Quantitative Social Science Methods, I, Lecture Notes: Model Evaluation

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August 17, 2020

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How Do You Know Which Model is Better?

Robust Standard Errors

A Better Way to Use Robust SEs: An Application

Evaluation by Out-of-Sample Forecast

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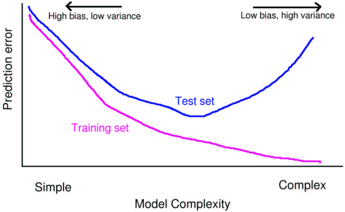

See Trevor Hastie et al. 2001. *The Elements of Statistical Learning*, Springer, Fig 7.1

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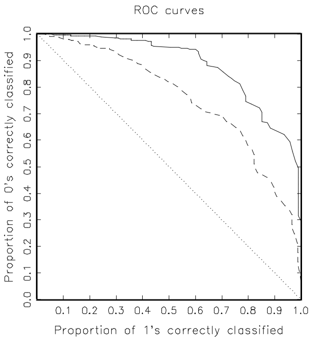
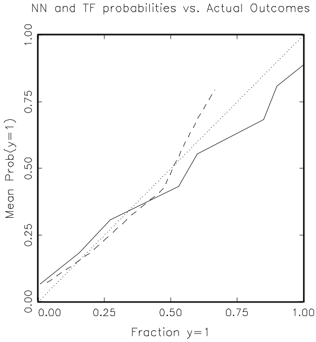
  - Otherwise, one model is better than the other in only given specificed ranges of C

ROC Curve In Sample





In-sample ROC on left

(from Gary King and Langche Zeng. "Improving Forecasts of State Failure," *World Politics*, 2001)

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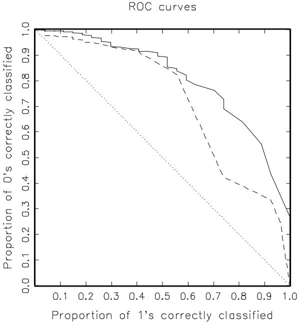

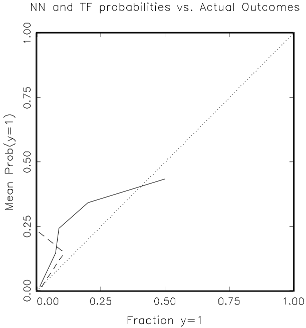

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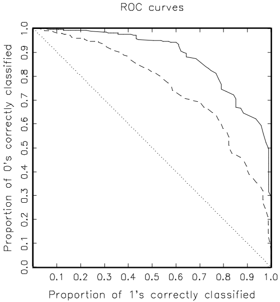

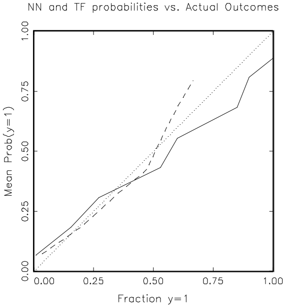

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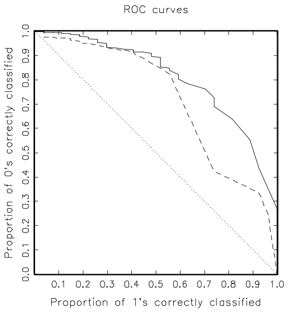

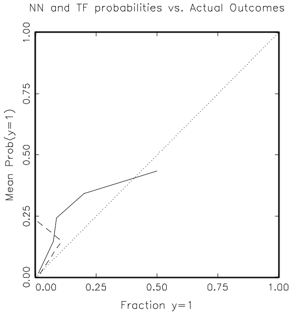

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- When SEs and RSEs differ:
  - Best case
    - Some QOIs (β in regression): unbiased but inefficient
    - Other QOIs ($\Pr(Y > 0.6)$): biased
  - Worst case
    - Misspecification is more widespread
    - All QOIs are biased
- Are: A good test for misspecification

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Variance matrix unconstrained (includes covariances)

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Still allows for heteroskedasticity

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$$\Sigma = \sigma^2 \begin{pmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{pmatrix}$$

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Standard linear-normal regression model assumptions

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- But: The model, sims, QOI estimates are still wrong

