# Quantitative Social Science Methods, I, Lecture Notes: Binary Outcome Models

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Linear Probability, Logit, Probit Models

Interpreting Functional Forms

Alternative Interpretations of Binary Models

General Rules for Presenting and Interpreting Statistical Results

# Linear Probability Model

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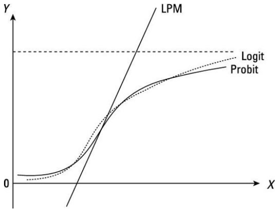

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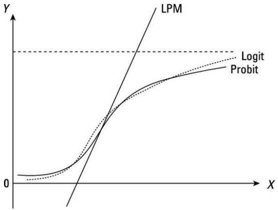


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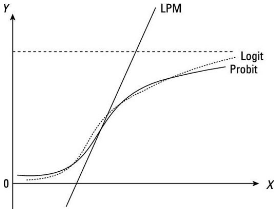


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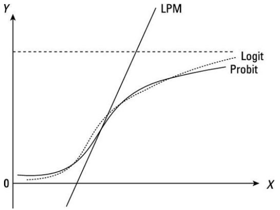


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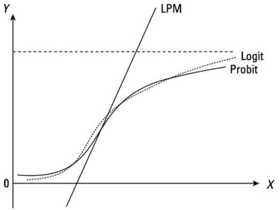


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- Unlikely to get uncertainties right

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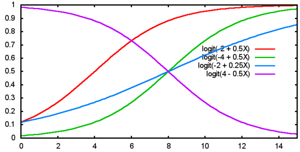

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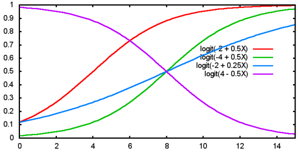


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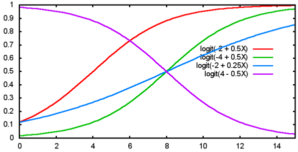


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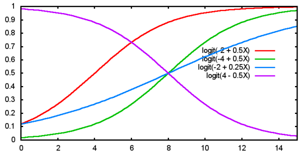


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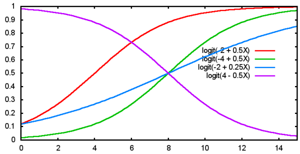


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- Could be more flexible; OK for now

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How to interpret $\hat{\beta}$? What's the QOI?

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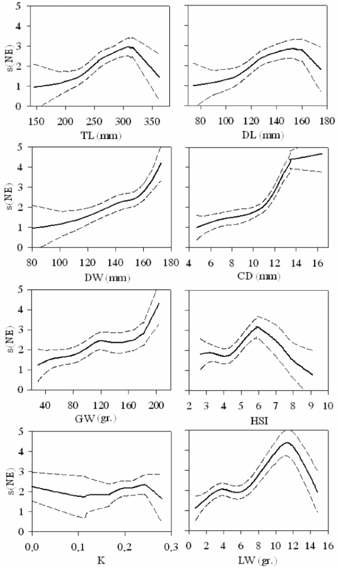
# Graphics to Interpret Functional Forms



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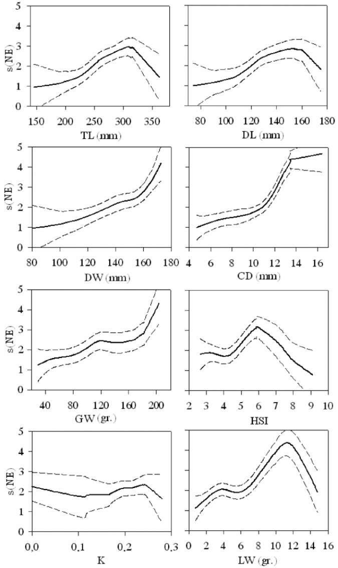


