$	$\overset{\text{avg}}{=} 0$		
Focus on SATE rather than PATE	$= 0$	$= 0$		
Weighting for nonrandom sampling	$= 0$	$= ?$		
Large sample size	$\longrightarrow ?$	$\longrightarrow ?$	$\longrightarrow ?$	$\longrightarrow ?$
Random treatment assignment			$\overset{\text{avg}}{=} 0$	$\overset{\text{avg}}{=} 0$
Complete blocking			$= 0$	$= ?$

Effects of Design Components on Estimation Error

$\Delta = \Delta_S + \Delta_T = (\Delta_{S_X} + \Delta_{S_U}) + (\Delta_{T_X} + \Delta_{T_U})$

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Focus on SATE rather than PATE	$= 0$	$= 0$		
Weighting for nonrandom sampling	$= 0$	$= ?$		
Large sample size	$\longrightarrow ?$	$\longrightarrow ?$	$\longrightarrow ?$	$\longrightarrow ?$
Random treatment assignment			$\overset{\text{avg}}{=} 0$	$\overset{\text{avg}}{=} 0$
Complete blocking			$= 0$	$= ?$
Exact matching			$= 0$	$= ?$

Effects of Design Components on Estimation Error

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Weighting for nonrandom sampling	$= 0$	$= ?$		
Large sample size	$\longrightarrow ?$	$\longrightarrow ?$	$\longrightarrow ?$	$\longrightarrow ?$
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Assumption				

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Large sample size	$\longrightarrow ?$	$\longrightarrow ?$	$\longrightarrow ?$	$\longrightarrow ?$
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Complete blocking			$= 0$	$= ?$
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Assumption				
No selection bias	$\overset{\text{avg}}{=} 0$	$\overset{\text{avg}}{=} 0$		

Effects of Design Components on Estimation Error

$\Delta = \Delta_S + \Delta_T = (\Delta_{S_X} + \Delta_{S_U}) + (\Delta_{T_X} + \Delta_{T_U})$

	Δ_{S_X}	Δ_{S_U}	Δ_{T_X}	Δ_{T_U}
Design Choice				
Random sampling	$\overset{\text{avg}}{=} 0$	$\overset{\text{avg}}{=} 0$		
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Focus on SATE rather than PATE	$= 0$	$= 0$		
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Large sample size	$\longrightarrow ?$	$\longrightarrow ?$	$\longrightarrow ?$	$\longrightarrow ?$
Random treatment assignment			$\overset{\text{avg}}{=} 0$	$\overset{\text{avg}}{=} 0$
Complete blocking			$= 0$	$= ?$
Exact matching			$= 0$	$= ?$
Assumption				
No selection bias	$\overset{\text{avg}}{=} 0$	$\overset{\text{avg}}{=} 0$		
Ignorability				$\overset{\text{avg}}{=} 0$

Effects of Design Components on Estimation Error

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Weighting for nonrandom sampling	$= 0$	$= ?$		
Large sample size	$\longrightarrow ?$	$\longrightarrow ?$	$\longrightarrow ?$	$\longrightarrow ?$
Random treatment assignment			$\stackrel{\text{avg}}{=} 0$	$\stackrel{\text{avg}}{=} 0$
Complete blocking			$= 0$	$= ?$
Exact matching			$= 0$	$= ?$
Assumption				
No selection bias	$\stackrel{\text{avg}}{=} 0$	$\stackrel{\text{avg}}{=} 0$		
Ignorability				$\stackrel{\text{avg}}{=} 0$
No omitted variables				$= 0$

Comparing Blocking (i.e., before) and Matching (i.e., after)

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- Adding blocking (on pretreatment vars related to outcome) to random assignment: as or more efficient, and never biased

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  - Exact matching, unlike blocking: dependent on good matches in already-collected data
  - Worst case scenario: matching on wrong vars (like regression adjustment) can increase bias
- Adding matching to a parametric model: reduces model dependence and bias, and sometimes variance too
- Quiz: Which is preferable: Matching or Blocking?