For RSEs to help: Everything has to be Juuuussttt Right

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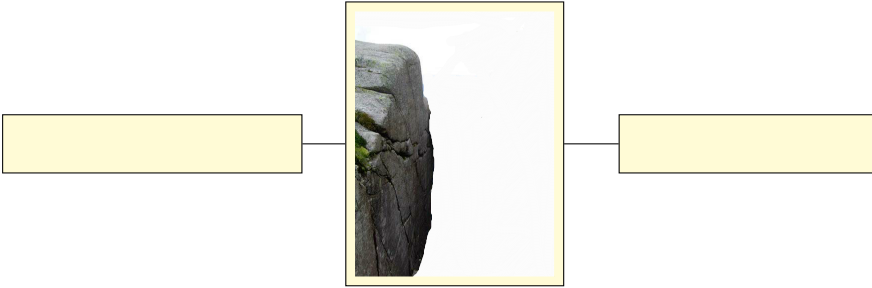
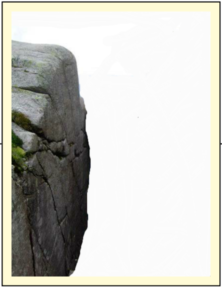


For RSEs to help: Everything has to be Juuuussttt Right

Model Misspecified



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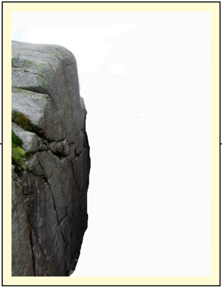
Model Misspecified

(point estimates biased)



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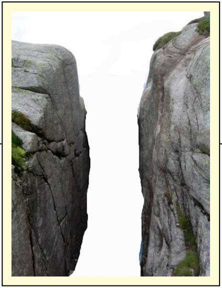

Model Correct

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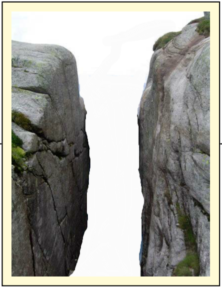

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Goldilocks Region

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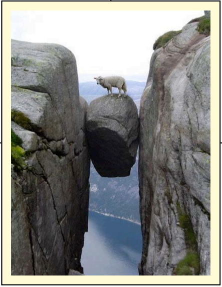

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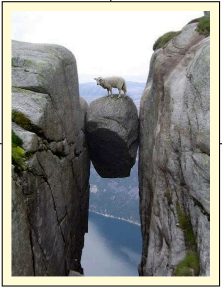
Goldilocks Region



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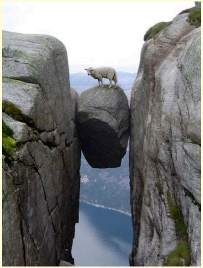
Goldilocks Region

but not so much as to bias everything else



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How Do You Know Which Model is Better?

Robust Standard Errors

A Better Way to Use Robust SEs: An Application

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Box-Cox Transformation makes Y Normal

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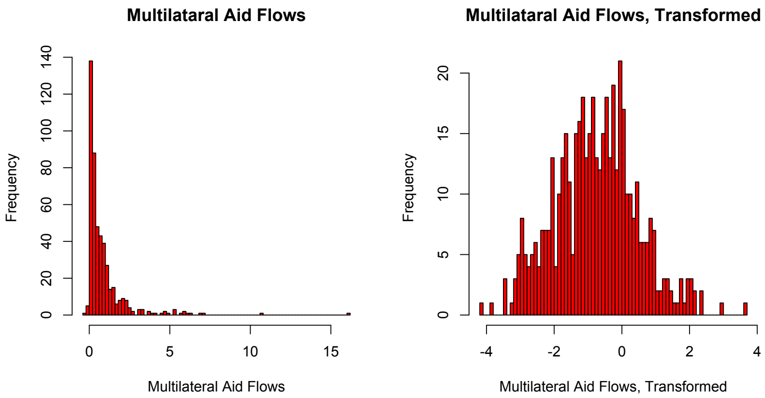

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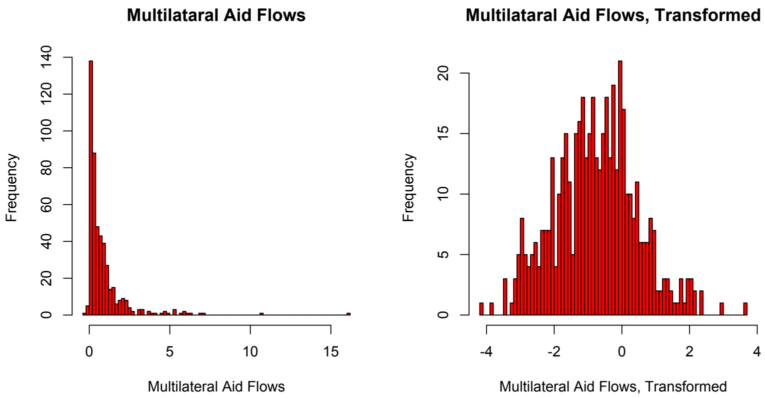

Quiz: Is this a good test of the new specification?