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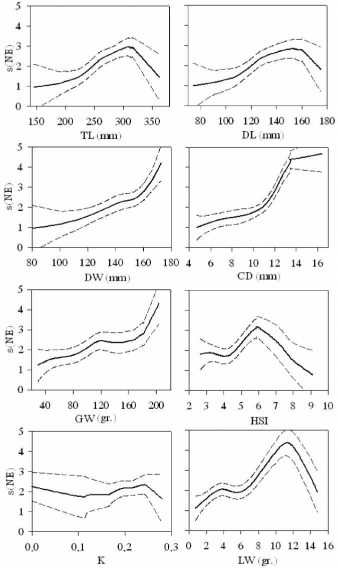


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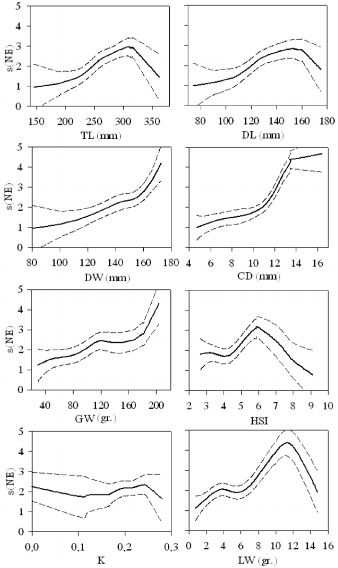


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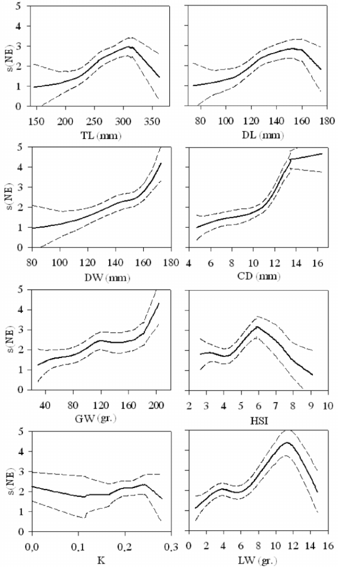


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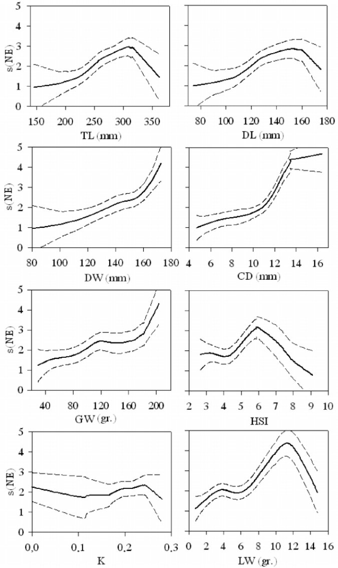


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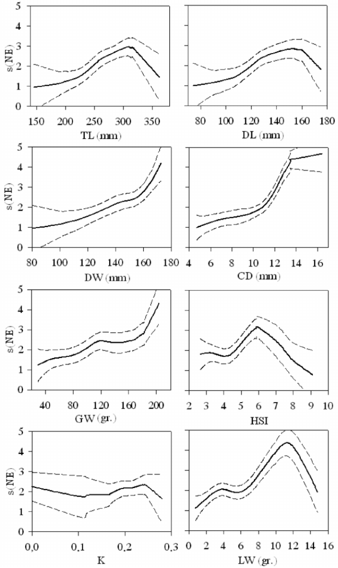


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- Be creative; choose graphs for impact

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- Easy to communicate
- Difficult to be comprehensive

# Fitted Values to Interpret Functional Forms

- Calculate fitted values given selected values of $X$, $X_c$ for "typical" people, person types, regional representatives, stereotypes, etc.
- Compute $\hat{\theta}_c = g(X_c, \hat{\beta})$
- An example for logit: $\hat{\pi}_c = \frac{1}{1+e^{-X_c\hat{\beta}}}$

| Sex | Age | Home | Income | Pr(vote) |
|---|---|---|---|---|
| Male | 20 | Chicago | \$33,000 | 0.20 |
| Female | 27 | New York City | \$43,000 | 0.28 |
| Male | 50 | Madison, WI | \$55,000 | 0.72 |
| ⋮ | | | | |