Components of Causal Estimation Error

Research Designs

Issues in Ideal Designs

The Benefits of Major Research Designs: Overview

	Δ_{S_X}	Δ_{S_U}	Δ_{T_X}	Δ_{T_U}
Ideal experiment	$\rightarrow 0$	$\rightarrow 0$	$= 0$	$\rightarrow 0$
Randomized clinicial trials (Limited or no blocking)	$\neq 0$	$\neq 0$	$\overset{\text{avg}}{=} 0$	$\overset{\text{avg}}{=} 0$
Randomized clinicial trials (Full blocking)	$\neq 0$	$\neq 0$	$= 0$	$\overset{\text{avg}}{=} 0$
Social Science Field Experiment (Limited or no blocking)	$\neq 0$	$\neq 0$	$\rightarrow 0$	$\rightarrow 0$
Survey Experiment (Limited or no blocking)	$\rightarrow 0$	$\rightarrow 0$	$\rightarrow 0$	$\rightarrow 0$
Observational Study (Representative data set, Well-matched)	≈ 0	≈ 0	≈ 0	$\neq 0$
Observational Study (Unrepresentative but partially, correctable data, well-matched)	≈ 0	$\neq 0$	≈ 0	$\neq 0$
Observational Study (Unrepresentative data set, Well-matched)	$\neq 0$	$\neq 0$	≈ 0	$\neq 0$

The Benefits of Major Research Designs: Overview

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- $\rightarrow 0$: $E(Q) = 0$ & $\lim_{n \to \infty} \text{Var}(Q) = 0$

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Social Science Field Experiment (Limited or no blocking)	$\neq 0$	$\neq 0$	$\longrightarrow 0$	$\longrightarrow 0$
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- $\longrightarrow 0$: $E(Q) = 0$ & $\lim_{n \to \infty} \text{Var}(Q) = 0$
- $\stackrel{\text{avg}}{=} 0$: $E(Q) = 0$

The Ideal Experiment (according to the paper)

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- Random selection from well-defined population

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- Random selection from well-defined population
- large n

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- Random selection from well-defined population
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- blocking on all known confounders

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- $E(\Delta_{S_X}) = 0, \quad \lim_{n\to\infty} V(\Delta_{S_X}) = 0$

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- $\Delta_{T_X} = 0$

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- $\Delta_{T_X} = 0$
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- Quiz: Is there an even more ideal experiment?

The Ideal Experiment (according to the paper)

- Random selection from well-defined population
- large n
- blocking on all known confounders
- random treatment assignment within blocks
- $E(\Delta_{S_X}) = 0, \quad \lim_{n\rightarrow\infty} V(\Delta_{S_X}) = 0$
- $E(\Delta_{S_U}) = 0, \quad \lim_{n\rightarrow\infty} V(\Delta_{S_U}) = 0$
- $\Delta_{T_X} = 0$
- $E(\Delta_{T_U}) = 0, \quad \lim_{n\rightarrow\infty} V(\Delta_{T_U}) = 0$
- Quiz: Is there an even more ideal experiment?
- Hint: How can we make $\Delta_{S_X} = 0$?

An Even More Ideal Experiment (not in the paper)

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- Begin with a well-defined population

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- Begin with a well-defined population
- New feature: Define sampling strata based on cross-classification of all known confounders

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- New feature: Define sampling strata based on cross-classification of all known confounders
- Random sampling within strata
- (if strata sample $\propto$ population size, no weights needed)

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- Begin with a well-defined population
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- Random sampling within strata
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- Begin with a well-defined population
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- $\Delta_{S_X} = 0$

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- Begin with a well-defined population
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- Random sampling within strata
- (if strata sample $\propto$ population size, no weights needed)
- large n
- blocking on all known confounders
- random treatment assignment within blocks
- $\Delta_{S_X} = 0$
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- $\Delta_{T_X} = 0$

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- $\Delta_{T_X} = 0$
- $E(\Delta_{T_U}) = 0$, $\lim_{n\to\infty} V(\Delta_{T_U}) = 0$
- Wait, why wasn't this in the paper?

Randomized Clinical Trials (Little or no Blocking)

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- nonrandom selection

Randomized Clinical Trials (Little or no Blocking)

- nonrandom selection
- small n

Randomized Clinical Trials (Little or no Blocking)

- nonrandom selection
- small n
- little or no blocking

Randomized Clinical Trials (Little or no Blocking)

- nonrandom selection
- small n
- little or no blocking
- random treatment assignment

Randomized Clinical Trials (Little or no Blocking)

- nonrandom selection
- small n
- little or no blocking
- random treatment assignment
- $\Delta_{S_X} \neq 0$

Randomized Clinical Trials (Little or no Blocking)

- nonrandom selection
- small n
- little or no blocking
- random treatment assignment
- $\Delta_{S_X} \neq 0$
- $\Delta_{S_U} \neq 0$

Randomized Clinical Trials (Little or no Blocking)

- nonrandom selection
- small n
- little or no blocking
- random treatment assignment
- $\Delta_{S_X} \neq 0$
- $\Delta_{S_U} \neq 0$
- $E(\Delta_{T_X}) = 0$

Randomized Clinical Trials (Little or no Blocking)