Transformation of Y: Removes Heteroskedasticty

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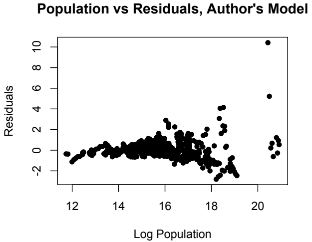

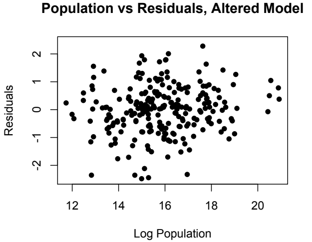

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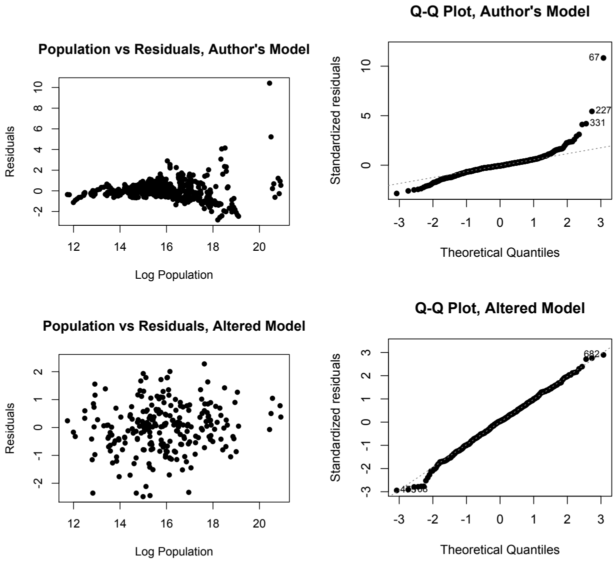

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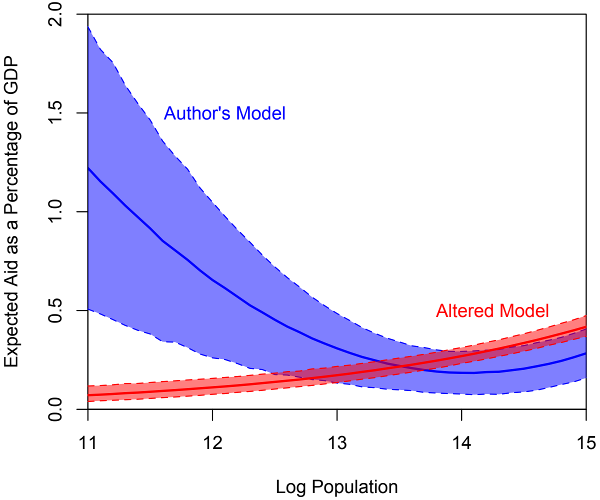

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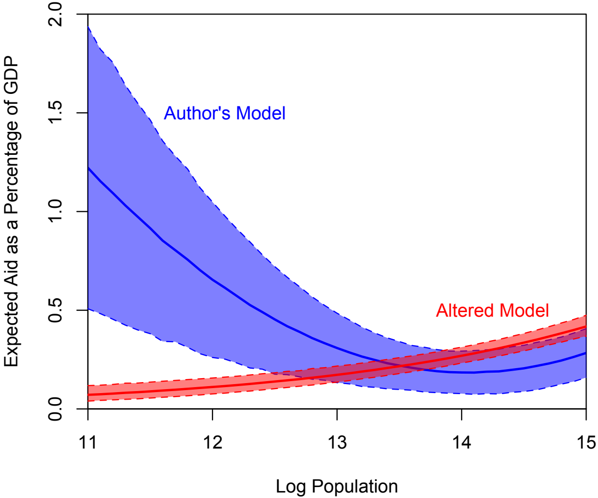

Quiz: Why are the CIs smaller for the correctly specified model?