- Include a measure of uncertainty (standard error, confidence interval, etc.) — for any quantity but a probability
- Easy to communicate
- Difficult to be comprehensive
- Better by simulation: point and uncertainty estimation

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- Example:

| Variable | From | | To | First Difference |
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| Sex | Male | $\longrightarrow$ | Female | .05 |
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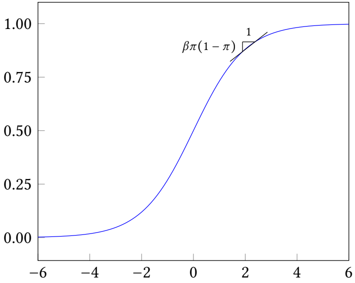

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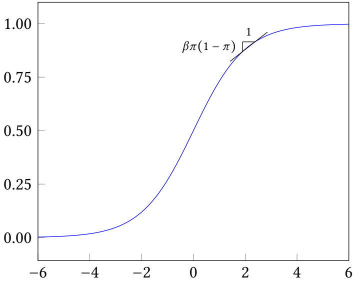


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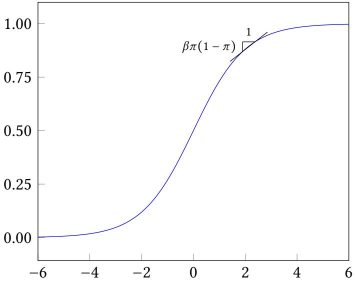


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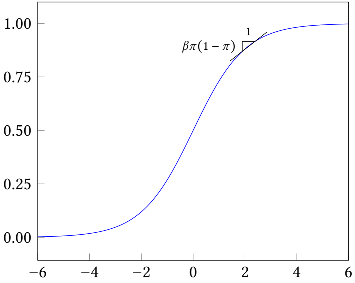


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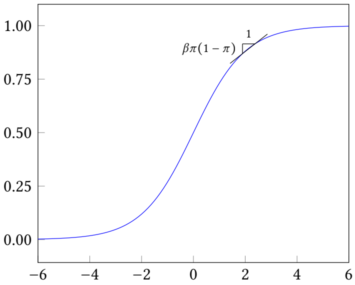


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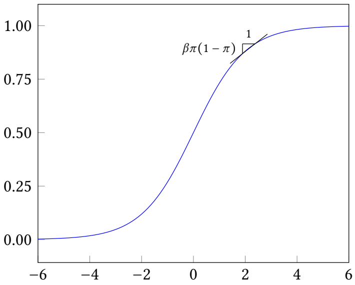


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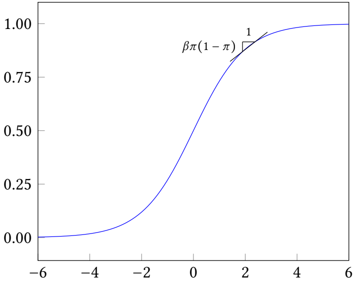


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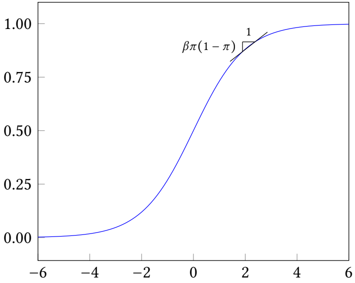


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Linear Probability, Logit, Probit Models

Interpreting Functional Forms

Alternative Interpretations of Binary Models

General Rules for Presenting and Interpreting Statistical Results

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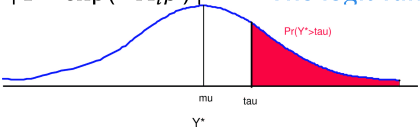

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Linear Probability, Logit, Probit Models

Interpreting Functional Forms

Alternative Interpretations of Binary Models

General Rules for Presenting and Interpreting Statistical Results

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| Education × Time | –.003* |
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| Constant | –9.257** |
| Number | 7,021 |
| –2 log-likelihood | 7,215.231 |
| Goodness of fit | 6,789.45 |
| Cox & Snell $R^2$ | .212 |
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| Overall correct classification (%) | 73.96 |

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- Reading: King, Tomz, Wittenberg, "Making the Most of Statistical Analyses: Improving Interpretation and Presentation" *American Journal of Political Science* (2000).