- nonrandom selection
- small n
- little or no blocking
- random treatment assignment
- $\Delta_{S_X} \neq 0$
- $\Delta_{S_U} \neq 0$
- $E(\Delta_{T_X}) = 0$
- $E(\Delta_{T_U}) = 0$

Randomized Clinical Trials (Full Blocking)

Randomized Clinical Trials (Full Blocking)

- nonrandom selection

Randomized Clinical Trials (Full Blocking)

- nonrandom selection
- small n

Randomized Clinical Trials (Full Blocking)

- nonrandom selection
- small n
- Full blocking

Randomized Clinical Trials (Full Blocking)

- nonrandom selection
- small n
- Full blocking
- random treatment assignment

Randomized Clinical Trials (Full Blocking)

- nonrandom selection
- small n
- Full blocking
- random treatment assignment
- $\Delta_{S_X} \neq 0$

Randomized Clinical Trials (Full Blocking)

- nonrandom selection
- small n
- Full blocking
- random treatment assignment
- $\Delta_{S_X} \neq 0$
- $\Delta_{S_U} \neq 0$

Randomized Clinical Trials (Full Blocking)

- nonrandom selection
- small n
- Full blocking
- random treatment assignment
- $\Delta_{S_X} \neq 0$
- $\Delta_{S_U} \neq 0$
- $\Delta_{T_X} = 0$

Randomized Clinical Trials (Full Blocking)

- nonrandom selection
- small n
- Full blocking
- random treatment assignment
- $\Delta_{S_X} \neq 0$
- $\Delta_{S_U} \neq 0$
- $\Delta_{T_X} = 0$
- $E(\Delta_{T_U}) = 0$

Social Science Field Experiment

Social Science Field Experiment

- nonrandom selection

Social Science Field Experiment

- nonrandom selection
- large n

Social Science Field Experiment

- nonrandom selection
- large n
- limited or no blocking

Social Science Field Experiment

- nonrandom selection
- large n
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- random treatment assignment

Social Science Field Experiment

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- large n
- limited or no blocking
- random treatment assignment
- $\Delta_{S_X} \neq 0$ or change PATE to SATE and $\Delta_{S_X} = 0$

Social Science Field Experiment

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- random treatment assignment
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- $\Delta_{S_U} \neq 0$ or change PATE to SATE and $\Delta_{S_U} = 0$

Social Science Field Experiment

- nonrandom selection
- large n
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- $\Delta_{S_U} \neq 0$ or change PATE to SATE and $\Delta_{S_U} = 0$
- $E(\Delta_{T_X}) = 0, \quad \lim_{n\to\infty} V(\Delta_{T_X}) = 0$

Social Science Field Experiment

- nonrandom selection
- large n
- limited or no blocking
- random treatment assignment
- $\Delta_{S_X} \neq 0$ or change PATE to SATE and $\Delta_{S_X} = 0$
- $\Delta_{S_U} \neq 0$ or change PATE to SATE and $\Delta_{S_U} = 0$
- $E(\Delta_{T_X}) = 0, \quad \lim_{n\to\infty} V(\Delta_{T_X}) = 0$
- $E(\Delta_{T_U}) = 0, \quad \lim_{n\to\infty} V(\Delta_{T_U}) = 0$

Survey Experiment

Survey Experiment

- random selection

Survey Experiment

- random selection
- large n

Survey Experiment

- random selection
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Components of Causal Estimation Error

Research Designs

Issues in Ideal Designs

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- Quiz: Astronomers never randomize; is astronomy a science?

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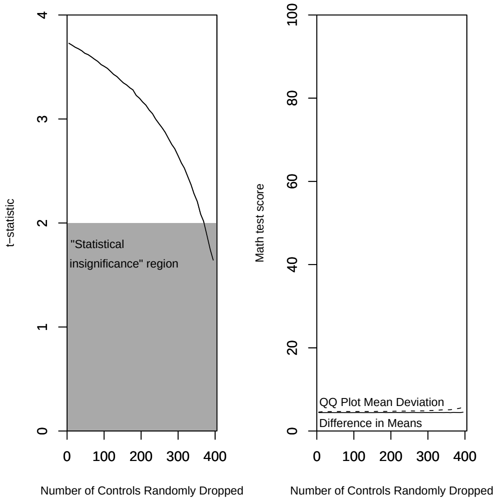
  - randomization balances on average: any one random assignment is not balanced exactly (which is why its better to block)

The Balance Test Fallacy in Matching Research



Quiz: randomly dropping observations reduces imbalance??

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- Quiz: Should we match on vars that do not influence Y?