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    3. Draw $\tilde{\gamma}$ from the multivariate normal (many times)

$$\tilde{\gamma} \sim N\left(\hat{\gamma}, \hat{V}(\hat{\gamma})\right)$$

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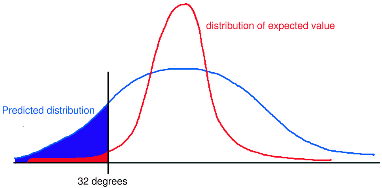

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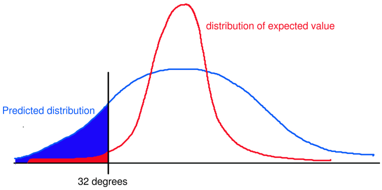


  - Predicted: $\Pr(\text{Temperature} < 32^\circ)$ tomorrow

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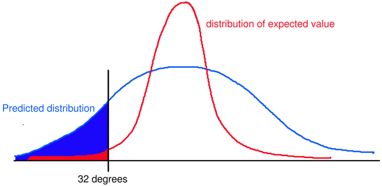
- Quiz: As $n \longrightarrow \infty$, does the variance go to zero?
  - Predicted: no
  - Expected: yes
- Example




  - Predicted: $\Pr(\text{Temperature} < 32^\circ)$ tomorrow
  - Expected: $\Pr(\text{Average Temperature} < 32^\circ)$ tomorrow

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- E.g.: histogram, average, variance, percentile values, etc.

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- When $E(Y_c) = \theta_c$: we *may* skip steps d–e. E.g., simulating $\pi_i$ in logit model. If you're unsure, do it anyway!

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- To save computation time, and improve approximation:
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    - In each case, $\eta$ is unbounded: estimate it, simulate from it, and reparameterize back to the scale you care about.

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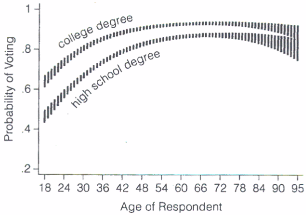
FIGURE 1 Probability of Voting by Age

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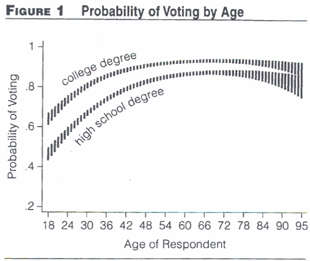


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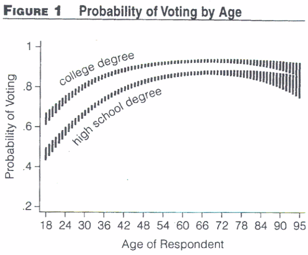


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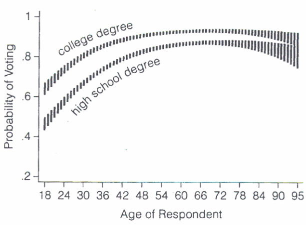
**Figure 1** Probability of Voting by Age




Vertical bars indicate 99-percent confidence intervals

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- Run logistic regression
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# Replication of Rosenstone and Hansen

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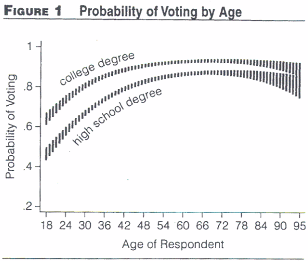


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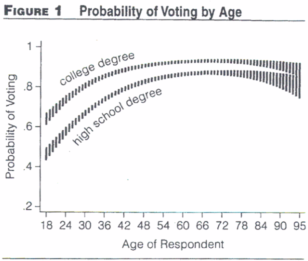

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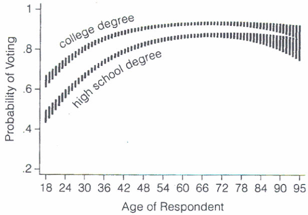


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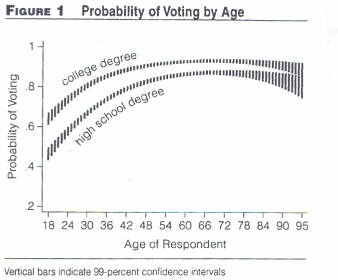


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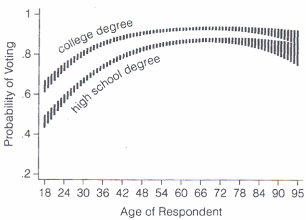


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- Repeat for other ages and college

# Replication of Garrett (King, Tomz and Wittenberg 2000)

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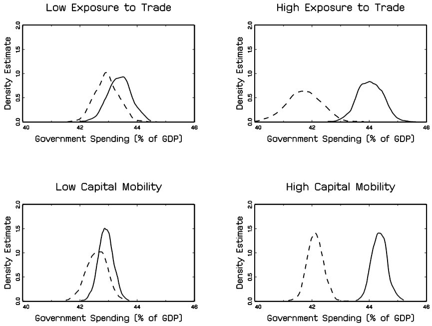

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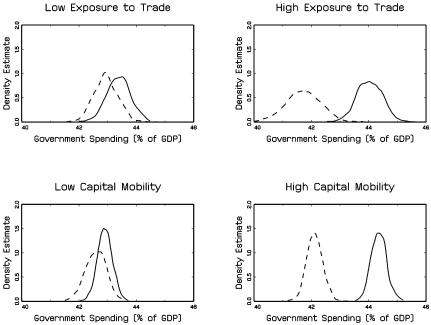


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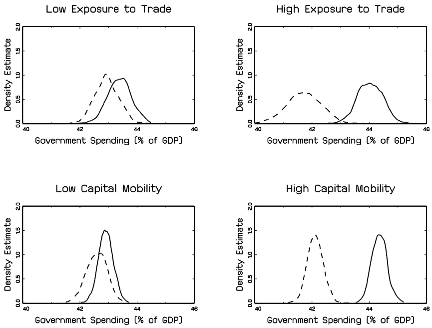


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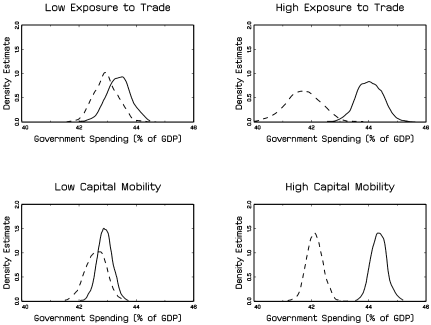


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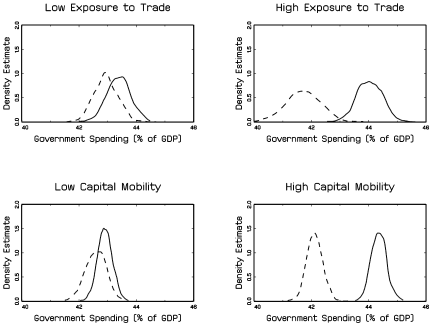


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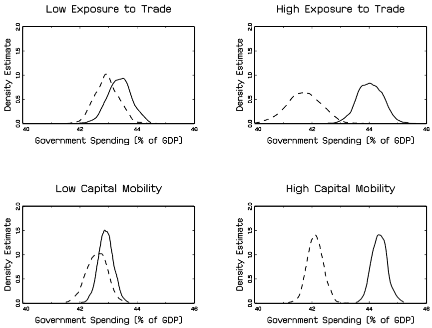


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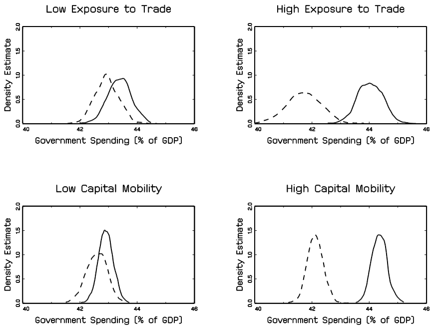


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- Quiz: How can we summarize this with less real estate?